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A Toy Model of Colour Screening in the Proton

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Abstract

In hadronic collisions, the mini-jet cross section is formally divergent in the limit $p_\perp \to 0$. We argue that this divergence is tamed by some effective colour correlation length scale of the hadron. A toy model of the hadronic structure is introduced, that allows an estimate of the screening effects, and especially their energy dependence.

The perturbative parton–parton interaction cross section in a hadronic collision is divergent roughly like

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} \propto \frac{\alpha_\mathrm{s}^2(p_\perp^2)}{p_\perp^4} \tag{1}$$

for $p_\perp \to 0$. This is to be convoluted with parton distributions $f_i(x, Q^2 \approx p_\perp^2)$ to give a hadronic interaction cross section. The dominant contributions come from scattering by t-channel gluon exchange. With an artificial lower cut-off scale $p_{\perp\mathrm{min}}$,

$$\sigma_\mathrm{int}(p_{\perp\mathrm{min}}) = \sum_{i,j,k,l} \int \mathrm{d}x_1\, f_i(x_1, Q^2) \int \mathrm{d}x_2\, f_j(x_2, Q^2) \int_{p_{\perp\mathrm{min}}^2} \mathrm{d}p_\perp^2\, \frac{\mathrm{d}\hat{\sigma}_{ij\to kl}(\hat{s} = x_1 x_2 s)}{\mathrm{d}p_\perp^2} \; . \tag{2}$$

The jet/mini-jet cross section is twice this, since each scattering gives two jets in the lowest-order approximation considered here. Studying e.g. Tevatron collider energies, $\sqrt{s} \approx 2$ TeV, $\sigma_\mathrm{int}(p_{\perp\mathrm{min}})$ exceeds the total $\mathrm{p\overline{p}}$ cross section σ_tot for $p_{\perp\mathrm{min}} \lesssim 3$ GeV. Since 3 GeV still is well above the Λ_QCD scale of ~ 0.2 GeV, there is no obvious reason why perturbation theory should have broken down, and one would seem to be in trouble. The resolution of this paradox probably comes in several steps, and at least in two.

First of all, the mini-jet cross section above is inclusive. Thus, if an event contains two parton–parton interactions [1], it counts twice in $\sigma_\mathrm{int}(p_{\perp\mathrm{min}})$ but only once in σ_tot. Thereby $\sigma_\mathrm{int}(p_{\perp\mathrm{min}}) > \sigma_\mathrm{tot}$ becomes allowed. Multiple parton–parton interactions is the concept that, based on the composite nature of hadrons, indeed several parton pairs may scatter in a typical hadron–hadron collision [2]. Over the years, evidence for this mechanism has accumulated [3], such as the direct observation by CDF [4]. The events studied experimentally, with two parton pairs at reasonably large $p_\perp$, only form the tip of the iceberg, however. One may expect that most interactions are at lower $p_\perp$, where they do not produce visible jets, but only contribute to the underlying event structure. As such, they are then believed to be at the origin of a number of key features, like the broad multiplicity distributions, the significant forward–backward multiplicity correlations, and the pedestal effect under jets [2].

While the effects of multiple interactions on event properties are smaller for lower-$p_\perp$ scatterings, in the models we studied so far the interaction rate increases faster with decreasing $p_\perp$ than the effect per scattering, so there is no stability in the limit $p_{\perp\mathrm{min}} \to 0$.

The second necessary aspect is likely that a regularization of the jet cross section should occur at small $p_\perp$ from the fact that the incoming hadrons are colour singlets — unlike the coloured partons assumed in the divergent perturbative calculations — and that therefore the colour charges should screen each other in the $p_\perp \to 0$ limit. Thus $p_{\perp\mathrm{min}}$ could take on a physics meaning, roughly, as the inverse of some colour screening length in the hadron. Of course, one would not expect a sharp cut-off of the mini-jet cross section at $p_{\perp\mathrm{min}}$, but rather a smooth dampening. Nevertheless the $p_{\perp\mathrm{min}}$ parameter provides a useful first approximation. Fits to data typically give $p_{\perp\mathrm{min}} \approx 2$ GeV.

