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$B \to \pi\pi$ Decay in QCD

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Abstract

A new method is suggested to calculate the $B \to \pi\pi$ hadronic matrix elements from QCD light-cone sum rules. To leading order in α_s and $1/m_b$, the sum rule reproduces a factorizable matrix element, in accordance with the prediction of the QCD factorization approach. Whereas the QCD factorization can only take into account the nonfactorizable corrections induced by hard-gluon exchanges, the method suggested here also allows a systematic inclusion of soft-gluon effects. In this paper, I concentrate on the latter aspect and present a calculation of the nonfactorizable soft-gluon exchange contributions to $B \to \pi\pi$. The result, including twist 3 and 4 terms, is suppressed by one power of $1/m_b$ with respect to the factorizable amplitude. Despite its numerical smallness, the soft effect is at the same level as the $O(\alpha_s)$ correction to the QCD factorization formula for $B \to \pi\pi$. The method suggested here can be applied to matrix elements of different topologies and operators and to various other B-decay channels. I also comment on the earlier applications of QCD sum rules to nonleptonic decays of heavy mesons.

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1 Introduction

Nonleptonic two-body B decays such as $B \to \pi\pi$ [1] are very important tools for studying the mechanism of CP-violation (recent overviews of theory and experimental perspectives can be found in Ref. [2, 3]). In order to fully explore the data on these decays one needs reliable theoretical predictions on hadronic matrix elements of the operators entering the effective weak Hamiltonian

$$H_W = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \left\{ \left(c_1(\mu) + \frac{c_2(\mu)}{3} \right) O_1(\mu) + 2 c_2(\mu) \widetilde{O}_1(\mu) + ... \right\} . \tag{1}$$

The operators relevant for $B \to \pi\pi$ are

$$O_1 = (\bar{d}\Gamma_\mu u)(\bar{u}\Gamma^\mu b) , \quad \widetilde{O}_1 = (\bar{d}\Gamma_\mu \frac{\lambda^a}{2} u)(\bar{u}\Gamma^\mu \frac{\lambda^a}{2} b) , \tag{2}$$

and the penguin operators denoted by ellipses (for a comprehensive review of the effective Hamiltonian see, e.g., Ref. [4]). In the above $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$, $Tr(\lambda^a \lambda^b) = 2\delta^{ab}$, and $O_2 = (\bar{u}\Gamma_\mu u)(\bar{d}\Gamma^\mu b)$ has been Fierz transformed:

$$O_2 = \frac{1}{3} O_1 + 2\widetilde{O}_1 . \tag{3}$$

In Eq.(1), $\mu \sim m_b$ is the normalization scale, and all hard gluon effects at distances shorter than $1/\mu$ are taken into account in perturbative QCD in a form of the Wilson coefficients $c_{1,2,...}$. The remaining contributions at distances $> 1/\mu$ have to be included in the hadronic matrix elements, such as $\langle \pi\pi | O_i | B \rangle$. Even an approximate calculation of these matrix elements is a very difficult problem for the available QCD methods, in particular, for the lattice QCD. Additional complications arise due to the fact that, as a rule, several quark topologies (emission, annihilation, penguin, etc.) contribute to each matrix element (a general analysis can be found in Ref. [5]).

The method of QCD light-cone sum rules (LCSR) [6, 7, 8] offers a possibility to calculate exclusive hadronic amplitudes by matching them, via dispersion relations and quark-hadron duality, with the correlation functions of quark currents. These functions are evaluated in the spacelike region using OPE near the light-cone. Among other applications, various heavy-to-light hadronic form factors have been obtained by this method. Importantly, the predicted form factors include both soft (end-point) and hard rescattering contributions and reveal a regular $1/m_b$ expansion (for reviews of the method and applications, see Refs. [9, 10]). In this paper a new LCSR technique is suggested to calculate hadronic matrix elements for two-body nonleptonic B decays. The sum rule is obtained from the light-cone expansion of a three-point correlation function. As a study case we consider the $\bar{B}_d^0 \to \pi^+\pi^-$ channel.

One of the main objectives of this work is to quantitatively assess the factorization approximation for $B \to \pi\pi$. Recently, the QCD factorization approach has been developed [11] for two-body nonleptonic B decays. It was shown that in the limit of the heavy b quark mass the decay amplitudes can be treated in QCD using the framework of perturbative factorization for exclusive processes [12], with the heavy mass playing the same role as

the large momentum transfer. In the limit $m_b \to \infty$, the effects violating factorization in $B \to \pi\pi$ are suppressed either by powers of $\alpha_s(m_b)$ or by powers of Λ_{QCD}/m_b.

The $B \to \pi\pi$ amplitude obtained below from LCSR has a very similar structure to the one predicted from QCD factorization. It contains a factorizable part, where the $B \to \pi$ form factor is itself given by LCSR, including both soft and hard contributions. Furthermore, and it is the main result of this paper, the sum rule approach allows one to calculate the effects of soft gluons that violate factorization. In LCSR, these effects are generated by higher-twist contributions to the correlation function and are therefore under control. Hard nonfactorizable effects correspond to $O(\alpha_s)$ corrections to the same correlation function, but their estimate demand technically complicated two-loop calculations. The new method suggested here avoids certain problems of the earlier applications of QCD sum rules to nonleptonic D and B decays [13, 14, 15]. Moreover, we demonstrate that while the use of the light-cone OPE yields a $1/m_b$ behavior of soft higher-twist effects, the expansion in local operators employed before fails to reproduce the $1/m_b$ suppression.

This paper contains a description of the method and the first results for $B \to \pi\pi$. We confine ourselves to the contributions of the current-current operators $O_{1,2}$ in the emission topology. Analysis of other topologies (annihilation, penguin) and operators (quark and gluon penguins), as well as applications of the same method to other B decay channels will be published elsewhere.

The content of this paper is as follows. Section 2 contains a derivation of the LCSR for $B \to \pi\pi$. In Section 3 the factorizable part of the hadronic matrix element $\langle \pi^-\pi^+|O_1|\bar{B}_d^0\rangle$ is reproduced and the soft-gluon nonfactorizable effects determined by $\langle \pi^-\pi^+|\widetilde{O}_1|\bar{B}_d^0\rangle$ are calculated. Using these results, in Section 4 we discuss the influence of the soft nonfactorizable correction on the QCD factorization formula. Section 5 is devoted to the comments on the earlier use of QCD sum rules for nonleptonic decays of heavy mesons. A summary is given in Section 6.

2 The method

In what follows, we concentrate on the $\bar{B}_d^0 \to \pi^+\pi^-$ hadronic matrix elements of the operators O_1 and $\widetilde{O}_1$. As a starting object for the derivation of LCSR we choose the following vacuum-pion correlation function:

$$F_\alpha^{(O)}(p,q,k) = -\int d^4x\, e^{i(p-q)x} \int d^4y\, e^{i(p-k)y} \langle 0 \mid T\{j_{\alpha 5}^{(\pi)}(y) O(0) j_5^{(B)}(x)\} \mid \pi^-(q)\rangle\,, \qquad (4)$$

where $j_{\alpha 5}^{(\pi)} = \bar{u}\gamma_\alpha\gamma_5 d$ and $j_5^{(B)} = m_b \bar{b} i\gamma_5 d$ are the quark currents interpolating π and B mesons, respectively, and O is either O_1 or $\widetilde{O}_1$. The correlator (4) is a function of three independent momenta, which are chosen to be q, $p-k$ and k. The pion is on-shell, $q^2 = m_\pi^2$. We will work in the chiral limit and set $m_\pi = 0$ everywhere, unless there is a chirally enhanced combination $m_\pi^2/(m_u + m_d)$.

