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# Total Cross Sections with Virtual Photons[1]

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**Abstract**

A model for total cross sections with virtual photons is presented, in particular $\gamma^*$p and $\gamma^*\gamma^*$ cross sections are considered. Our approach is based on an existing model for photoproduction, which subdivides the total cross section into three distinct event classes: direct, VMD and anomalous [1]. In the region of large photon virtualities, the Deep Inelastic Scattering processes (up to $\mathcal{O}(\alpha_s)$ corrections) are obtained. Hence, the model provides a smooth transition between the two regions. By the breakdown into different event classes, a complete picture of all event properties is the ultimate goal.

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# 1 Introduction

PYTHIA is a general-purpose event generator of high-energy particle physics reactions. A full description of models and processes implemented are found in Ref. [2] with relevant update notes on the PYTHIA webpage. Some aspects specific to $\gamma\gamma$ physics are summarized in Ref. [3] and will briefly be reviewed here to illustrate the modifications and improvements done to also treat virtual photons. The model used so far, incorporating Leading Order (LO) hard scattering processes, as well as elastic, diffractive, low-$p_\perp$ and multiple parton–parton scattering for specific event classes, considers only real incoming photons with a separate treatment of the Deep Inelastic Scattering (DIS) region, $\mathrm{e}\gamma \to \mathrm{e}\mathbf{X}$ [1]. Here, the two extreme scenarios will be merged in order to obtain a description that smoothly interpolates between the two regions.

A model for jet production with virtual photons has been described in detail elsewhere [4]. Photon flux factors are convoluted with matrix elements involving either direct or resolved photons and, for the latter, with parton distributions of the photon. The direct and single-resolved matrix elements are depending on the virtuality of the photon and the virtual resolved photons are dampened with dipole factors in the parton distributions. The range of uncertainty in the modeling of the resolved component was explored, eg. parton distribution sets of the photon, scale choice in the parton distributions, longitudinal contributions etc.

In this report we will limit ourself to the discussion of mixing different event classes to obtain total cross sections with virtual photons, $\gamma^*\mathrm{p}$ and $\gamma^*\gamma^*$. The extension to virtual photons will eventually also be made for elastic scattering and diffractive events. This will be described in a future publication [5] together with a more detailed description of the model presented here.

# 2 Event Classes

Traditionally, different descriptions are used for virtual and real photons. Virtual photons in the DIS region are normally described as devoid of any structure, while for the real ones, the possibility of hadronic-like fluctuations play an important rôle. In the region of intermediate $Q^2$, it should be possible to find a description starting from either extreme. Then the language may not always be unique, i.e. a given Feynman diagram may be classified in different ways. In the following, we will develop one specific approach, where the main idea is to classify events by the hardest scale involved.

We begin by a reminder on the models for photoproduction and DIS, before embarking on the generalization also to intermediate virtualities in $\gamma^*\mathrm{p}$ processes. The $\gamma\gamma$, $\gamma^*\gamma$ and $\gamma^*\gamma^*$ processes thereafter follow by an application of the same rules.

## 2.1 Photoproduction

To first approximation, the photon is a point-like particle. However, quantum mechanically, it may fluctuate into a (charged) fermion–antifermion pair. The fluctuations $\gamma \leftrightarrow \mathrm{q}\overline{\mathrm{q}}$ can interact strongly and therefore turn out to be responsible for the major part of the $\gamma\mathrm{p}$ and $\gamma\gamma$ total cross sections. The total rate of $\mathrm{q}\overline{\mathrm{q}}$ fluctuations is not perturbatively calculable, since low-virtuality fluctuations enter a domain of non-perturbative QCD physics. It is therefore customary to split the spectrum of fluctuations into a low-virtuality and a high-virtuality part. The former part can be approximated by a sum over low-mass vector-meson states, customarily (but not necessarily) restricted to the lowest-lying vector multiplet. Phenomenologically, this Vector Meson Dominance (VMD) ansatz turns out to be very successful in describing a host of data. The high-virtuality part, on the other hand, should be in a perturbatively calculable domain. Based on the above separation, three main classes of interacting photons can be distinguished: direct, VMD and anomalous photons, corresponding to the following event classes in $\gamma$p events [1]:

1. The VMD processes, where the photon turns into a vector meson before the interaction, and therefore all processes allowed in hadronic physics may occur. This includes elastic and diffractive scattering as well as low-$p_\perp$ and high-$p_\perp$ non-diffractive events.
2. The direct processes, where a bare photon interacts with a parton from the proton.
3. The anomalous processes, where the photon perturbatively branches into a $\mathrm{q\overline{q}}$ pair, and one of these (or a daughter parton thereof) interacts with a parton from the proton.