One key issue is the energy dependence of $p_{\perp\mathrm{min}}$; this may be relevant e.g. for comparisons of jet rates at different Tevatron energies, and even more for any extrapolation to LHC energies. The question actually is more pressing now than at the time of the study in [2], since nowadays parton distributions are known to be rising more steeply at small x than the flat $xf(x)$ behaviour normally assumed for small Q^2 before HERA. This translates into a more dramatic energy dependence of the multiple-interactions rate for a fixed $p_{\perp\mathrm{min}}$, and thence to a charged multiplicity rising faster than data [5]. Based on such considerations, the $p_{\perp\mathrm{min}}$ in the PYTHIA program [6] was made explicitly energy-dependent

some time ago:

$$p_{\perp\min} = (1.9\ \mathrm{GeV}) \left(\frac{s}{1\ \mathrm{TeV}^2} \right)^{\epsilon} , \tag{3}$$

with $\epsilon = 0.08$. This value is picked to agree with the s^{ϵ} behaviour assumed in the parameterization of the total cross section in hadron–hadron collisions [7] that, via reggeon phenomenology, should relate to the behaviour of parton distributions at small x and Q^2. A study of the energy dependence of $\mathrm{d}N_{\mathrm{charged}}/\mathrm{d}\eta|_{\eta=0}$ also confirms that an ϵ in this range gives sensible agreement with data [8].

In the following, we will develop an extremely simple model for the proton structure, where the amount of screening may be studied [9]. Our basic approach is to assign an exclusive parton configuration to each proton. This configuration is — for a given evolution scale Q — defined in terms of the longitudinal momentum fraction x values, the transverse coordinate positions and colour charges of 'all' the partons present in a proton. The traditional inclusive parton-distribution picture is obtained as the average over many such proton 'snapshots'. A simple plane wave represents the exchange of a gluon between two colliding hadrons. Longitudinal coordinates of the partons are not specified, so the exchanged gluon is only characterized by its transverse momentum. The exclusive proton picture then allows a comparison of a coherent versus an incoherent sum of colour charges coupling to the plane wave. It is this ratio, coherent over incoherent, that will provide our measure of screening between partons in the proton. In particular, the long wavelength limit then explicitly corresponds to a vanishing of interactions between two colourless hadrons. As the collision energy is increased, more partons at smaller x values become accessible and the screening effects increase in importance.

We certainly know this ansatz to be wrong in many of its details, but still hope that it will catch enough of the spirit to provide some insight. It is clear that a full-fledged description — e.g. with lattice QCD calculations — is way beyond the current capability, so toy models is all that could be offered today. The approach we present here should be viewed as only one possible aspect or formulation of the required dampening effect. The vanishing net colour charge of hadrons has been used in models since long [10], as has the spatial size of the colour singlet wave function [11]. Our model also has some similarities with the dense-packing view of the proton [12], although without the possibility of parton recombinations. In ultrarelativistic heavy ion collisions, dense-packing saturation in combination with the perturbative QCD cross section has been used to derive an s-dependent $p_{\perp\min}$ [13]. Among alternatives, one could mention the possibility of a nontrivial structure of the QCD vacuum [14], the solution of a Dyson-Schwinger equation for the gluon propagator [15], the introduction of an effective gluon mass from lattice QCD results [16], and the possibility of several hard scatterings within a single parton chain [17], e.g. formulated in terms of non-integrated structure functions and the Linked Dipole Model description of non-$p_\perp$-ordered chains [18].