The four-momentum $k \neq 0$ introduced in Eq. (4) is unphysical because in reality there is no external momentum flow from the weak operator vertex. This momentum is added deliberately, otherwise in the correlator the quark states before and after the b-quark decay will have one and the same four-momentum, and the dispersion relation in

the variable of this momentum squared will contain, below the pole of the ground state B meson, a continuum of light "parasitic" contributions. At $k \neq 0$, the momentum $p-q$ in the B channel is independent of $P \equiv p-k-q$, the total momentum of the state formed after the b quark decay. As we shall see below, the procedure is designed in such a way that in the final dispersion relation the 4-momentum k automatically vanishes in the $B \to \pi\pi$ ground-state contribution.

The decomposition of the correlation function (4) in independent momenta is straightforward and contains four invariant amplitudes:

$$F_\alpha^{(O)} = (p-k)_\alpha F^{(O)} + q_\alpha \widetilde{F}_1^{(O)} + k_\alpha \widetilde{F}_2^{(O)} + \epsilon_{\alpha\beta\lambda\rho} q^\beta p^\lambda k^\rho \widetilde{F}_3^{(O)} \,. \tag{5}$$

In what follows only the amplitude $F^{(O)}$ is relevant. The correlation function (4) at $k \neq 0$ is, in fact, a $2 \to 2$ scattering amplitude. Therefore, counting independent kinematical invariants, one has to add two additional variables P^2 and p^2 to the four external momenta squared $q^2 = 0$, $(p-q)^2$, k^2, and $(p-k)^2$.

The correlation function is calculated in QCD by expanding the T-product of three operators, two currents and O, near the light-cone $x^2 \sim y^2 \sim (x-y)^2 \sim 0$. For this expansion to be valid, the kinematical region should be carefully chosen, in order to stay far away from hadronic thresholds in both channels of π and B currents. Consequently, the external momenta squared $(p-q)^2$ and $(p-k)^2$, as well as the kinematical invariant P^2 have to be taken spacelike and large. Simultaneously, to simplify calculations, it is possible to put the remaining external momentum squared k^2 and the kinematical invariant p^2 to zero having in mind that hadronic states in the channels with four-momenta k and p contain heavy b quark and, therefore, the corresponding thresholds are located far enough from the zero points. To summarize, our choice of the kinematical region in (4) is

$$q^2 = p^2 = k^2 = 0 \,, \quad |(p-k)^2| \sim |(p-q)^2| \sim |P^2| \gg \Lambda_{QCD}^2 \,. \tag{6}$$

In this region the light-cone expansion of the correlation function (4) is applicable and the result is obtained in terms of hard scattering amplitudes convoluted with the pion light-cone distribution amplitudes of different twist. The actual calculation is described in the next section. At this point, it suffices to represent the result of the QCD calculation in a generic form of a dispersion relation in the variable $(p-k)^2$:

$$F_{QCD}^{(O)}((p-k)^2, (p-q)^2, P^2) = \frac{1}{\pi} \int\limits_0^\infty ds \; \frac{\mathrm{Im} F_{QCD}^{(O)}(s, (p-q)^2, P^2)}{s-(p-k)^2} \,. \tag{7}$$

To proceed with the derivation, we obtain a corresponding hadronic dispersion relation, inserting in Eq. (4) a complete set of intermediate states with the pion quantum numbers:

$$F^{(O)}((p-k)^2, (p-q)^2, P^2) = \frac{i f_\pi \Pi_{\pi\pi}^{(O)}((p-q)^2, P^2)}{-(p-k)^2} + \int\limits_{s_h^{(\pi)}}^\infty ds \; \frac{\rho_h^{(\pi)}(s, (p-q)^2, P^2)}{s-(p-k)^2} \,, \tag{8}$$

where the lowest, one-pion contribution contains the pion decay constant

$$\langle 0 | \bar{u} \gamma_\alpha \gamma_5 d | \pi(p-k) \rangle = i f_\pi (p-k)_\alpha \,, \tag{9}$$

and the hadronic matrix element

$$\Pi^{(O)}_{\pi\pi}((p-q)^2,P^2) = i\int d^4x\; e^{i(p-q)x}\langle\pi^-(p-k)|T\{O(0)j_5^{(B)}(x)\}|\pi^-(q)\rangle\,, \qquad (10)$$

which itself is a two-point correlation function. The spectral function $\rho_h^{(\pi)}$ in Eq. (8) accounts for the contributions of excited states and continuum in the pion channel, $s_h^{(\pi)}$ being the lowest threshold. Subtraction terms in both dispersion relations (7) and (8) are omitted in anticipation of the Borel transformation which will remove them anyway. Physically, the two-point function $\Pi^{(O)}_{\pi\pi}$ defined in Eq. (10) describes a scattering process. In more detail, the pion undergoes a deep elastic scattering due to a combined action of the current $j_5^{(B)}$ and operator O, with a total spacelike momentum transfer P^2. To obtain $\Pi^{(O)}_{\pi\pi}$ we follow the usual derivation procedure of QCD sum rules [16] and match the dispersion relation (8) with the result (7) of the light-cone expansion at large $|(p-k)^2|$. Furthermore, the integral over $\rho_h^{(\pi)}$ is approximated using the quark-hadron duality:

$$\int\limits_{s_h^{(\pi)}}^{\infty} ds\;\frac{\rho_h^{(\pi)}(s,(p-q)^2,P^2)}{s-(p-k)^2} = \frac{1}{\pi}\int\limits_{s_0^\pi}^{\infty} ds\;\frac{\mathrm{Im}F^{(O)}_{QCD}(s,(p-q)^2,P^2)}{s-(p-k)^2}\,, \qquad (11)$$

where s_0^π is an effective threshold parameter. After the Borel transformation in the variable $(p-k)^2$ the result reads

$$\Pi^{(O)}_{\pi\pi}((p-q)^2,P^2) = -\frac{i}{\pi f_\pi}\int\limits_0^{s_0^\pi} ds\; e^{-s/M^2}\mathrm{Im}_s F^{(O)}_{QCD}(s,(p-q)^2,P^2)\,. \qquad (12)$$

Having at hand an expression for $\Pi^{(O)}_{\pi\pi}((p-q)^2,P^2)$ valid at large spacelike P^2, we perform an analytic continuation to large timelike values of P^2, keeping the variable $(p-q)^2$ fixed. This procedure is very similar to the evaluation of the timelike asymptotics of the pion electromagnetic form factor from a QCD calculation at large spacelike momentum transfer. For $\Pi^{(O)}_{\pi\pi}$ a natural and convenient point of the analytic continuation is $P^2=m_B^2$ where the invariant mass of two pions is equal to the B meson mass which is sufficiently large, $m_B\sim m_b\gg\Lambda_{QCD}$. The analytic continuation of Eqs. (10) and (12) yields a relation

$$\begin{aligned}\Pi^{(O)}_{\pi\pi}((p-q)^2,m_B^2) &= i\int d^4x\; e^{i(p-q)x}\langle\pi^-(p-k)\pi^+(-q)|T\{O(0)j_5^{(B)}(x)\}|0\rangle\\ &= \frac{-i}{\pi f_\pi}\int\limits_0^{s_0^\pi} ds\; e^{-s/M^2}\mathrm{Im}_s F^{(O)}_{QCD}(s,(p-q)^2,m_B^2) \qquad (13)\end{aligned}$$

for the hadronic matrix element which is crossing-related to the one defined in Eq. (10) and corresponds to an amplitude of two-pion production with a large invariant mass m_B^2. This process is very similar to $\gamma^*\to\pi^+\pi^-$ but has a more complicated underlying quark-current structure. Note also that the analytic continuation to timelike P^2 can generate in $\Pi^{(O)}_{\pi\pi}((p-q)^2,m_B^2)$ a complex phase, which should be identified, within the accuracy of our calculation, with the phase of the final-state rescattering of two pions.