The total photoproduction cross section can then be written as $\sigma_{\mathrm{tot}}^{\gamma\mathrm{p}} = \sigma_{\mathrm{VMD}}^{\gamma\mathrm{p}} + \sigma_{\mathrm{direct}}^{\gamma\mathrm{p}} + \sigma_{\mathrm{anomalous}}^{\gamma\mathrm{p}}$.

Total hadronic cross sections show a characteristic fall-off at low energies and a slow rise at higher energies. This behaviour can be parameterized by the form

$$\sigma_{\mathrm{tot}}^{AB}(s) = X^{AB} s^{\epsilon} + Y^{AB} s^{-\eta} \tag{1}$$

for $A + B \to X$. The powers $\epsilon$ and $\eta$ are universal, with fit values [6] $\epsilon \approx 0.0808, \eta \approx 0.4525$, while the coefficients $X^{AB}$ and $Y^{AB}$ are process-dependent. Equation (1) can be interpreted within Regge theory, but for the purpose of our study we can merely consider it as a convenient parameterization.

The VMD part of the $\gamma$p cross section is an obvious candidate for a hadronic description. The diagonal VMD model suggests:

$$\sigma_{\mathrm{VMD}}^{\gamma\mathrm{p}}(s) = \sum_{V=\rho^0,\omega,\phi,\mathrm{J}/\psi} \frac{4\pi\alpha_{\mathrm{em}}}{f_V^2}\, \sigma_{\mathrm{tot}}^{V\mathrm{p}}(s) \approx 53.4 s^{\epsilon} + 115 s^{-\eta} \quad [\mu\mathrm{b}]\ , \tag{2}$$

with the $f_V$ determined from data [7]. The $V$p cross sections can be parameterized assuming an additive quark model [1] and adding the vector meson contributions, we arrive at the above numbers (with $s$ in GeV$^2$).

There is no compelling reason that such an ansatz should hold also for the total $\gamma$p cross section, but empirically a parameterization according to eq. (1) does a good job [6]. For instance, such parameterization predicted the high-energy behaviour of the cross section, close to what was then measured by H1 and ZEUS. Thus VMD corresponds to approximately 80 % of the total $\gamma$p cross section at high energies, with the remaining 20 % then shared among the direct and anomalous event classes.

Introducing a cut-off parameter $k_0$ to separate the low- and high-virtuality parts of the $\mathrm{q\overline{q}}$ fluctuations, the anomalous contribution can be written as

$$\sigma_{\mathrm{ano}}^{\gamma\mathrm{p}}(s) = \frac{\alpha_{\mathrm{em}}}{2\pi} \sum_{\mathrm{q}} 2e_{\mathrm{q}}^2 \int_{k_0^2}^{\infty} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, \sigma^{\mathrm{q\overline{q}p}}(s;k_\perp) = \frac{\alpha_{\mathrm{em}}}{2\pi} \sum_{\mathrm{q}} 2e_{\mathrm{q}}^2 \int_{k_0^2}^{\infty} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, \frac{k_{V(\mathrm{q\overline{q}})}^2}{k_\perp^2}\, \sigma^{V(\mathrm{q\overline{q}})\mathrm{p}}(s) \tag{3}$$

where the prefactor and integral over $dk_\perp^2/k_\perp^2$ corresponds to the probability for the photon to split into a $q\overline{q}$ state of transverse momenta $\pm k_\perp$. The cross section for this $q\overline{q}$ pair to scatter against the proton, $\sigma^{q\overline{q}p}$, need to be modeled. Based only on geometrical scaling arguments, one could expect a decrease roughly like $1/k_\perp^2$ which motivates the second equality. The $k_{V(q\overline{q})}$ is a free parameter introduced for dimensional reasons. It could be associated with the typical $k_\perp$ inside the vector meson $V$ formed from a $q\overline{q}$ pair: $\rho^0 \approx \omega$ for u and d, $\phi$ for s, $J/\psi$ for c. As a reasonable ansatz, one could guess $k_{V(q\overline{q})} \approx m_V/2 \approx m_\rho/2$. Fits to the total cross section at not too high energies, with a large VMD and a small direct contributions subtracted, give corresponding numbers, $k_{V(q\overline{q})} \approx 0.41$ GeV for a $k_0 \approx 0.5$ GeV.

To leading order, the direct events come in two kinds: QCD Compton $\gamma q \to qg$ (QCDC) and boson-gluon fusion $\gamma g \to q\overline{q}$ (BGF). The cross sections are divergent in the limit $k_\perp \to 0$ for the outgoing parton pair. Therefore a lower cut-off is required, but no other specific model assumptions.