In our model, the Q^2 evolution of the parton distributions is given by the conventional DGLAP equations [19]. As usual, the starting configuration requires nonperturbative input. A valence-like ansatz at a small Q_0 scale [20] limits the number of required free parameters. We have therefore used the GRV distributions as a reference [21], where the three valence quarks together with two 'valence gluons' almost completely define the proton at the small scale $Q_0 = 0.48$ GeV. In our simulation, these five partons are first selected according to a single-particle distribution

$$f(x) \propto x^{\alpha}(1-x)^{\beta} , \tag{4}$$

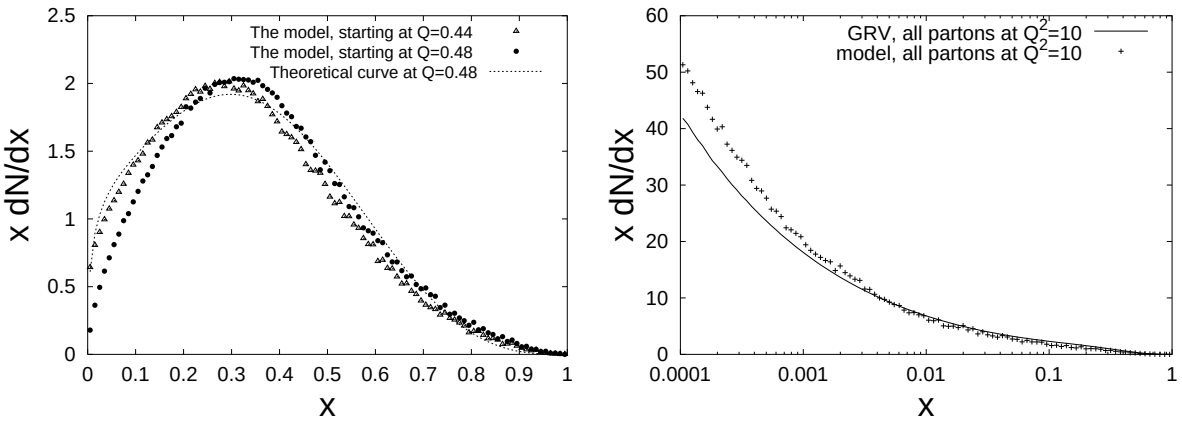

Figure 1: Parton momentum distribution, summed over quarks and gluons. Our simulation is compared with the GRV parameterization [21]. (a) At the small scale $Q_0 = 0.48$ GeV. (b) Evolved to $Q^2 = 10$ GeV^2, with a cut so that branchings do not produce partons below $x_{min} = 10^{-4}$.

with tuned parameters $\alpha = -0.4$ and $\beta = 1.2$. Thereafter all five x_i are normalized so that their sum corresponds to unity:

$$(x_i)_{\rm norm} = \frac{x_i}{\sum_{j=1}^{5} x_j} \; . \tag{5}$$

This means that the original distribution is pushed towards the middle, and it is especially difficult to tune the low-x tail, at least with the simple $f(x)$ ansatz above. A reduction of the Q_0 scale from 0.48 to 0.44 GeV helps improve the small-x behaviour, Fig. 1a.

From there on, the evolution follows the standard leading-order DGLAP formalism, expressed in terms of Monte Carlo-generated branchings of partons, $\mathrm{q} \to \mathrm{qg}$, $\mathrm{g} \to \mathrm{gg}$ and $\mathrm{g} \to \mathrm{q\overline{q}}$. Unfettered, such an approach gives an infinity of gluons in the $x \to 0$ limit. For the exclusive approach we have in mind, this is not so physical. In the study of scatterings above some $p_{\perp\rm min}$ scale, only partons above some related $x_{\rm min}$ scale can at all interact. Partons below $x_{\rm min}$ then have a dubious existence. To simplify the picture, we assume they are not resolved, i.e. that only branchings producing both daughters above $x_{\rm min}$ are allowed. This reduces the amount of evolution, but not as dramatically as might be guessed at first glance. In Fig. 1b we see that, in a typical case, the parton density with an $x_{\rm min}$ constraint is at most 20% above the no-constraints normal evolution in the region $x > x_{\rm min}$, while the former obviously vanishes for $x < x_{\rm min}$. Disagreements are more visible — but also less interesting for us — in individual parton species, especially for sea quarks, which are not part of our simple ansatz at Q_0 but are in the GRV one.