The next step in the derivation procedure employs analytic properties of the two-pion amplitude (13) in the remaining spacelike variable $(p-q)^2$. Inserting in Eq. (13)

a complete set of hadronic states with the B meson quantum numbers one obtains the following dispersion relation:

$$\Pi^{(O)}_{\pi\pi}((p-q)^2, m_B^2) = \frac{f_B m_B^2 \langle \pi^-(p)\pi^+(-q)|O|B(p-q)\rangle}{m_B^2 - (p-q)^2} + \int\limits_{s_h^B}^{\infty} ds' \frac{\rho_h^{(B)}(s')}{s' - (p-q)^2}\,, \qquad (14)$$

where the standard definition of the B-meson decay constant $\langle B|j_5|0\rangle = m_B^2 f_B$ is used. Note that in the ground-state contribution the auxiliary momentum k vanishes, due to simultaneous conditions $(p-q-k)^2 = m_B^2$ and $(p-q)^2 = m_B^2$, so that the on-shell $B \to \pi\pi$ matrix element of the operator O is recovered. In the spectral density of excited and continuum states k remains nonzero and depends on the value of $s' > m_B^2$. At large $|(p-q)^2|$ the relation (14) is matched with the result of the QCD calculation given by the r.h.s. of Eq. (13) which is rewritten in a form of a dispersion relation:

$$\Pi^{(O)}_{\pi\pi}((p-q)^2, m_B^2) = \frac{-i}{\pi^2 f_\pi} \int\limits_0^{s_0^\pi} ds\, e^{-s/M^2} \int\limits_{m_b^2}^{R(s,m_b^2,m_B^2)} \frac{ds'}{s' - (p-q)^2}\, \mathrm{Im}_{s'}\mathrm{Im}_s F^{(O)}_{QCD}(s, s', m_B^2). \qquad (15)$$

The upper limit R of the integration in s' depends, in general, on s and on $P^2 = m_B^2$. At this point one again makes use of quark-hadron duality and approximates the dispersion integral over the spectral density $\rho_h^{(B)}$ by the $s' \geq s_0^B$ part of the dispersion integral (15), where s_0^B is the effective threshold in the B channel. Finally, the second Borel transformation with respect to the variable $(p-q)^2$ is performed. The resulting LCSR for the $\bar{B}_d^0 \to \pi^+\pi^-$ matrix element of the operator O reads:

$$\begin{aligned} & A^{(O)}(\bar{B}_d^0 \to \pi^+\pi^-) \equiv \langle \pi^-(p)\pi^+(-q)|O|B(p-q)\rangle \\ = \; & \frac{-i}{\pi^2 f_\pi f_B m_B^2} \int\limits_0^{s_0^\pi} ds\; e^{-s/M^2} \int\limits_{m_b^2}^{\bar{R}(s,m_b^2,m_B^2,s_0^B)} ds'\; e^{(m_B^2 - s')/M'^2} \mathrm{Im}_{s'}\, \mathrm{Im}_s F^{(O)}_{QCD}(s, s', m_B^2)\,, \qquad (16) \end{aligned}$$

where $\bar{R}$ is the upper limit after the duality subtraction in the dispersion integral.

3 $\bar{B}_d^0 \to \pi^+\pi^-$ hadronic matrix elements

3.1 Factorizable part

The remaining task is to calculate $F^{(O)}_{QCD}$ for $O = O_1, \widetilde{O}_1$. At the diagrammatical level, there are four topologically different contributions to the correlation function (4), corresponding to four possible combinations of $\bar{u}$ and d quark-field operators in the pion distribution amplitude $\langle 0 \mid \bar{u}_\alpha(z_1) d_\beta(z_2) \mid \pi^- \rangle$, where $z_1 = 0$ or y, and $z_2 = x$ or y (α, β are the spinor indices). Drawing the quark diagrams we find that each contribution yields a $B \to \pi\pi$ matrix element with a certain quark topology: emission ($z_1 = 0$, $z_2 = x$), annihilation ($z_1 = 0$, $z_2 = y$), penguin ($z_1 = y$, $z_2 = x$) and penguin annihilation ($z_1 = z_2 = y$). In what follows we will concentrate on the emission topology adding a subscript E to the corresponding quantities. Some of the diagrams contributing to $F^{(O)}_{\alpha,E}$

are shown in Figs. 1,2. In the case of the operator O_1 there are diagrams, where the quarks belonging to the heavy-light currents do not interact with the quarks of the light-quark currents. The leading-order diagram of this type is shown in Fig. 1a. To single out the contribution of this and all other "factorizable" diagrams, we insert an intermediate vacuum state between the weak currents of the operator O_1. Eq. (4) is then converted into a product of two disconnected two-point correlation functions:

$$F_{\alpha E}^{(O_1)}(p,q,k) = \left(i \int d^4y \; e^{i(p-k)y} \langle 0 \mid T\{ j_{\alpha 5}^{(\pi)}(y) \bar{d}(0) \gamma_\mu \gamma_5 u(0) \} \mid 0 \rangle \right) \times \left(i \int d^4x \; e^{i(p-q)x} \langle 0 \mid T\{ \bar{u}(0) \gamma^\mu b(0) j_5^{(B)}(x) \} \mid \pi^-(q) \rangle \right), \tag{17}$$

where, due to spin-parity conservation, $\Gamma_\mu \to \gamma_\mu \gamma_5$ ($\Gamma_\mu \to \gamma_\mu$) in the first (second) weak current. To proceed further, we insert complete sets of intermediate states with the pion and B-meson quantum numbers in the first and second correlators in Eq. (17), respectively. After that, the matrix element $\langle \pi^-(p) \pi^+(-q) | O | B(p-q) \rangle$ is extracted in the usual factorized form:

$$A_E^{(O_1)}(\bar{B}_d^0 \to \pi^+ \pi^-) = i f_\pi f_{B\pi}^+(0) m_B^2 \,, \tag{18}$$

where $f_{B\pi}^+$ is the $B \to \pi$ form factor defined as

$$\langle \pi(q) | \bar{u} \gamma_\mu b | B(p+q) \rangle = 2 f_{B\pi}^+(p^2) q_\mu + (f_{B\pi}^+(p^2) + f_{B\pi}^-(p^2)) p_\mu \,. \tag{19}$$

and taken at $p^2 = m_\pi^2 \simeq 0$. Correction of $O(m_\pi^2/m_B^2)$ to Eq. (18) is neglected in the chiral limit adopted here.

Within the sum rule method, it is possible to calculate the r.h.s. of Eq. (18) from Eq. (17), with a certain accuracy. In leading-order, $F_{\alpha E}^{(O_1)}$ is given by the diagram in Fig. 1a. The answer is easily obtained by forming the pion distribution amplitude $\langle 0 \mid \bar{u}_\alpha(0) d_\beta(x) \mid \pi^- \rangle$, contracting the remaining quark fields in both correlation functions and using free-quark propagators. Integrating over the coordinates x, y we obtain:

$$F_E^{(O_1)} = -\frac{m_b^2 f_\pi}{4\pi^2} q \cdot (p-k) \ln\left(-(p-k)^2\right) \int_0^1 du \frac{\varphi_\pi(u)}{m_b^2 - (p - q(1-u))^2} \,. \tag{20}$$

In the above, the logarithm corresponds to the bare-loop approximation of the first (vacuum-vacuum) correlation function in Eq. (17) [1], whereas the integral represents the second (vacuum-pion) correlation function. In the latter function, only the leading twist-2 part is retained, and the pion distribution amplitude φ_π is defined in a standard way:

$$\langle 0 | \bar{u}(0) \gamma_\mu \gamma_5 d(x) | \pi(q) \rangle = i q_\mu f_\pi \int_0^1 du \; e^{-iuq \cdot x} \varphi_\pi(u,\mu) \,, \tag{21}$$

where

$$\varphi_\pi(u,\mu) = 6u(1-u) \left[1 + \sum_{n=2,4,..} a_n(\mu) C_n^{3/2}(2u-1) \right] , \tag{22}$$

[1] The divergent part of the loop vanishes after the Borel transformation and is therefore omitted from Eq. (20).