The subdivision of the photon into three different components leads to the existence of three times three event classes in $\gamma\gamma$ events. By symmetry, the 'off-diagonal' combinations appear pairwise, so for real photons the number of distinct classes is only six. These are,

1. VMD×VMD: both photons turn into hadrons, and the processes are therefore the same as allowed in hadron–hadron collisions.
2. VMD×direct: a bare photon interacts with the partons of the VMD photon.
3. VMD×anomalous: the anomalous photon perturbatively branches into a $q\overline{q}$ pair, and one of these (or a daughter parton thereof) interacts with a parton from the VMD photon.
4. Direct×direct: the two photons directly give a quark pair, $\gamma\gamma \to q\overline{q}$. Also lepton pair production is allowed, $\gamma\gamma \to \ell^+\ell^-$, but will not be considered by us.
5. Direct×anomalous: the anomalous photon perturbatively branches into a $q\overline{q}$ pair, and one of these (or a daughter parton thereof) directly interacts with the other photon.
6. Anomalous×anomalous: both photons perturbatively branch into $q\overline{q}$ pairs, and subsequently one parton from each photon undergoes a hard interaction.

Most of the above classes above are pretty much the same as allowed in $\gamma p$ events, since the interactions of a VMD or anomalous photon and those of a proton are about the same. Only the direct×direct class offer a new hard subprocess.

The main parton-level processes that occur in the above classes are:

- The 'direct' processes $\gamma\gamma \to q\overline{q}$ only occur in class 4.
- The '1-resolved' processes $\gamma q \to qg$ and $\gamma g \to q\overline{q}$ occur in classes 2 and 5.
- The '2-resolved' processes $qq' \to qq'$ (where $q'$ may also represent an antiquark), $q\overline{q} \to q'\overline{q}'$, $q\overline{q} \to gg$, $qg \to qg$, $gg \to q\overline{q}$ and $gg \to gg$ occur in classes 1, 3 and 6.

The VMD, direct and anomalous classes have so far been considered separately. The complete physics picture presumably would provide smooth transitions between the various possibilities. To understand the relation between the processes, consider the simple graph of Fig. 1a. There two transverse momentum scales, $k_\perp$ and $p_\perp$, are introduced (we first consider the case $Q^2 = 0$). Here $k_\perp$ is related to the $\gamma \to q\overline{q}$ vertex while $p_\perp$ is the hardest QCD $2 \to 2$ subprocess of the ladder between the photon and the proton. (Further softer partons in the ladders are omitted for clarity.) The allowed phase space can then conveniently be represented by a two-dimensional plane, Fig. 1b. The region $k_\perp < k_0$ corresponds to a small transverse momentum at the $\gamma \to q\overline{q}$ vertex, and thus to

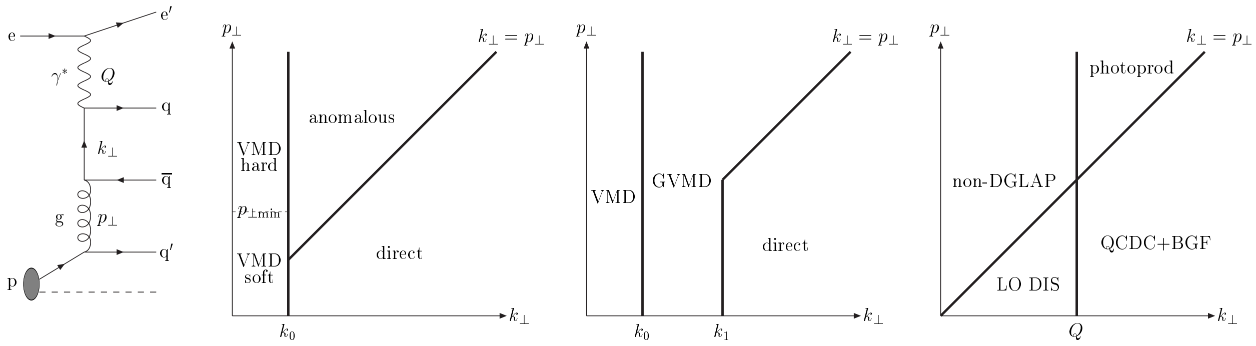


Figure 1: (a) Schematic graph for a hard $\gamma^*$p process, illustrating the concept of three different scales. (b) The allowed phase space for this process when $Q^2 = 0$, with one subdivision into event classes. (c) An alternative classification of the phase space in (b), which better takes into account unitarization effects. (d) Event classification in the large-$Q^2$ limit.