The kinematics of a hard scattering requires that $x_1 x_2 = \hat{s}/s \geq 4p_\perp^2/s$. For collisions at central rapidities this suggests $x_{\rm min} \simeq 2p_\perp/E_{\rm CM}$. Away from the center, $x_{\rm min}$ would be smaller on one side, down to $4p_\perp^2/E_{\rm CM}^2$, and correspondingly larger on the other, which to some extent should cancel out. Therefore we will have as standard scenario that $x_{\rm min}$ should increase proportionately to the $p_\perp$ considered, but will include as alternative a fixed $x_{\rm min}$. Furthermore, we will make the association that $E_{\rm CM} \propto 1/x_{\rm min}$, for $x_{\rm min}$ evaluated at some fixed reference $p_\perp$ like 1 GeV, but allow as alternative that the relation could be more like $E_{\rm CM} \propto 1/\sqrt{x_{\rm min}}$.

Next we consider the assignment of transverse position coordinates. The original five partons are here selected according to a Gaussian shape, with a radius of 0.7 fm

in each of the two transverse dimensions. This gives a minimal correlation between the position coordinates of these five partons, and thus probably errs on the side of simplicity. Attempts with somewhat more correlated forms gave fairly similar results, however, so the Gaussian ansatz is not so critical.

The spatial extent of a fluctuation like q $\to$ qg $\to$ q, g $\to$ gg $\to$ g and g $\to$ q$\overline{\mathrm{q}}$ $\to$ g is of order $1/Q$ according to the uncertainty relation. Therefore, when a DGLAP branching occurs at a scale Q, the two daughters are assumed to have time to fluctuate a distance of this order away from their production coordinates before they branch in their turn, or are probed by our plane gluon wave. The daughter partons are assumed to move out in opposite directions from the production vertex, with a uniform azimuthal distribution. The two distances are picked independently of each other, uniformly between 0 and $1/Q$, alternatively between 0 and $2/Q$. We will see that this choice is relevant, whereas results are essentially equivalent e.g. for exponentially dampened distributions with the same mean. For a parton which takes the momentum fraction z in a branching, an additional factor $\sqrt{1-z}$ is included, giving an approximate $p_\perp/Q$ dampening factor for the separation in transverse rather than longitudinal direction. Here the relation $p_\perp = \sqrt{1-z}Q$ follows in light-cone kinematics assuming the mother and recoiling partons to be massless. It is of some relevance that a very soft gluon emission does not affect the position of the hard parton unduly, since else multiple soft gluons could lead to a too rapid increase of the proton radius with Q. For the final result here, however, the $\sqrt{1-z}$ factor turns out not to be crucial.

Finally we come to the colour space picture. Here we have picked a simple planar representation, with the three primary colours placed in a triangle around the origin. Thus the full phase information is lost; i.e. it is possible to ensure that the proton state is colour neutral but not that it is in a singlet. This certainly is a major simplification of the real world, and one which it is difficult to estimate the impact of. A gluon emission corresponds to a quark changing colour, e.g. $\mathrm{q}(r) \to \mathrm{q}(b) + \mathrm{g}(r\overline{b})$. The emitted gluon thus carries one colour and a different anticolour, i.e. the two colour diagonal gluon states are not populated; another simplification. Further, the ratio of gluon to quark charge is $\sqrt{3}$ versus $\sqrt{N_C/C_F} = 3/2$ in QCD. Also in a gluon branching to two, a new colour is picked, e.g. $\mathrm{g}(r\overline{b}) \to \mathrm{g}(r\overline{g}) + \mathrm{g}(g\overline{b})$, in order to avoid diagonal gluons. The gluon branching to quarks is unambiguous, $\mathrm{g}(r\overline{b}) \to \mathrm{q}(r) + \overline{\mathrm{q}}(\overline{b})$. For colour assignments in the starting configuration, the three valence quarks are first picked to be $r + g + b$, and then the two gluons are handled as if emitted from two of the quarks, picked at random.