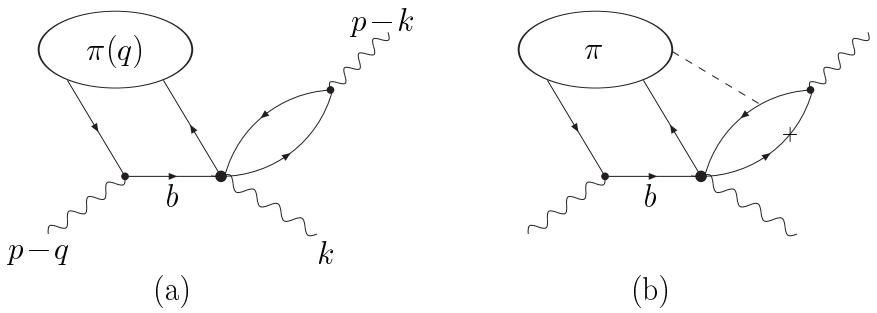

Figure 1: *Diagrams corresponding (a) to the leading-order of the correlation function (4) for $O = O_1$; (b) to the higher-twist soft-gluon nonfactorizable contribution for $O = \widetilde{O}_1$. Solid, dashed and wavy lines represent quarks, gluons, and external momenta, respectively. Thick points denote the weak interaction vertices, and ovals the pion distribution amplitudes. The cross indicates the point of gluon emission in the second similar diagram.*

$C_n^{3/2}$ are the Gegenbauer polynomials and a_n are multiplicatively renormalizable coefficients depending on the normalization scale μ. Following the procedure described in Sect. 2 and applying, subsequently, duality with Borel transformation in the pion channel, analytic continuation $P^2 \to m_B^2$ and duality with Borel transformation in the B-meson channel, one obtains:

$$\begin{aligned} A_{E,tw2}^{(O_1)}(\bar{B}_d^0 \to \pi^+\pi^-) &= im_B^2 \left(\frac{1}{4\pi^2 f_\pi} \int_0^{s_0^\pi} ds \left(1 - \frac{s}{m_B^2}\right) e^{-s/M^2} \right) \\ &\quad \times \left(\frac{m_b^2 f_\pi}{2 f_B m_B^2} \int_{u_0^B}^1 \frac{du}{u}\, e^{\,m_B^2/M'^2 - m_b^2/uM'^2} \varphi_\pi(u,\mu_b) \right), \end{aligned} \tag{23}$$

where $u_0^B = m_b^2/s_0^B$ and μ_b is an appropriate scale. In this expression, the first bracket, up to a small $O(s/m_B^2) \sim O(s_0^\pi/m_B^2)$ correction, is approximately equal to f_π, because the integral over s gives the quark-loop (leading order) term of the SVZ sum rule [16] for f_π^2. The second bracket in Eq. (23) coincides with the leading twist 2 term in the LCSR [8] for the form factor $f_{B\pi}^+$. Thus, the factorizable matrix element (18) is restored at the leading-order level. Factorizable corrections in both correlation functions entering Eq. (17) can be added, improving the accuracy of Eq. (23). The LCSR for the form factor $f_{B\pi}^+$ [17] contains, in addition to the twist 2 term, the twist 3, 4 contributions corresponding to two- and three-particle distribution amplitudes of the pion. The numerically important twist 3 contribution is proportional to the chirally enhanced coefficient $m_\pi^2/(m_u + m_d)$. In addition, the $O(\alpha_s)$ radiative correction to the twist 2 part has been calculated [18, 19]. A certain part of this correction [19] corresponds to the hard rescattering mechanism. Summarizing, the hadronic matrix element of the operator O_1 in the emission topology is given by Eq. (18) where $f_{B\pi}^+$ itself is calculated from LCSR with an available accuracy.

3.2 Nonfactorizable effects

In the case of the operator O_1, nonfactorizable corrections to the correlation function (4) emerge at a two-gluon level and are neglected here. Calculation of the matrix element $\langle\pi^-\pi^+|\widetilde{O}_1|B\rangle$ brings nonfactorizable effects at a one-gluon level into the game. The relevant correlation function $F_\alpha^{(\widetilde{O}_1)}$ receives contributions of hard gluon exchanges which, at $O(\alpha_s)$, correspond to the diagrams in Fig.2. These two-loop diagrams can be calculated in QCD perturbation theory, but this task is beyond the scope of this paper. In future, it is important to obtain this part of the correlation function, not only for the sake of improving the accuracy of the LCSR, but also for other important reasons. First, at $O(\alpha_s)$, the matrix element should partially compensate the scale-dependence of the short-distance coefficients $c_{1,2}$ in the effective weak Hamiltonian. Second, the analytic continuation of these two-loop contributions in P^2 will generate an imaginary part, yielding a final-state rescattering phase in the decay amplitude. Third, it will be interesting to compare the $O(\alpha_s)$ part of the LCSR with the hard nonfactorizable corrections obtained in the QCD factorization approach [11]. In fact, there is a certain correspondence between the diagrams in Fig. 2 and the quark diagrams contributing to the hard-scattering kernels of the QCD factorization. More specifically, the diagrams in Fig. 2a,b, where the hard gluon is exchanged between the quarks participating in the weak interaction, are of the non-spectator type, whereas the diagrams in Fig. 2c correspond to the hard spectator contribution. In particular, it would be instructive to check if at $m_b \to \infty$ the form factor $f_{B\pi}^+$ factorizes out in the non-spectator part of LCSR, as it does in the QCD factorization formula. [2]

There is another class of nonfactorizable effects contributing to $F^{(\widetilde{O}_1)}$, with on-shell gluons emitted from the quarks of the pion current and absorbed in the pion distribution amplitude, as shown in Fig. 1b. In terms of the light-cone expansion these contributions are of a higher twist, starting from twist 3. The diagrams in Fig. 1b are calculated employing the light-cone expansion of the quark propagator [20]:

$$S(x,0) \equiv -i\langle 0|T\{q(x)\bar{q}(0)\}|0\rangle = \frac{\Gamma(d/2)\ \not{x}}{2\pi^2(-x^2)^{d/2}}$$
$$+ \frac{\Gamma(d/2-1)}{16\pi^2(-x^2)^{d/2-1}}\int_0^1 dv\Big\{(1-v)\ \not{x}\sigma_{\mu\nu}G^{\mu\nu}(vx) + v\sigma_{\mu\nu}G^{\mu\nu}(vx)\ \not{x}\Big\} + ..., \quad (24)$$

where $G_{\mu\nu} = g_s G^a_{\mu\nu}(\lambda^a/2)$, and d is the space-time dimension. In the above, the fixed-point gauge for the gluon field is adopted and only the terms proportional to the one gluon-field strength are shown, which is the accuracy we need here.

The relevant quark-antiquark-gluon distribution amplitudes are defined by the matrix elements [21]

$$\langle 0|\bar{u}(0)\sigma_{\mu\nu}\gamma_5 G_{\alpha\beta}(vy)d(x)|\pi(q)\rangle = if_{3\pi}[(q_\alpha q_\mu g_{\beta\nu} - q_\beta q_\mu g_{\alpha\nu})$$
$$-(q_\alpha q_\nu g_{\beta\mu} - q_\beta q_\nu g_{\alpha\mu})]\int \mathcal{D}\alpha_i\,\varphi_{3\pi}(\alpha_i,\mu)e^{-iq(x\alpha_1+yv\alpha_3)}\,, \quad (25)$$

[2] Differences may arise due to the fact that in the LCSR approach one of the pions and B meson are represented by the integrals over quark spectral densities. In these integrals, quark transverse momenta up to $O(s_0^\pi)$ and $O(s_0^B)$, respectively, are taken into account, whereas in QCD factorization these hadrons are described by collinear light-cone distribution amplitudes.