VMD processes. For $k_\perp > k_0$, the events are split along the diagonal $k_\perp = p_\perp$. If $k_\perp > p_\perp$, the hard $2 \to 2$ process of Fig. 1a is $\gamma \mathrm{g} \to \mathrm{q}\overline{\mathrm{q}}$, and the lower part of the graph is part of the leading log QCD evolution of the gluon distribution inside the proton. These events are direct ones. If $p_\perp > k_\perp$, on the other hand, the hard process is $\overline{\mathrm{q}}\mathrm{q}' \to \overline{\mathrm{q}}\mathrm{q}'$, and the $\gamma \to \mathrm{q}\overline{\mathrm{q}}$ vertex builds up the quark distribution inside a photon. These events are thus anomalous ones.

What complicates the picture is that an event may contain several interactions, once one considers an incoming particle as a composite object with several partons that may interact, more or less independently of each other, with partons from the other incoming particle. Such a multiple parton–parton interaction scenario is familiar already from pp physics [8]. Here the jet cross section, above some $p_{\perp\mathrm{min}}$ scale of the order of 2 GeV, increases faster with energy than the total cross section. Above an energy of a few hundred GeV the calculated jet cross section is larger than the observed total one. Multiple interactions offers a solution to this apparent paradox, by squeezing a larger number of jet pairs into the average event, a process called unitarization or eikonalization. The perturbative jet cross section can then be preserved, at least down to $p_{\perp\mathrm{min}}$, but in the reinterpreted inclusive sense. At the same time, the unitarization plays a crucial rôle in taming the growth of the total cross section.

The composite nature of hadrons also fills another function: it regularizes the singularity of perturbative cross sections, such as $\mathrm{qg} \to \mathrm{qg}$, in the limit $p_\perp \to 0$. Perturbative calculations assume free colour charges in the initial and final states of the process, while the confinement in hadrons introduces some typical colour neutralization distance. It is the inverse of this scale that appears as some effective cutoff scale $p_{\perp\mathrm{min}} \simeq 2$ GeV, most likely with a slow energy dependence [9]. One possible parameterization is

$$p_{\perp\mathrm{min}}(s) = (1.9\ \mathrm{GeV}) \left( \frac{s}{1\ \mathrm{TeV}^2} \right)^{\epsilon} , \tag{4}$$

with the same $\epsilon$ as in eq. (1), since the rise of the total cross section with energy via Regge theory is related to the small-$x$ behaviour of parton distributions and thus to the density of partons.

Now, if an event contains interactions at several different $p_\perp$ scales, standard practice is to classify this event by its hardest interaction. With this prescription, the cross section for an event of scale $p_\perp$ is the naive jet cross section at this $p_\perp$ scale *times* the probability that the event contained no interaction at a scale above $p_\perp$. The latter defines a form factor, related to probability conservation. At large $p_\perp$ values the probability of having an even larger $p_\perp$ is small, i.e. the form factor is close to unity, and the perturbative cross section is directly preserved in the event rate. At lower $p_\perp$ values, the likelihood of a larger $p_\perp$ is increased, i.e. the form factor becomes smaller than unity, and the rate of events classified by this $p_\perp$ scale falls below the perturbative answer.

We expect this picture to hold also for the VMD part of the photon, since this is clearly in the domain of hadronic physics. Thus, in the VMD domain $k_\perp < k_0$, the region of large $p_\perp$ in Fig. 1b is populated according to perturbation theory, though with nonperturbative input to the parton distributions. The region of smaller $p_\perp$ is suppressed, since the form factor here drops significantly below unity.

As one moves away from the "pure" VMD states, such as the $\rho^0$, much of the same picture could well hold. Interactions at a larger $k_\perp$ value could be described in terms of some $\rho'$ state. The uncertainty relation gives us that a state of virtuality $\simeq k_\perp$ has a maximal size $\simeq 1/k_\perp$ and thus spans an area $\propto 1/k_\perp^2$. Such a state could undergo one or several interactions of the anomalous-type or remain as a "low-$p_\perp$" direct event. It is then reasonable to assume that the unitarized cross section is proportional to the area of the state interacting with the proton, i.e. a (kind of) geometrical scaling. The colour neutralization distance inside a more virtual photon state is also reduced, so that the interactions in general tend to be weakened by interference effects not included in the simple perturbative cross sections. This could then be the origin for a geometrical scaling like the one in eq. (3).

Calculating the perturbative anomalous cross section in the region $p_\perp > \max(k_\perp, p_{\perp\mathrm{min}}(s))$, the geometric scaling answer is exceeded for some region $k_\perp \lesssim k_1$, with $k_1 \approx 2-4$ GeV (higher for higher energies). Only for $k_\perp > k_1$ is the jet cross section dropping below the geometric scaling one. At these larger $k_\perp$ values, the direct rate dominates over the anomalous. As a convenient but rather arbitrary choice, for subsequent studies we put $k_1 = p_{\perp\mathrm{min}}(s)$, with the latter given by eq. (4).