An effort is thus made to retain overall vanishing colour for all partons in a proton that are seen by an exchanged gluon. In case of multiple interactions, it could be argued that only the 'first' exchange would be between singlet hadrons, while 'subsequent' ones would be between already coloured hadrons. Clearly, the imposition of such a time ordering can be questioned but, even if taken at face value, colour screening could still provide a significant dampening of the naive perturbative answer.

To summarize, an explicit parton configuration of the proton is constructed for any value of the evolution parameter Q above some low Q_0 starting scale. Each parton k is characterized by its momentum fraction x_k, its transverse position coordinate $\mathbf{r}_k$ and its two-dimensional colour charge q_k. If such a proton is probed by a gluon plane wave function of transverse momentum $\mathbf{p}_\perp$, we may define a ratio

$$A = \frac{|\sum_k q_k e^{i\mathbf{r}_k\mathbf{p}_\perp}|^2}{\sum_k |q_k|^2} \tag{6}$$

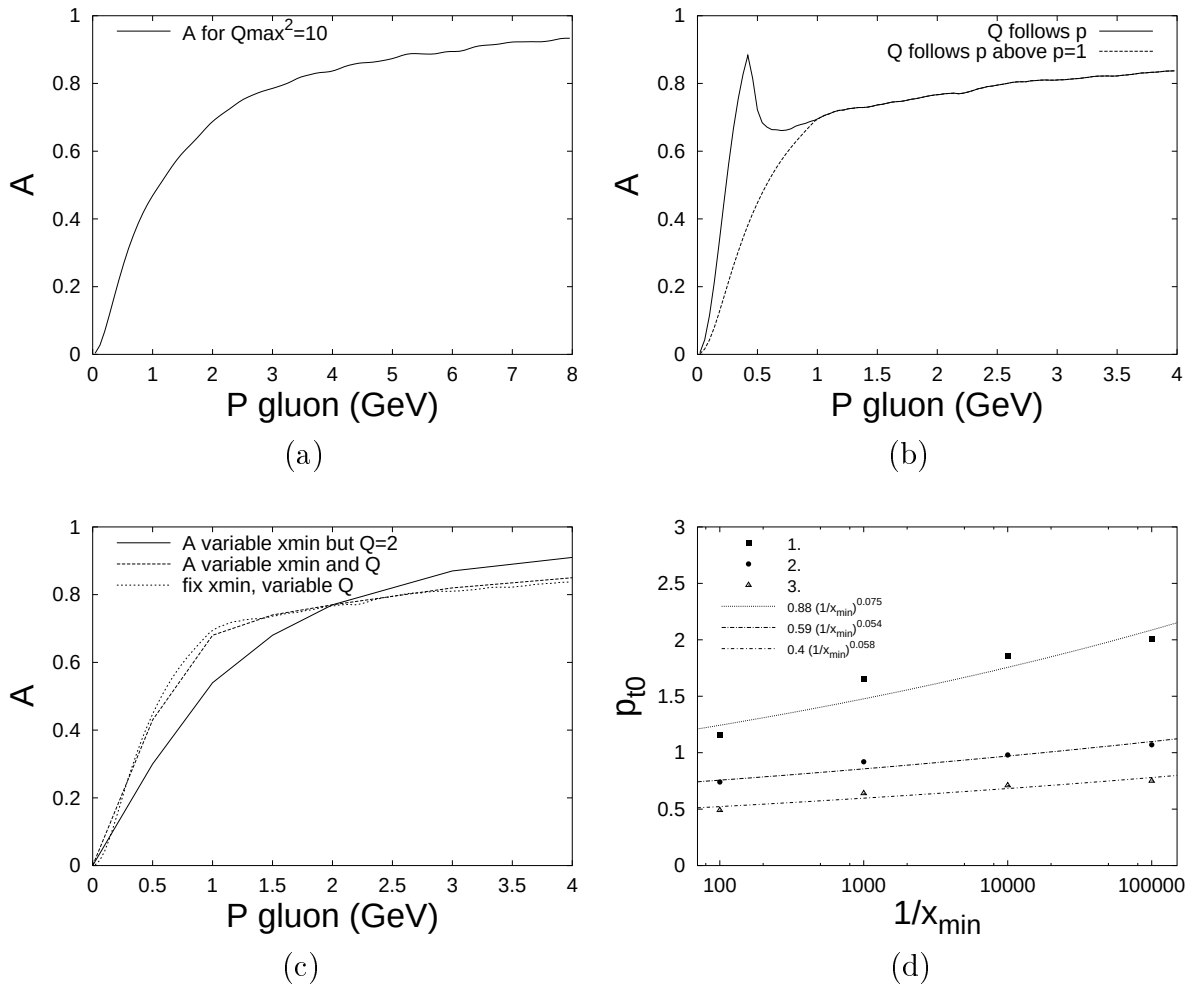

Figure 2: (a-c) The amount of screening, A in eq. 6, as a function of the probing gluon (transverse) momentum $P = p_\perp$. (a) For a proton at fixed evolution scale $Q^2 = 10$ GeV^2, and fixed $x_{\rm min} = 10^{-3}$. (b) For a proton where the proton evolution scale $Q = p_\perp$ for $p_\perp \geq 0.44$ GeV, alternatively 1 GeV, again with $x_{\rm min} = 10^{-3}$. (c) Full: variable $x_{\rm min} = 10^{-3} p_\perp$ and fixed $Q = 2$ GeV. Dashed: variable $x_{\rm min} = 10^{-3} p_\perp$ and $Q = \max(p_\perp, 1\ {\rm GeV})$. Dotted: fix $x_{\rm min} = 10^{-3}$ and variable $Q = \max(p_\perp, 1\ {\rm GeV})$. (d) The $x_{\rm min}$ dependence of the $p_{\perp 0}$ effective cut-off. See text for a explanation of the three scenarios.