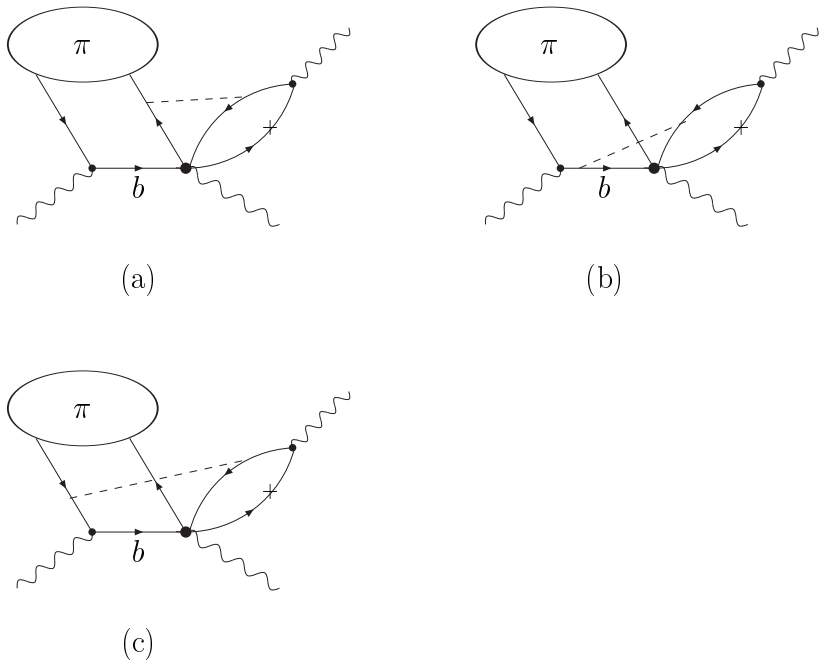

Figure 2: *Diagrams corresponding to the $O(\alpha_s)$ nonfactorizable contributions to the correlation function (4) for $O = \widetilde{O}_1$.*

$$\langle 0|\bar{u}(0)i\gamma_\mu \widetilde{G}_{\alpha\beta}(vy)d(x)|\pi(q)\rangle = q_\mu \frac{q_\alpha x_\beta - q_\beta x_\alpha}{qx} f_\pi \int \mathcal{D}\alpha_i \widetilde{\varphi}_{\|}(\alpha_i,\mu) e^{-iq(x\alpha_1 + yv\alpha_3)}$$
$$+ (g^\perp_{\mu\alpha} q_\beta - g^\perp_{\mu\beta} q_\alpha) f_\pi \int \mathcal{D}\alpha_i \widetilde{\varphi}_\perp(\alpha_i,\mu) e^{-iq(x\alpha_1 + yv\alpha_3)}, \qquad (26)$$

and the one analogous to Eq. (26) with $i\gamma_\mu \to \gamma_\mu\gamma_5$, $\widetilde{G}_{\alpha\beta} \to G_{\alpha\beta}$ and $\widetilde{\varphi}_{\|,\perp} \to \varphi_{\|,\perp}$. In the above, $\widetilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\rho\lambda}G^{\rho\lambda}$, $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta\,(1-\alpha_1-\alpha_2-\alpha_3)$, and $g^\perp_{\alpha\beta} = g_{\alpha\beta} - (x_\alpha q_\beta + x_\beta q_\alpha)/qx$. The pion distribution amplitude $\varphi_{3\pi}$ is of twist 3, whereas $\widetilde{\varphi}_{\|,\perp}$ and $\varphi_{\|,\perp}$ are of twist 4. In what follows we suppress the μ dependence for brevity.

A straightforward calculation of the two diagrams in Fig. 1b yields the following answer for the twist 3 contribution:

$$F^{(\widetilde{O}_1)}_{E,tw3} = \frac{m_b f_{3\pi}}{4\pi^2} \int_0^1 dv \int \mathcal{D}\alpha_i \; \frac{\varphi_{3\pi}(\alpha_i)}{(m_b^2 - (p-q)^2(1-\alpha_1))(-P^2 v\alpha_3 - (p-k)^2(1-v\alpha_3))}$$
$$[(2-v)(q\cdot k) + 2(1-v)q\cdot(p-k)]\,(q\cdot(p-k))\,, \qquad (27)$$

which is then easily transformed into a form of the dispersion integral (7):

$$F^{(\widetilde{O}_1)}_{E,\ tw3} = -\frac{m_b f_{3\pi}}{16\pi^2} \int_0^\infty \frac{ds}{s-(p-k)^2} \int_{s/(s-P^2)}^1 \frac{du}{m_b^2 - (p-q)^2 u}$$

$$\times \int\limits_{s/(s-P^2)}^{u} \frac{dv}{v^2}\,\varphi_{3\pi}(1-u,u-v,v)\left[s+\frac{m_b^2}{u}\left(2v-\frac{s}{s-P^2}\right)\right], \qquad (28)$$

For simplicity, all terms vanishing after Borel transformations are omitted in this expression. In the part of Eq. (28) which is dual to a pion, the upper limit of the first integral is $s_0^\pi \ll |P^2| \sim m_b^2$. Hence, the resulting expression for the two-pion matrix element $\Pi_{\pi\pi}^{(\widetilde{O}_1)}((p-q)^2,P^2)$ can be simplified by expanding it in powers of s/P^2, and neglecting very small $O(s/P^2)\sim O(s_0^\pi/P^2)$ corrections:

$$\Pi_{\pi\pi,\ tw3}^{(\widetilde{O}_1)}((p-q)^2,P^2) = \frac{im_b^3 f_{3\pi}}{8\pi^2 f_\pi}\int\limits_0^{s_0^\pi} ds\ e^{-s/M^2}\int\limits_0^1 \frac{du}{u(m_b^2-(p-q)^2u)}$$
$$\times\int\limits_0^u \frac{dv}{v}\varphi_{3\pi}(1-u,u-v,v)\left\{1+O\left(s_0^\pi/P^2\right)\right\}. \qquad (29)$$

The analytic continuation $P^2\to m_B^2$ is then trivial. The twist 4 contribution to $\Pi_{\pi\pi}^{(\widetilde{O}_1)}$ has a structure similar to Eq. (29) but a bulky expression not shown here for brevity. Finally, the LCSR for the $\bar{B}_d^0\to\pi^+\pi^-$ matrix element of the operator $\widetilde{O}_1$ is obtained applying to $\Pi_{\pi\pi}^{(\widetilde{O}_1)}((p-q)^2,m_B^2)$ the duality approximation and Borel transformation in the B channel. The result is:

$$A_E^{(\widetilde{O}_1)}(\bar{B}_d^0\to\pi^+\pi^-) = im_B^2\left(\frac{1}{4\pi^2 f_\pi}\int\limits_0^{s_0^\pi} ds\ e^{-s/M^2}\right)\left(\frac{m_b^2}{2f_B m_B^4}\int\limits_{u_0^B}^1 \frac{du}{u} e^{\,m_B^2/M'^2-m_b^2/uM'^2}\right.$$
$$\times\left[\frac{m_b f_{3\pi}}{u}\int\limits_0^u\frac{dv}{v}\,\varphi_{3\pi}(1-u,u-v,v)+f_\pi\int\limits_0^u\frac{dv}{v}\,\left[3\widetilde{\varphi}_\perp(1-u,u-v,v)\right.\right.$$
$$\left.\left.-\left(\frac{m_b^2}{uM^2}-1\right)\frac{\Phi_1(1-u,v)}{u}\right]+\left(\frac{m_b^2}{uM'^2}-\frac{s}{M'^2}-1\right)\frac{\Phi_2(u)}{u^2}\right]\left.\right)\left\{1+O(s_0^\pi/m_B^2)\right\}, \qquad (30)$$

where the following definitions are introduced:

$$\frac{\partial\Phi_1(w,v)}{\partial w} = \widetilde{\varphi}_\perp(w,1-w-v,v)+\widetilde{\varphi}_\parallel(w,1-w-v,v), \quad \frac{\partial\Phi_2(v)}{\partial v} = \Phi_1(1-v,v). \qquad (31)$$

The asymptotic form of the pion distribution amplitudes in Eq. (30) is given by [21]

$$\varphi_{3\pi}(\alpha_i) = 360\alpha_1\alpha_2\alpha_3^2, \quad \tilde{\varphi}_\perp(\alpha_i) = 10\delta^2\alpha_3^2(1-\alpha_3), \quad \tilde{\varphi}_\parallel(\alpha_i) = -40\delta^2\alpha_1\alpha_2\alpha_3\,. \qquad (32)$$

More complete expressions containing scale-dependent nonasymptotic terms can be found in Ref. [9].