The final scenario is illustrated in Fig. 1c. The bulk of the cross section, in the region $k_\perp < k_1$, is now described by the photon interacting as dense, hadronic states, VMD for $k_\perp \lesssim k_0$ and Generalized VMD (GVMD) for $k_0 \lesssim k_\perp \lesssim k_1$. The total VMD cross section is given by the pomeron-type ansatz, while the jet cross section can be obtained from the parton distributions of the respective vector meson state. Correspondingly, the GVMD states have a total cross section based on Pomeron considerations and a jet cross section now based on the anomalous part of the parton distributions of the photon. In principle, an eikonalization should be performed for each GVMD state separately, but in practice that would be overkill. Instead the whole region is represented by one single state per quark flavour, with a jet production given by the full anomalous part of the photon distributions.

Thus, post facto, the approximate validity of a Regge theory ansatz for $\sigma_{\mathrm{tot}}^{\gamma\mathrm{p}}$ is making sense. Above $k_1$ only the direct cross section need be considered, since here the anomalous one is negligibly small, at least in terms of total cross sections. (As noted above, we have actually chosen to lump it with the other GVMD contributions, so as not to lose the jet rate itself.)

## 2.2 Deeply Inelastic Scattering

At not too large $Q^2$, the Deeply Inelastic Scattering of a high-energy charged lepton off a proton target, involves a single photon exchange between a beam lepton and a target quark. The double-differential ep $\to$ e$\mathbf{X}$ cross-section for DIS can be expressed in terms of the total cross-section for virtual transverse (T) and longitudinal (L) photons [10]: $\mathrm{d}^2\sigma/\mathrm{d}y/\mathrm{d}Q^2 = f_{\gamma/\mathrm{e}}^{\mathrm{T}}(y,Q^2)\sigma_{\mathrm{T}}(y,Q^2) + f_{\gamma/\mathrm{e}}^{\mathrm{L}}(y,Q^2)\sigma_{\mathrm{L}}(y,Q^2)$ , where $f_{\gamma/\mathrm{e}}^{\mathrm{T,L}}$ are the transverse and longitudinal fluxes [4].

The total virtual photon-proton cross section can be related to the proton structure function $F_2$ by [11]

$$\sigma_{\mathrm{tot}}^{\gamma^*\mathrm{p}} \equiv \sigma_{\mathrm{T}} + \sigma_{\mathrm{L}} \simeq \frac{4\pi^2\alpha_{\mathrm{em}}}{Q^2} F_2(x,Q^2) = \frac{4\pi^2\alpha_{\mathrm{em}}}{Q^2} \sum_{\mathrm{q}} e_{\mathrm{q}}^2 \left\{ xq(x,Q^2) + x\overline{q}(x,Q^2) \right\} \ . \tag{5}$$

where the last equality is valid for the parton model to lowest order. Such an interpretation is not valid in the limit $Q^2 \to 0$, where gauge invariance requires $F_2(x,Q^2) \to 0$ so that $\sigma_{\mathrm{tot}}^{\gamma^*\mathrm{p}}$ remains finite. We will replace the DIS description by a photoproduction one in this limit. Hence, at small photon virtualities, the DIS process $\gamma^*\mathrm{q} \to \mathrm{q}$ should be constructed vanishingly small as compared to the contribution from the interaction of the hadronic component of the photon. To obtain a well-behaved DIS cross section in this limit, a $Q^4/(Q^2+m_\rho^2)^2$ factor is introduced. Here $m_\rho$ is some non-perturbative hadronic parameter, for simplicity identified with the $\rho$ mass. Then, in the parton model, eq. (5) modifies to a DIS cross section

$$\sigma_{\mathrm{DIS}}^{\gamma^*\mathrm{p}} \simeq \frac{4\pi^2\alpha_{\mathrm{em}}Q^2}{(Q^2+m_\rho^2)^2} \sum_{\mathrm{q}} e_{\mathrm{q}}^2 \left\{ xq(x,Q^2) + x\overline{q}(x,Q^2) \right\} \ . \tag{6}$$

For numerical studies, the available parton distribution parameterizations for the proton have some lower limit of applicability in both $x$ and $Q^2$. For values below these minimal ones, the parton distributions are frozen at the lower limits.

In DIS, the photon virtuality $Q^2$ introduces a further scale to the process in Fig. 1a. The traditional DIS region is the strongly ordered one, $Q^2 \gg k_\perp^2 \gg p_\perp^2$, where DGLAP-style evolution [12] is responsible for the event structure. As above, ideology wants strong ordering, while real life normally is based on ordinary ordering $Q^2 > k_\perp^2 > p_\perp^2$. Then the parton-model description of $F_2(x,Q^2)$ in eq. (5) is a very good first approximation. The problems come when the ordering is no longer well-defined, i.e. either when the process contains several large scales or when $Q^2 \to 0$. In these regions, an $F_2(x,Q^2)$ may still be defined by eq. (5), but its physics interpretation is not obvious.