between a coherent and an incoherent sum of the colour charges. Note that we assume the exchanged gluon to be a superposition of all possible colour-anticolour pairs, so that we do not need to single out a special direction in colour space.

The screening effect is most easily observed if one fixes the Q of the probed proton and then varies the $p_\perp$ of the probing gluon, Fig. 2a. The suppression factor A then has to vanish quadratically as $p_\perp \to 0$, but in the interesting $p_\perp$ range the dominant aspect is instead the slower rise to the asymptotic limit $A = 1$. The choice of a fixed Q scale is representative for the way parton distributions de facto are used with Q_0 as scale for $Q < Q_0$. For popular distributions like the current CTEQ and MRST ones [22], where $Q_0^2 \geq 1$ GeV^2, the inclusion of a suppression factor then significantly modifies the picture for $Q < Q_0$. Actually Q_0 has tended to come down with time; some years ago $Q_0^2 \geq 4$ GeV^2 was the norm. Also remember that the cross section suppression is A^2, i.e.

with one factor of A for each hadron beam.

A conventional choice, but not a unique one, is to pick the Q scale of parton distributions to agree with the $p_\perp$ scale of the hard interaction. Then the approach $A \to 1$ is much slower at large $p_\perp$, since the branchings that occurred at scales Q not so much smaller than $p_\perp$ are not fully resolved, Fig. 2b. We remind that the proton seen by a gluon probe is a much more 'busy' place than the one seen by a photon in Deeply Inelastic Scattering processes, where the gluon content is not probed directly. Thus the destructive interference we introduce here does not invalidate the conventional structure function measurements and partonic interpretations. In hadronic physics, higher order perturbative corrections could partly contain, and partly mask, the physics of an A somewhat below unity.