A few comments are in order concerning LCSR (30). Firstly, the magnitude of the nonfactorizable effect predicted by this sum rule is determined by the two nonperturbative parameters $f_{3\pi}$ and δ^2. They are defined by the vacuum-pion matrix elements $\langle 0|\bar{u}\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}d|\pi\rangle$ and $\langle 0|\bar{u}g_s\widetilde{G}_{\alpha\mu}\gamma^\alpha d|\pi\rangle$, respectively. Secondly, at leading order in s_0^π/m_B^2, the matrix element (30) factorizes into f_π and an expression which has a typical

structure of the LCSR for a heavy-light form factor. Finally, the analytic continuation in P^2 does not produce an imaginary part in the nonfactorizable matrix element $A_E^{(\widetilde{O}_1)}$. This fact can be interpreted as an absence of a final-state rescattering phase induced by soft gluon exchanges.

3.3 Heavy quark limit

The heavy-quark mass behavior of the matrix elements (23) and (30) can be easily derived. To this end, one has to substitute in the LCSR the standard expansions of quantities dependent on the heavy quark mass:

$$m_B = m_b + \bar{\Lambda} \ , \quad s_0^B = m_b^2 + 2m_b\omega_0 \ , \quad M'^2 = 2m_b\tau \ , \quad f_B = \hat{f}_B/\sqrt{m_b}, \tag{33}$$

where $\bar{\Lambda}$, ω_0, τ, and $\hat{f}_B$ are m_b-independent quantities. One also has to take into account the end-point behavior of the light-cone distribution amplitudes, since at $m_b \to \infty$, $u_B^0 \to 1 - \omega_0/m_b$ and only the end-point regions of the momentum fraction u contribute to Eqs. (23) and (30). As a result we get:

$$A_{E,tw2}^{(O_1)}(\bar{B}_d^0 \to \pi^+\pi^-) \sim \sqrt{m_b}, \quad A_E^{(\widetilde{O}_1)}(\bar{B}_d^0 \to \pi^+\pi^-) \sim \frac{1}{\sqrt{m_b}} \, . \tag{34}$$

Thus, according to LCSR, the higher-twist nonfactorizable effects are suppressed by one power of $1/m_b$ with respect to the twist 2 factorizable amplitude.

3.4 Numerical estimates

The sum rule (30) is derived for a finite b-quark mass, and it is therefore possible to obtain a numerical estimate of the nonfactorizable matrix element $A_E^{(\widetilde{O}_1)}(\bar{B}_d^0 \to \pi^+\pi^-)$ and compare it with the factorizable approximation (18).

To specify the input, we take $f_\pi = 132$ MeV, $s_0^\pi = 0.7$ GeV^2 and $M^2 = 0.5 \div 1.2$ GeV^2 for the parameters of the pion channel. The value of the effective threshold and the interval of the Borel parameter are provided by the two-point QCD sum rule [16] for f_π. For the parameters of the B-meson channel and for the pion distribution amplitudes we adopt the same input as the one used in LCSR for the form factor $f_{B\pi}^+$ (for a recent update see Ref. [22]): $f_B = 180 \pm 30$ MeV, $m_b = 4.7 \mp 0.1$ GeV, $s_0^B = 35 \pm 2$ GeV^2 (the values obtained from the QCD sum rule for f_B), $\mu_b = \sqrt{m_B^2 - m_b^2} \simeq 2.4$ GeV, $M'^2 = 10 \pm 2$ GeV^2, $f_{3\pi}(\mu_b) = 0.0026$ GeV^2 [7, 23] and $\delta^2(\mu_b) = 0.17$ GeV^2 [24, 25]. The shapes of distribution amplitudes are varied between the BF ansatz of nonasymptotic coefficients [7] (where, in particular, $a_2(1\text{GeV}) = 0.44$, $a_4(1\text{GeV}) = 0.25$ and all $a_{n>4} = 0$) and purely asymptotic form. With this input one, in particular, obtains [22]

$$f_{B\pi}^+(0) = 0.28 \pm 0.5. \tag{35}$$

To quantify the magnitude of the nonfactorizable soft-gluon effect, we introduce the ratio:

$$\frac{\lambda_E(\bar{B}_d^0 \to \pi^+\pi^-)}{m_B} \equiv \frac{A_E^{(\widetilde{O}_1)}(\bar{B}_d^0 \to \pi^+\pi^-)}{A_E^{(O_1)}(\bar{B}_d^0 \to \pi^+\pi^-)}, \tag{36}$$

and obtain the following estimate

$$\lambda_E(\bar{B}_d^0 \to \pi^+\pi^-) = 0.05 \div 0.15 \text{ GeV}\,, \tag{37}$$

The quoted interval corresponds to the variation of all input parameters within adopted ranges and adding linearly the corresponding uncertainties. The parameter λ_E is independent of f_B and less sensitive to the input than the individual matrix elements in Eq. (36). The main uncertainty in λ_E is introduced by the nonperturbative parameters $f_{3\pi}$ and δ^2. They are extracted from two-point sum rules with a limited accuracy of about $\pm 30\%$.

The soft-gluon contribution to the matrix element $\langle \pi^+\pi^-|\widetilde{O}_1|\bar{B}_d^0\rangle$ turns out to be very small, at the level of $1 \div 2\%$ of the factorizable matrix element $\langle \pi^+\pi^-|O_1|\bar{B}_d^0\rangle$. In fact, this effect is at the same level as the twist 3,4 contributions of quark-antiquark-gluon distribution amplitudes to the LCSR for the form factor $f_{B\pi}^+$ [17].

4 Soft-gluon correction to the QCD factorization formula

For the $\bar{B}_d^0 \to \pi^+\pi^-$ amplitude, the QCD factorization formula reads [11]:

$$\begin{aligned}\mathcal{A}(\bar{B}_d^0 \to \pi^+\pi^-) \;\equiv\; & \langle \pi^-\pi^+|H_W|\bar{B}_d^0\rangle = i\frac{G_F}{\sqrt{2}} V_{ub}V_{ud}^* f_\pi f_{B\pi}^+(0) m_B^2 \\ & \times \left\{ c_1(\mu) + \frac{c_2(\mu)}{3} + \frac{\alpha_s}{9\pi} c_2(\mu) F(\mu) + O\left(\frac{\Lambda_{QCD}}{m_b}\right)\right\}\,, \end{aligned} \tag{38}$$

where, in order to simplify the discussion, we neglect the matrix elements of penguin operators/topologies. The hard nonfactorizable contributions of $O(\alpha_s)$ parametrized by F have been explicitly calculated [11], in a form of the convolution of hard-scattering kernels with the pion and B-meson distribution amplitudes [3].

The form factor $f_{B\pi}^+$ in Eq. (38) includes not only hard, but also soft contributions [4]. The latter are not accessible in QCD perturbation theory. Therefore, $f_{B\pi}^+$ represents an external input to the QCD factorization formula. Currently, this form factor is calculated in lattice QCD using certain extrapolations. It can also be obtained from LCSR, as discussed above. The results of both approaches generally agree with each other (see Ref. [29] for a recent comparison). The pion distribution amplitudes, at least their nonasymptotic terms, e.g., the coefficients a_n in Eq. (22), represent another set of inputs. They, in principle, can be determined from lattice QCD [30]. Another possibility to estimate and/or constrain the nonasymptotic parts of the pion distribution amplitudes is provided by fitting the LCSR prediction on the pion electromagnetic form factor [31] to experimental data. Yet another external input, used to calculate the hard spectator contribution to F in Eq. (38), is the B-meson distribution amplitude, which is currently taken from models.

The last term in Eq. (38) symbolizes contributions of soft nonfactorizable effects. Their magnitude cannot be calculated within the QCD factorization approach. In fact, the $1/m_b$

[3] For the $B \to D^{(*)}\pi$ a similar calculation has been done in Ref. [26].