Let us first consider a large $Q^2$, where a possible classification is illustrated in Fig. 1d. The regions $Q^2 > p_\perp^2 > k_\perp^2$ and $p_\perp^2 > Q^2 > k_\perp^2$ correspond to non-ordered emissions, that then go beyond DGLAP validity and instead have to be described by the BFKL [13] or CCFM [14] equations, see e.g. [15]. Normally one expects such cross sections to be small at large $Q^2$. The (sparsely populated) region $p_\perp^2 > k_\perp^2 > Q^2$ can be viewed as the interactions of a resolved (anomalous) photon.

The region $k_\perp^2 > Q^2 \gg 0$ and $k_\perp^2 > p_\perp^2$ contains the $\mathcal{O}(\alpha_{\mathrm{s}})$ corrections to the lowest-order (LO) DIS process $\gamma^*\mathrm{q} \to \mathrm{q}$, namely QCD Compton $\gamma^*\mathrm{q} \to \mathrm{qg}$ and boson-gluon fusion $\gamma^*\mathrm{g} \to \mathrm{q\overline{q}}$. These are nothing but the direct processes $\gamma\mathrm{q} \to \mathrm{qg}$ and $\gamma\mathrm{g} \to \mathrm{q\overline{q}}$ extended to virtual photons. The borderline $k_\perp^2 > Q^2$ is here arbitrary — also processes with $k_\perp^2 < Q^2$ could be described in this language. In the parton model, this whole class of events are implicitly included in $F_2$, and are related to the logarithmic scaling violations of the parton distributions. The main advantage of a separation at $k_\perp = Q$ thus comes from the matching to photoproduction. Also the exclusive modeling of events, with the attaching of parton showers of scale $Q^2$ to DIS events, is then fairly natural.

The DIS cross section thus is subdivided into $\sigma_{\mathrm{tot}}^{\gamma^*\mathrm{p}} \simeq \sigma_{\mathrm{DIS}}^{\gamma^*\mathrm{p}} = \sigma_{\mathrm{LO\,DIS}}^{\gamma^*\mathrm{p}} + \sigma_{\mathrm{QCDC}}^{\gamma^*\mathrm{p}} + \sigma_{\mathrm{BGF}}^{\gamma^*\mathrm{p}}$. The $\sigma_{\mathrm{DIS}}^{\gamma^*\mathrm{p}}$ is given by eq. (6), while the last two terms are well-defined by an integration of the respective matrix element [4]. When extended to small $Q^2$, these two terms will increase in importance, and one may eventually encounter an $\sigma_{\mathrm{LO\,DIS}}^{\gamma^*\mathrm{p}} < 0$, if calculated by a subtraction of the QCDC and BGF terms from the total DIS cross section. However, here we expect the correct answer not to be a negative number but an exponentially suppressed one, by a Sudakov form factor. This modifies the cross section:

$$\sigma_{\mathrm{LO\,DIS}}^{\gamma^*\mathrm{p}} = \sigma_{\mathrm{DIS}}^{\gamma^*\mathrm{p}} - \sigma_{\mathrm{QCDC}}^{\gamma^*\mathrm{p}} - \sigma_{\mathrm{BGF}}^{\gamma^*\mathrm{p}} \quad \longrightarrow \quad \sigma_{\mathrm{DIS}}^{\gamma^*\mathrm{p}} \exp\left( - \frac{\sigma_{\mathrm{QCDC}}^{\gamma^*\mathrm{p}} + \sigma_{\mathrm{BGF}}^{\gamma^*\mathrm{p}}}{\sigma_{\mathrm{DIS}}^{\gamma^*\mathrm{p}}} \right) . \tag{7}$$

Since we here are in a region where $\sigma_{\mathrm{LO\,DIS}}^{\gamma^*\mathrm{p}} \ll \sigma_{\mathrm{DIS}}^{\gamma^*\mathrm{p}}$, i.e. where the DIS cross section is no longer the dominant one, this change of the total DIS cross section is not essential. Even more, for $Q^2 \to 0$ we know that the direct processes should survive whereas the lowest-order DIS one has to vanish. Since eq. (6) ensures that $\sigma_{\mathrm{DIS}}^{\gamma^*\mathrm{p}} \to 0$ in this limit, it also follows that $\sigma_{\mathrm{LO\,DIS}}^{\gamma^*\mathrm{p}}$ does so.