More interesting is the behaviour at small $p_\perp$. We here note a spike in A at the scale $p_\perp = Q_0 = 0.44$ GeV, Fig. 2b. At this scale, the proton consists of the original five partons, and these are normally well separated in space. Above it, the partons emitted in the evolution process tend to have a more clustered spatial distribution and therefore screen more. Below it, there is no further evolution of the proton but only a straightforward destructive interference. Thus the spike is an artifact of how we discontinuously in Q go from a fixed proton picture to a very rapid evolution; remember that the evolution rate behaves like $\alpha_s(Q^2)\,\mathrm{d}Q^2/Q^2$ and thus is as largest just at the lower cut-off. In order to provide a smoother and physically maybe more sensible picture, we therefore do not probe the proton at scales below 1 GeV, i.e. put $Q = \max(p_\perp, 1\ \mathrm{GeV})$. The original Q_0 scale and evolution below 1 GeV still lives on in terms of a sensible $x/\mathbf{r}$/colour configuration at 1 GeV.

Fig. 2b is based on a fixed $x_{\min}$ scale. However, an $x_{\min}$ scaling $\propto p_\perp$ behaves very similarly, Fig. 2c, if the two are matched for $p_\perp = 1$ GeV. If instead Q is fixed, the shape is more changed.

We now turn to the prime objective of this paper, namely to study the energy dependence of $p_{\perp\min}$. Actually, the sharp cut-off $\theta(p_\perp - p_{\perp\min})$ is not very physical. A more likely behaviour is a dampening factor something like $p_\perp^2/(p_\perp^2 + p_{\perp 0}^2)$, where $p_{\perp 0}$ takes over the role of free parameter. With one such factor per incoming proton, the $1/p_\perp^4$ singularity of the perturbative parton-parton cross section is regularized.

Whereas the proposed dampening form does not completely reproduce the curves shown above, qualitatively there is agreement. Instead of doing a fit to the curve shape, $p_{\perp 0}$ is extracted as the $p_\perp$ value for which $A = 1/2$. This would also have been a sensible definition of a $p_{\perp\min}$, backed up by experience with the impact of the two regularization procedures on fits to data, where $p_{\perp 0} \approx p_{\perp\min}$ is obtained [2].

The resulting $p_{\perp 0}$ values are plotted as a function of $1/x_{\min}$ in Fig. 2d. Three scenarios are compared:

1. The proton probed at a fixed $Q = 2$ GeV scale, travel distance of a parton is distributed between 0 and $1/Q$ of the branching where it is produced.
2. Also proton probed at a fixed $Q = 2$ GeV scale, but travel distance twice as large.
3. The proton is probed at a running $Q = p_\perp$, travel distance between 0 and $2/Q$ as in 2.

In all these three cases, representative for several more studied by us, a clear energy-dependence of $p_{\perp 0}$ is observed. If one tries to make a fit to an exponential form $p_{\perp 0} \propto 1/x_{\min}^{\delta}$, typically one obtains a δ in the range 0.05 to 0.08. However, two comments are in order. One is that the curves tend to rise less steeply at higher energies than implied by the fit form. The other is that, if one attempts the association $1/x_{\min} \propto E_{\mathrm{CM}}$, then eq. (3) would have predicted $\delta = 2\epsilon = 0.16$, i.e. a much steeper slope. The alternative

association $1/x_{\rm min} \propto E_{\rm CM}^2$ would give nicer agreement, but we iterate that it may be the less plausible ansatz.

In summary, we have presented a very simpleminded model of the longitudinal momentum, transverse coordinates and colour structure of the proton, in order to be able to estimate colour screening effects at small $p_\perp$. Many of the details turn out to be less critical, but there are two key assumptions. The first is that the proton, by virtue of being a colour singlet, should couple to a soft gluon with reduced strength. The second is that, at higher energies, the proton can effectively be resolved into more partons that, in principle, can interact to give scatterings at a given $p_\perp$ scale. We show that these two assumptions lead to the conclusion that the perturbative ansatz for the cross section is only valid above some $p_{\perp\rm min}$ scale that increases with energy. If this increase is fitted to a form $p_{\perp\rm min} \propto E_{\rm CM}^\kappa$, any value κ in the range 0.05 to 0.16 could be obtained, which certainly overlaps with an experimentally acceptable behaviour. However, one lesson is that the ansatz in eq. (3) may tend to overestimate the rate of increase at large energies, and thereby e.g. lead to an underestimate of multiplicities at LHC energies. Conversely, the study of minimum bias events at LHC may help improve the understanding of very low-$p_\perp$ physics processes.

