LU TP 00–28

HIGHER–TWIST CONTRIBUTIONS TO F_L AND F_2 FROM INSTANTONS*

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We study the twist–4 contribution to the unpolarized nucleon structure functions, F_L and F_2, in an approach where the vacuum structure of QCD is described by a dilute medium of instantons ($\bar{\rho}/\bar{R} \ll 1$). The dominant power corrections come from the twist–4 quark–gluon operators and are of the order $1/(\bar{\rho}^2 Q^2)$. The contributions from four–quark operators are suppressed. The approach predicts large $1/Q^2$–corrections to F_L, in agreement with the results of QCD fits to the data.

The operator product expansion of QCD explains not only the scaling behavior of structure functions at asymptotically large Q^2 — including the logarithmic corrections, which can be computed using perturbation theory — but allows to classify also the power ($1/Q^2$–) corrections to scaling. The latter are determined by the expectation values of certain operators of non-leading twist (> 2) in the target hadron state [1]. While a fully quantitative theory combining logarithmic and power corrections is presently out of reach, it is interesting to study these higher–twist matrix elements in their own right. These are "universal" measures of the non-perturbative correlations of the quark and gluon fields inside hadrons, which appear also in other contexts, *e.g.* in the description of "intrinsic" heavy quark contributions to light hadron properties [2]. Knowledge of these matrix elements would be useful for a more precise extraction of the leading–twist parton distributions from the DIS data, and for a better understanding of parton–hadron duality [3].

* Presented at DIS 2000, Liverpool, April 25 – 30, 2000.
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Two basic types of twist–4 operators appear in the $1/Q^2$–power corrections to the second moments of the unpolarized structure functions (for details, see Refs.[1,7]). One are quark–gluon operators, which describe the influence in DIS of the non-perturbative gluon field in the target on the "struck" quark:

$$\left.\begin{array}{rcl} \langle p|\,\bar\psi_f\gamma_\alpha(D^\gamma F_{\gamma\beta})\psi_f\,|p\rangle &-& \text{trace} \;=\; 2\,A_f \\ \langle p|\,\bar\psi_f\gamma^\gamma\gamma_5 i\left(\nabla_\alpha\widetilde{F}_{\beta\gamma}+\widetilde{F}_{\alpha\gamma}\nabla_\beta\right)\psi_f\,|p\rangle &-& \text{trace} \;=\; 2\,B_f \end{array}\right\}\times\left(p_\alpha p_\beta-\frac{p^2}{4}g_{\alpha\beta}\right). \tag{1}$$

Here, $F_{\alpha\beta}$ ($\widetilde{F}_{\alpha\beta}$) denote the (dual) gluon field. The other are four–quark operators, describing the interference contribution to DIS from scattering off two different quarks in the target:

$$g^2\,\langle p|\,\bar\psi_f\frac{\lambda^a}{2}\gamma_\alpha\gamma_5\psi_f\;\bar\psi_{f'}\frac{\lambda^a}{2}\gamma_\beta\gamma_5\psi_{f'}\,|p\rangle \;\;=\;\; 2\,C_{ff'}\left(p_\alpha p_\beta-\frac{p^2}{4}g_{\alpha\beta}\right). \tag{2}$$

In this context two questions arise:

- What are the relevant non-perturbative scales determining the values of the twist–4 matrix elements, A, B and C?
- Are the matrix elements of both types of operators of equal magnitude or is one of them dominant?

It is interesting to address these questions from the point of view of the instanton vacuum, where one studies non-perturbative effects generated by a dilute "medium" of instantons and antiinstantons. These vacuum fluctuations are known to be responsible for the spontaneous breaking of the chiral symmetry of QCD and a variety of other effects; see [5] for a review. The properties of the instanton "medium" have been derived within a variational approach to the QCD ground state [4], and have also been extensively studied in lattice simulations [5]. A crucial property is that the average size of the instantons in the medium, $\bar\rho = (600\,\text{MeV})^{-1}$, is small compared to their average distance, $\bar R$, with $\bar\rho/\bar R \approx 1/3$ ("diluteness"). This ratio provides a small parameter which allows to classify non-perturbative effects generated by the instanton medium. For instance, the dynamical quark mass, M, which appears in the spontaneous breaking of chiral symmetry, is parametrically of the order $M \sim (\bar\rho/\bar R)^2\,\bar\rho^{-1}$. As a result, *e.g.* the pion weak decay constant is parametrically suppressed:

$$F_\pi^2 \;\sim\; \int\limits_{k^2<\bar\rho^{-2}}\frac{d^4k}{(2\pi)^4}\frac{M^2}{(k^2+M^2)^2} \;\sim\; M^2\log M\bar\rho \;\sim\; \left(\frac{\bar\rho}{\bar R}\right)^4\log\left(\frac{\bar\rho}{\bar R}\right)\bar\rho^{-2}. \tag{3}$$

Similar parametric estimates can be performed for other observables.

Following the same philosophy we can estimate also the magnitude of the twist–4 nucleon matrix elements, (1) and (2).[7,6] When computing the matrix element of the twist–4 quark–gluon operators, (1), the gauge field in the operator is replaced by the field of a single (anti–) instanton, which then interacts with the fermion fields in the operator through its fermionic zero modes. (The same zero–mode induced interaction is instrumental in the dynamical breaking of chiral symmetry.) One finds that the isoscalar nucleon matrix element is determined by an integral of the form

$$B_u + B_d \;\sim\; \int\limits_{k^2<\bar\rho^{-2}} \frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2+M^2)^2} \;\sim\; \bar\rho^{-2}. \tag{4}$$

Contrary to the pion decay constant, (3), this integral is quadratically divergent, so that its value is determined by the square of the ultraviolet cutoff — the inverse instanton size, $\bar\rho^{-1}$. A more quantitative estimate gives $B_u + B_d \approx 0.9\,\bar\rho^{-2} = (570\,\mathrm{MeV})^2$. The matrix element has a large numerical value, because of the smallness of the average instanton size. The corresponding matrix element of the first operator, (1), is accidentally suppressed, since $D^\gamma F_{\gamma\beta} \equiv 0$ in the field of one instanton. For the matrix elements of the four–fermionic operators, on the other hand, we find

$$C_{uu}, C_{dd}, C_{ud} \;\sim\; F_\pi^2 \;\sim\; \left(\frac{\bar\rho}{\bar R}\right)^4 \log\left(\frac{\bar\rho}{\bar R}\right)\bar\rho^{-2}. \tag{5}$$

These matrix elements are parametrically suppressed relative to that of the operator B, (4). In addition, they are suppressed in the coupling constant, g^2. Numerically, a value of $(50\,\mathrm{MeV})^2$ for the isoscalar matrix element was found in Ref.[7] — two orders of magnitude smaller than for the quark–gluon operator!

To summarize, the parametric order of the higher–twist matrix elements obtained from the instanton vacuum implies that:

- Matrix elements of twist–4 four–quark operators, (2), are parametrically (and numerically) small compared to those of quark–gluon operators, (1).

- The relevant scale for twist–4 quark–gluon operators is the square of the average instanton size, $\bar\rho^{-2} = (600\,\mathrm{MeV})^2$. The dominant power corrections are thus expected to be of the order $1/(\bar\rho^2 Q^2)$.

These findings are in agreement with the results of QCD fits to the data for F_L and F_2, which show a sizable $1/Q^2$ corrections to F_L consistent with (4). [7]

References

1. E. V. Shuryak and A. I. Vainshtein, Nucl. Phys. **B199** (1982) 451.
2. For the polarized case this has been discussed in M. V. Polyakov, A. Schafer and O. V. Teryaev, Phys. Rev. **D60** (1999) 051502.
3. X. Ji and P. Unrau, Phys. Rev. **D52** (1995) 72.
4. D. I. Diakonov and V. Y. Petrov, Nucl. Phys. **B245** (1984) 259.
5. T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70** (1998) 323.
6. J. Balla, M. V. Polyakov and C. Weiss, Nucl. Phys. **B510** (1998) 327.
7. B. Dressler, M. Maul and C. Weiss, Nucl. Phys. **B578** (2000) 293.