# 3 From Real to Virtual Photons

It is now time to try to combine the different aspects of the photon, to provide an answer that smoothly interpolates between the photoproduction and DIS descriptions, in a physically sensible way.

A virtual photon has a reduced probability to fluctuate into a vector meson state, and this state has a reduced interaction probability. This can be modeled with the traditional dipole factors [16] introduced to eq. (2). Similarly, the GVMD states are affected, where a relation $2k_\perp \simeq m$ is assumed.

The above generalization to virtual photons does not address the issue of longitudinal photons. Their interactions vanish in the limit $Q^2 \to 0$, but can well give a non-negligible contribution at finite $Q^2$ [17]. A common approach is to attribute the longitudinal cross section with an extra factor of $r_V = a_V Q^2/m_V^2$ relative to the transverse one [18], where $a_V$ is some unknown parameter to be determined from data. Such an ansatz only appears reasonable for moderately small $Q^2$, however, so following the lines of our previous study of jet production by virtual photons [4], we will try the two alternatives

$$r_1(m_V^2, Q^2) = a \frac{m_V^2 Q^2}{(m_V^2 + Q^2)^2} \qquad r_2(m_V^2, Q^2) = a \frac{Q^2}{(m_V^2 + Q^2)} \tag{8}$$

While $r_1$ vanishes for high $Q^2$, $r_2$ approaches the constant value $a$. The above VMD expressions are again extended to GVMD by the identification $m_V \approx 2k_\perp$. The GVMD cross section can then be written as

$$\sigma_{\mathrm{GVMD}}^{\gamma^*\mathrm{p}} = \frac{\alpha_{\mathrm{em}}}{2\pi} \sum_{\mathrm{q}} 2e_{\mathrm{q}}^2 \int_{k_0^2}^{k_1^2} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \left[1 + r_i(4k_\perp^2, Q^2)\right] \left( \frac{4k_\perp^2}{4k_\perp^2 + Q^2} \right)^2 \frac{k_{V(\mathrm{q\overline{q}})}^2}{k_\perp^2} \sigma^{V(\mathrm{q\overline{q}})\mathrm{p}}(W^2) . \tag{9}$$

The extrapolation to $Q^2 > 0$ is trivial for the direct processes, which coincide with the DIS QCDC and BGF processes. The matrix elements contain all the required $Q^2$ dependence, with a smooth behaviour in the $Q^2 \to 0$ limit. They are to be applied to the region $k_\perp > \max(k_1, Q)$ (and $k_\perp > p_\perp$, as usual).

Remains the LO DIS process. It is here that one could encounter an overlap and thereby double-counting with the VMD and GVMD processes. Comparing Fig. 1d with Fig. 1c, one may note that the region $p_\perp > k_\perp$ involves no problems, since we have made no attempt at a non-DGLAP DIS description but cover this region entirely by the VMD/GVMD descriptions. Also, if $Q > k_1$, then the region $k_1 < k_\perp < Q$ (and $k_\perp > p_\perp$) is covered by the DIS process only. So it is in the corner $k_\perp < k_1$ that the overlap can occur. If $Q^2$ is very small, the exponential factor in eq. (7) makes the DIS contribution too small to worry about. Correspondingly, if $Q^2$ is very big, the VMD/GVMD contributions are too small to worry about. Furthermore, a large $Q^2$ implies a Sudakov factor suppression of a small $k_\perp$ in the DIS description. If $W^2$ is large, the multiple-interaction discussions above are relevant for the VMD/GVMD states: the likelihood of an interaction at large $p_\perp$ will preempt the population of the low-$p_\perp$ region.

In summary, it is only in the region of intermediate $Q^2$ and rather small $W^2$ that we have reason to worry about a significant double-counting. Typically, this is the region where $x \approx Q^2/(Q^2 + W^2)$ is not close to zero, and where $F_2$ is dominated by the valence-quark contribution. The latter behaves roughly $\propto (1-x)^n$, with an $n$ of the order of 3 or 4. Therefore we will introduce a corresponding damping factor to the VMD/GVMD terms. The real damping might be somewhat different but, since small $W$ values are not our prime interest, we rest content with this approximate form.