[4] This important feature distinguishes the QCD factorization approach from other analyses of $B \to \pi\pi$ in perturbative QCD [27, 28].

suppression of these nonperturbative effects is just a conjecture based on analysing [11] the infrared behavior of perturbative diagrams. Therefore, in order to control the accuracy of QCD factorization at finite m_b, one has to estimate soft nonfactorizable corrections in a certain nonperturbative framework and confirm their $1/m_b$ suppression.

The LCSR approach described in the previous sections yields for the $\bar{B}_d^0 \to \pi^+\pi^-$ amplitude an expression very similar to Eq. (38):

$$\mathcal{A}(\bar{B}_d^0 \to \pi^+\pi^-)_{LCSR} = i\frac{G_F}{\sqrt{2}} V_{ub}V_{ud}^* f_\pi [f_{B\pi}^+(0)]_{LCSR}\, m_B^2 \Bigg\{ c_1(\mu) + \frac{c_2(\mu)}{3} + 2c_2(\mu)\left(\frac{\lambda_E(\bar{B}_d^0 \to \pi^+\pi^-)}{m_B} + \sum_{i=A,P,PA} \frac{\lambda_i(\bar{B}_d^0 \to \pi^+\pi^-)}{m_B} + O(\alpha_s) \right) \Bigg\}, \tag{39}$$

In the nonfactorizable part, in addition to the soft correction in the emission topology calculated in Sect. 3, we indicate the contributions to the hadronic matrix element of $\widetilde{O}_1$ which still have to be calculated. In particular, the terms proportional to $\lambda_{A,P,PA}$ denote soft nonfactorizable corrections due to annihilation, penguin, and penguin annihilation topologies. We tacitly assume that all these effects are at least of $O(1/m_b)$. That has to be confirmed by a direct calculation. Finally, $O(\alpha_s)$ in Eq. (39) indicates the contribution of hard nonfactorizable effects originating from the diagrams in Fig.2 and from analogous diagrams of other topologies. All these terms can be calculated one by one from the underlying correlation function (4) as it was done for λ_E . Moreover, the calculation is performed using the same framework and input as for the form factor $[f_{B\pi}^+(0)]_{LCSR}$. We stress that in LCSR there is no need to introduce the B-meson distribution amplitude because the heavy meson is interpolated by a current.

The completion of Eq. (39) is a difficult calculational task. Before it is fulfilled, one can use the evaluated $\sim \lambda_E/m_B$ term interpreting it as a soft correction to the QCD factorization formula, that is, replacing $O(\Lambda_{QCD}/m_b) \to 2c_2\lambda_E/m_B$ in Eq. (38). To compare the soft and hard nonfactorizable corrections, we take our estimate (37) and calculate the short-distance part of Eq. (38) numerically. For consistency, we adopt $\mu = \mu_b$, adjusting the QCD factorization formula to the characteristic scale at which the hadronic matrix elements have been calculated from LCSR. Furthermore, we use $c_{1,2}(\mu_b)$ in NLO and in the NDR scheme, $\alpha_s(\mu_b) = 0.279$ corresponding to $\alpha_s(m_Z) = 0.118$, and the expression for F given in Ref. [11]. In the latter, we adopt for the integral $\int_0^1 \Phi_B(\xi)/\xi = m_B/\lambda_B$ over the B meson distribution amplitude $\Phi_B(\xi)$ the same parameter $\lambda_B = 0.3$ GeV as in Ref. [11]. The following estimates are obtained for the separate terms in Eq. (38):

$$\left[c_1(\mu_b) + \frac{c_2(\mu_b)}{3}\right] + \left[\frac{\alpha_s}{9\pi} c_2(\mu_b) F(\mu_b)\right] + \left[2c_2(\mu_b)\frac{\lambda_E(\bar{B}_d^0 \to \pi^+\pi^-)}{m_B}\right] = [1.03] + [(-0.007 \div +0.01) + 0.03i] - [0.005 \div 0.015], \tag{40}$$

where the interval for the real part in the second bracket on r.h.s. corresponds to varying $\varphi_\pi(u,\mu_b)$ between the BF and asymptotic form in the expression for F. [5] This interval

[5] We parenthetically notice that the real part of the coefficient F in Eq. (38) is quite sensitive to the shape of $\varphi_\pi(u)$, whereas the imaginary part is independent of this shape, being simply proportional to the normalization of the distribution amplitude.

has only a small correlation with the range in the third bracket on r.h.s. of Eq. (40). The latter corresponds to the variation of λ_E in Eq. (37) and is to a large extent caused by the uncertainty in the nonperturbative parameters $f_{3\pi}$ and δ^2.

We find that the hard and soft nonfactorizable corrections are of the same order and their overall effect varies within $\pm 2\%$ of the factorizable amplitude. Thus, in the $\bar{B}_d^0 \to \pi^+\pi^-$ channel both effects are very small and do not influence the decay amplitude, which is predominantly factorizable. The situation is different in the case of the $c_2 + c_1/3$ combination relevant for the colour-suppressed $B \to \pi^0\pi^0$ decay:

$$\left[c_2(\mu_b) + \frac{c_1(\mu_b)}{3}\right] + \left[\frac{\alpha_s}{9\pi} c_1(\mu_b) F(\mu_b)\right] + \left[2c_1(\mu_b)\frac{\lambda_E(\bar{B}_d^0 \to \pi^+\pi^-)}{m_B}\right]$$
$$= [0.103] + [(0.03 \div -0.04) - 0.104i] + [0.021 \div 0.064]\,. \qquad (41)$$

Here the nonfactorizable corrections have a noticeable impact on the amplitude, both on its magnitude and phase.

We conclude that soft nonfactorizable effects are as important as the $O(\alpha_s)$ nonfactorizable corrections in the QCD factorization formula. To be cautious, the numbers presented in Eqs. (40), (41) cannot yet be considered as final predictions to be used in the phenomenological analysis. Adding to λ_E the remaining soft effects, due to annihilation and penguins, will most probably cause further changes in the decay amplitudes. One also has to stress that the soft and hard effects in Eqs. (40) and (41) are calculated in different approaches. It is more safe and consistent to estimate all nonfactorizable effects using one and the same method, and the LCSR approach offers such a possibility.

5 Earlier applications of QCD sum rules

There have been several attempts to calculate nonleptonic decays of heavy mesons using OPE and QCD sum rules [13, 14, 15] and focusing on nonfactorizable matrix elements. An important prediction of this approach, as emphasized in Ref. [9], is the channel-dependence of nonfactorizable effects. However, certain complications manifested themselves in the derivation procedure of the sum rules. The original method [13] of four-point correlation functions, applied to two-body D decays, was plagued by a presence of light-hadron states in the dispersion relation for the heavy-meson channel. These states, as explained in Sect. 2, originate due to the absence of an external momentum in the b-quark decay vertex. A different procedure suggested in Ref. [14] for $B \to D\pi$ (and used in Ref. [15] for other channels) avoids this problem. If we apply the latter procedure to $B \to \pi\pi$, the starting point is then a two-point hadronic matrix element $M_{B\pi} \equiv i \int d^4x \, \exp(ipx) \langle \pi | T\{j_{\alpha 5}^{(\pi)}(x) O(0)\} | B \rangle$. The expansion of the operator product in a series of local operators,

$$T\{j_{\alpha 5}^{(\pi)}(x) O(0)\} = \sum_k C_{\alpha k}(x) O_k^{eff}(0), \qquad (42)$$

reduces $M_{B\pi}$ to a set of simpler matrix elements, $\langle \pi | O_k^{eff} | B \rangle$, which have to be calculated separately using, e.g., QCD sum rules. Only the lowest-dimension operator is usually taken into account for $O = O_{1,2}$. Matching the result of this calculation with the dispersion

relation for $M_{B\pi}$ in the channel of the current $j^{(\pi)}_{\alpha 5}$, one finally obtains the two-pion matrix element $\langle \pi\pi|O|B\rangle$. This estimate is rather ambiguous, because quark-hadron duality cannot be applied to the hadronic matrix elements involved in this procedure. The method suggested in this paper avoids any unwanted contamination of the dispersion relation, due to the auxiliary external four-momentum in the b -decay vertex. With this additional element, the sum rule for the hadronic matrix element is directly obtained from the initial three-point correlation function, as demonstrated in Sect. 2. One therefore does not need to follow the two-stage method of effective operators.