References

[1] P.V. Landshoff and J.C. Polkinghorne, Phys. Rev. **D18** (1978) 3344;
C. Goebel, D.M. Scott and F. Halzen, Phys. Rev. **D22** (1980) 2789.

[2] T. Sjöstrand and M. van Zijl, Phys. Rev. **D36** (1987) 2019.

[3] AFS Collaboration, T. Åkesson et al., Z. Phys. **C34** (1987) 163;
UA2 Collaboration, J. Alitti et al., Phys. Lett. **B268** (1991) 145.

[4] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. **79** (1997) 584.

[5] UA5 Collaboration, R.E. Ansorge et al., Z. Phys. **C43** (1989) 357.

[6] T. Sjöstrand, Computer Phys. Commun. **82** (1994) 74;
T. Sjöstrand, P. Edén, C. Friberg, L. Lönnblad, G. Miu, S. Mrenna and E. Norrbin, LU TP 00–30 [hep-ph/0010017], to appear in Computer Phys. Commun.

[7] A. Donnachie and P.V. Landshoff, Phys. Lett. **B296** (1992) 227.

[8] P. Bartalini et al., in 'Proceedings of the Workshop on Standard Model Physics (and More) at the LHC', eds. G. Altarelli and M.L. Mangano, CERN 2000–004, p. 293.

[9] J. Dischler, LU TP 00-01 [hep-ph/0001237].

[10] F.E. Low, Phys. Rev. **D12** (1975) 163.

[11] J.F. Gunion and D.E. Soper, Phys. Rev. **D15** (1977) 2617.

[12] L.V. Gribov, E.M. Levin and M.G. Ryskin, Phys. Rep. **100** (1983) 1;
A.H. Mueller and J. Qiu, Nucl. Phys. **B268** (1986) 427.

[13] K.J. Eskola, K. Kajantie, P.V. Ruuskanen and K. Tuominen, Nucl. Phys. **B570** (2000) 379;
K.J. Eskola, K. Kajantie and K. Tuominen, hep-ph/0009246.

[14] P.V. Landshoff and O. Nachtmann, Z. Phys. **C35** (1987) 405.

[15] J.R. Cudell and D.A. Ross, Nucl. Phys. **B359** (1991) 247.

[16] J.E. Mandula and M. Ogilvie, Phys. Lett. **B185** (1987) 127.

[17] M. Jacob and P.V. Landshoff, Mod. Phys. Lett. **A1** (1986) 657.

[18] G. Gustafson and G. Miu, LU TP 99–43 [hep-ph/0002278].

[19] G. Altarelli and G. Parisi, Nucl. Phys. Rev. **B126** (1977) 298;
Yu.L. Dokshitzer, Sov. Phys. JETP **46** (1977) 641.

[20] G. Parisi and R. Petronzio, Phys. Lett **62B** (1976) 331;
V.A Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, JETP Lett. **24** (1976) 341; Ann. Phys. (N.Y.) **105** (1977) 276;
M. Glück and E. Reya: Nucl.Pys. **B130** (1977) 76.

[21] M. Glück, E. Reya and A. Vogt, Z. Phys **C67** (1995) 433.

[22] CTEQ Collaboration, H.L. Lai et al., Eur. Phys. J. **C12** (2000) 375;
A.D. Martin, R.G. Roberts, W.J. Stirling and R.S Thorne, Eur. Phys. J. **C14** (2000) 133.