In total, we have now arrived at our ansatz for all $Q^2$

$$\sigma_{\mathrm{tot}}^{\gamma^* \mathrm{p}} = \sigma_{\mathrm{DIS}}^{\gamma^* \mathrm{p}} \, \exp\left( - \frac{\sigma_{\mathrm{dir}}^{\gamma^* \mathrm{p}}}{\sigma_{\mathrm{DIS}}^{\gamma^* \mathrm{p}}} \right) + \sigma_{\mathrm{dir}}^{\gamma^* \mathrm{p}} + \left( \frac{W^2}{Q^2 + W^2} \right)^n \sigma_{\mathrm{res}}^{\gamma^* \mathrm{p}} \; . \tag{10}$$

To keep the terminology reasonably compact, also for the $\gamma^*\gamma^*$ case below, we use res as shorthand for the resolved VMD plus GVMD contributions and dir as shorthand for the QCDC and BGF processes. The DIS and GVMD terms are given by eqs. (6) and (9), respectively, and the QCDC and BGF terms by direct integration of the respective matrix elements for the region $k_\perp > \max(k_1, Q)$. (The VMD contribution is obtained from eq. (2) with the appropriate dipole and polarization factors applied.)

The extension to $\gamma^*\gamma^*$ follows from the $\gamma^*\mathrm{p}$ formalism above, but now with (up to) five scales to keep track of: $p_\perp$, $k_{\perp 1}$, $k_{\perp 2}$, $Q_1$ and $Q_2$. First consider the three by three classes present already for real photons, which remain nine distinct ones for $Q_1^2 \neq Q_2^2$. Each VMD or GVMD state is associated with its dipole damping factor and its correction factor for the longitudinal contribution. The QCDC and BGF matrix elements involving one direct photon on a VMD or a GVMD state explicitly contain the dependence on the direct photon virtuality, separately given for the transverse and the longitudinal contributions. Also the direct$\times$direct matrix elements are known for the four possible transverse/longitudinal combinations.

To this should be added the new DIS processes that appear for non-vanishing $Q^2$, when one photon is direct and the other resolved, i.e. VMD or GVMD. For simplicity, first assume that one of the two photons is real, $Q_2^2 = 0$. For large $Q_1^2$, this DIS contribution $\sigma_{\mathrm{DIS}\times\mathrm{res}}^{\gamma^*\gamma}(Q_1^2)$ can be given a parton-model interpretation similarly to eq. (5) and (6). Note that this is only the resolved part of $\sigma_{\mathrm{tot}}^{\gamma^*\gamma}$. The direct contribution from $\gamma^*\gamma \to \mathrm{q}\overline{\mathrm{q}}$ comes in addition, but can be neglected in the leading-order definition of $F_2^\gamma$. We will therefore use parton distribution parameterizations for the resolved photon, like SaS 1D [19], to define the $\sigma^{\gamma^*\gamma}_{\mathrm{DIS}\times\mathrm{res}}(Q_1^2)$. Then eq. (10) generalizes to

$$\sigma^{\gamma^*\gamma}_{\mathrm{tot}} = \sigma^{\gamma^*\gamma}_{\mathrm{DIS}\times\mathrm{res}} \exp\left(-\frac{\sigma^{\gamma^*\gamma}_{\mathrm{dir}\times\mathrm{res}}}{\sigma^{\gamma^*\gamma}_{\mathrm{DIS}\times\mathrm{res}}}\right) + \sigma^{\gamma^*\gamma}_{\mathrm{dir}\times\mathrm{res}} + \sigma^{\gamma^*\gamma}_{\mathrm{res}\times\mathrm{dir}} + \sigma^{\gamma^*\gamma}_{\mathrm{dir}\times\mathrm{dir}} + \left(\frac{W^2}{Q_1^2+W^2}\right)^n \sigma^{\gamma^*\gamma}_{\mathrm{res}\times\mathrm{res}} \,. \quad (11)$$

The large-$x$ behaviour of a resolved photon does not agree with that of the proton, but for simplicity we will stay with $n = 3$.

Finally, the generalization to both photons virtual gives an extra term $\sigma^{\gamma^*\gamma^*}_{\mathrm{res}\times\mathrm{DIS}}$ times its corresponding exponential factor as in eq. (7) and eq. (11). When $Q_1^2 \gg Q_2^2$ the expression for $\sigma^{\gamma^*\gamma^*}_{\mathrm{tot}}(W^2, Q_1^2, Q_2^2)$ can be related to the structure function of a virtual photon, $F_2^{\gamma^*}(x, Q^2 = Q_1^2, P^2 = Q_2^2)$, where $x = Q_1^2/(Q_1^2 + Q_2^2 + W^2)$.

# 4 Summary and Outlook

A brief description of a model for real and virtual photons has been presented. We have developed one specific approach, where the main idea is to classify events by the hardest scale involved. This is not an economical route, since it leads to many event classes. For studies e.g. of the total cross section in the intermediate-$Q^2$ region, it is cumbersome and not necessarily better than existing approaches [20]. However, by the breakdown into distinct event classes, the road is open to provide a (more or less) complete picture of all event properties. It is this latter aspect that has then guided the model development. A more detailed description can be found in a future publication [5]. Some results was presented on the Working group meeting in April 2000 [21].

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