Apart from these essential improvements, there is also a very important difference between LCSR derived in this paper and sum rules obtained before. At all stages of the calculational procedure described in Sect. 2,3 we use the light-cone OPE. In contrast, the sum rules obtained from four-point correlation functions [13] are based on the short-distance expansion in local condensates. Similarly, the method of effective operators [14] employs a truncated short-distance expansion (42). This might be a reasonable approximation for a specific kinematics of $B \to D\pi$ considered in Ref. [14], where the pion energy does not scale with the heavy mass (see also a discussion on this point in the second paper of Ref.[11]). However, as already emphasized in Ref. [14], if the non-spectator hadron carries a large energy $\sim m_b$, as in $B \to \pi\pi$, the resummation of local operators with the same twist is necessary. In fact, such a resummation is implicitly performed within the procedure of the light-cone OPE employed in Sect. 3.

It is instructive to demonstrate what exactly happens if one employs a truncated short-distance expansion, instead of the light-cone one, in the correlation function (4). We consider again the diagram in Fig. 1b with the effective operator $\widetilde{O}_1$, and perform a short-distance expansion similar to the one in Eq. (42):

$$i\int d^4y\; e^{i(p-k)y} T\{j^{(\pi)}_{\alpha 5}(y)\widetilde{O}_1(0)\} = C_{\alpha\mu\lambda\rho}(p-k)\bar{u}(0)\Gamma^\mu \widetilde{G}^{\lambda\rho}(0) b(0)\,, \tag{43}$$

retaining only one local operator with the lowest dimension. To obtain the corresponding Wilson coefficient $C_{\alpha\mu\lambda\rho}$ one simply has to put $v \to 0$ in the argument of the gluon field in the quark propagator (24). This local limit corresponds to the quark propagating in the static gluon field, a usual approximation for the condensates in the conventional QCD sum rules [16]. Thus, in Eq. (43) an infinite series of quark-gluon operators of the same twist containing derivatives of the gluon field is neglected. Physically, this approximation, corresponding to a vanishing four-momentum of the soft gluon, cannot be justified, because in the diagram of Fig. 1b the momentum of the on-shell gluon is in the range $0 < v\alpha_3 q < q$, where q is of the order of the virtual quark momenta.

After substituting the expansion (43) back in the correlation function (4), we proceed using the light-cone expansion of the product $O_{eff}(0) j^{(B)}_5(x)$ and confining ourselves by the twist 3 contribution. The result is given by the same expression as in Eq. (27), but with $v = 0$ in the denominator:

$$\left(F^{(\widetilde{O}_1)}_{E,tw3}\right)_{\text{short dist.}} = \frac{m_b f_{3\pi}}{4\pi^2(-(p-k)^2)}\int \mathcal{D}\alpha_i\; \frac{\varphi_{3\pi}(\alpha_i)}{(m_b^2-(p-q)^2(1-\alpha_1))} \times \left[\frac{3}{2}(q\cdot k) + q\cdot(p-k)\right](q\cdot(p-k))\,. \tag{44}$$

We emphasize that in this modified procedure, the short-distance expansion has only been partially used. Nevertheless, there are important changes in the resulting expression (44) as compared with Eq. (27). The dependence of $F_{\alpha E}^{(\widetilde{O}_1)}$ on the variable P^2 drops out, and the dependence on $(p-k)^2$ reduces to a simple factor $1/(p-k)^2$, with an imaginary part proportional to $\delta(s-(p-k)^2)$. It is therefore difficult to attribute certain duality counterparts to the contributions of the pion and hadronic continuum, while matching the short-distance version of $F_{\alpha E}^{(\widetilde{O}_1)}$ with the dispersion relation (8). Let us, nevertheless, continue the derivation of the sum rule using Eq. (44) and assuming a complete pion dominance in the dispersion relation. Following the same procedure as described in Sect. 2, we obtain the hadronic matrix element

$$A_E^{(\widetilde{O}_1)}(\bar{B}_d^0 \to \pi^+\pi^-)_{\text{short dist.}} = -\frac{if_{3\pi}}{16\pi^2 f_\pi}\left(\frac{m_b^3}{2f_B}\right)\int\limits_{u_0^B}^1 \frac{du}{u^2} e^{\,m_B^2/M'^2 - m_b^2/uM'^2}$$

$$\times \int\limits_0^u dv\ \varphi_{3\pi}(1-u,u-v,v)\left(3-\frac{m_B^2}{m_b^2}u\right)\,, \tag{45}$$

where only the twist 3 part is taken into account. We have checked that a very similar result is obtained employing the two-step procedure [14] based on the effective operators, if one uses the approximation (43) for the expansion (42) and calculates the $B\to\pi$ matrix element of the effective operator separately, using the LCSR.

In the heavy quark limit, evaluated using Eq. (33), the r.h.s. of Eq. (45) is proportional to $m^{3/2}$, enhanced by one power of m_b in comparison with the twist-2 factorizable matrix element (23). Note that the latter is obtained from the diagram in Fig. 1a using the free-quark propagators, and there is no difference between short-distance and light-cone expansion in the quark-loop diagram. The situation closely resembles QCD sum rules for heavy-to-light form factors based on the local condensate expansion. In these sum rules, the momenta of soft quarks and gluons forming vacuum condensates are also neglected yielding an enhancement of condensate terms at $m_b\to\infty$ as compared with the leading-order perturbative terms (for more details, see the discussion in Ref. [32]). Numerically, the nonfactorizable correction λ_E/m_B calculated from Eq. (45) is almost an order of magnitude larger than the estimate (37) and has a different sign. Note that the sign difference in the nonfactorizable matrix element obtained using the new LCSR procedure is a consequence of the P^2 dependence in the correlation function and emerges after the analytic continuation $(P^2<0)\to m_B^2$.

We conclude that even a limited use of the short-distance expansion in the correlation function (4) yields a sum rule, where it is difficult to separate the ground-state contribution applying the usual duality approximation and where the higher-twist terms are enhanced at $m_b\to\infty$ with respect to the leading twist-2 term.

6 Conclusions

In this paper, a new method has been suggested to calculate the hadronic matrix elements relevant for two-body nonleptonic B decays. The approach is based on the light-cone expansion of a three-point corelation function and on the analytic continuation from the

spacelike to the timelike region. The predicted hadronic matrix element contains a factorizable part as well as soft and hard nonfactorizable corrections. All these contributions can be calculated within one framework using a common input.

The nonfactorizable soft-gluon exchange correction calculated for the $B \to \pi\pi$ channel is our main result. Importantly, the $1/m_b$ suppression of this effect has been explicitly reproduced, in agreement with the expectation of the QCD factorization approach.

The accuracy of the LCSR method is limited, as usual in QCD sum rules, since one has to rely on the quark-hadron duality, both in the pion and B channels. Nevertheless, one may argue that the duality approximation works reasonably well having in mind, e.g. an agreement of the LCSR predictions for the $B \to \pi$ form factors with the lattice QCD results. It is also important that uncertainties of the sum rule predictions can be assessed by varying the input parameters within the currently adopted limits. These parameters are universal, therefore, their accuracy can be improved in future by direct determinations on the lattice or by fitting QCD sum rules for experimentally measured hadronic characteristics to the data.

The same method, with minor modifications, can be used for all other channels where B meson decays either into two light hadrons, or into one charmed hadron (or charmonium state) and a light hadron. The important examples in the latter case are $B \to D^{(*)}\pi, J/\psi K^{(*)}$ decays. The only limitation is that at least one of the light-quark hadrons in the final state has to be a pseudoscalar or vector meson, for which the light-cone distribution amplitudes are better known.

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