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# Chiral Limit Prediction for $\varepsilon'_K/\varepsilon_K$ at NLO in $1/N_c$ [1]

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**Abstract**

We report on a calculation of $\varepsilon'_K/\varepsilon_K$ at next-to-leading in the $1/N_c$ expansion and to lowest order in Chiral Perturbation Theory. We discuss the short-distance matching, the scale and scheme dependence as well as the long-distance short-distance matching. We include the two known chiral corrections to our result and discuss further order $p^4$ corrections to it.

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# 1 Introduction

Recently, direct CP violation in the Kaon system has been unambiguously established by the KTeV [1] experiment at Fermilab and by the NA48 [2] experiment at CERN. Their results together with the previous NA31 [3] and E731 [4] measurements produce the present world average

$$\mathrm{Re}\,(\varepsilon'_K/\varepsilon_K) \;=\; (19.3 \pm 2.4)\cdot 10^{-4}\,. \tag{1}$$

Further reduction of the statistical error to the order of $1\cdot 10^{-4}$ is expected.

A lot of effort has been put in the theoretical side to get a Standard Model prediction for this quantity in the last twenty five years. Recent reviews and predictions are [5, 6, 7, 8, 9, 10, 11]. Here, we would like to report on a calculation of this quantity in the chiral limit and next-to-leading (NLO) order in $1/N_c$ [12]. We also discuss how it changes when the known chiral corrections, i.e. final state interactions (FSI) and $\pi_0-\eta$ mixing are included. Comments on the different approaches to obtain the contributions from $Q_6$ and $Q_8$ to $\varepsilon'_K$ will also be given in the Summary.

Direct CP-violation in $K\to\pi\pi$ decays amplitudes is parameterized by

$$\begin{aligned}\frac{\varepsilon'_K}{\varepsilon_K} &= \frac{1}{\sqrt{2}}\left[\frac{A\,[K_L\to(\pi\pi)_{I=2}]}{A\,[K_L\to(\pi\pi)_{I=0}]}\right.\\ &- \left.\frac{A\,[K_S\to(\pi\pi)_{I=2}]}{A\,[K_S\to(\pi\pi)_{I=0}]}\right]\,.\end{aligned} \tag{2}$$

We want here to predict $K\to\pi\pi$ at NLO in $1/N_c$ and to lowest order in the chiral expansion.

In the isospin symmetry limit, $K\to\pi\pi$ invariant amplitudes can be decomposed into definite isospin quantum numbers as $[A\equiv -iT]$

$$\begin{aligned} i\,A[K^0\to\pi^0\pi^0] &\equiv \frac{a_0}{\sqrt{3}}\,e^{i\delta_0}-\frac{2\,a_2}{\sqrt{6}}\,a_2\,e^{i\delta_2}\,,\\ i\,A[K^0\to\pi^+\pi^-] &\equiv \frac{a_0}{\sqrt{3}}\,e^{i\delta_0}+\frac{a_2}{\sqrt{6}}\,e^{i\delta_2}\end{aligned} \tag{3}$$

with $\delta_0$ and $\delta_2$ the FSI phases.

To lowest order in Chiral Perturbation Theory (CHPT), i.e. order $e^0p^2$ and $e^2p^0$, strong and electromagnetic interactions between $\pi$, $K$, $\eta$ and external sources are described by

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\mathrm{tr}\,(\mathrm{u}_\mu\mathrm{u}^\mu+\chi_+) + \mathrm{e}^2\tilde{\mathrm{C}}_2\mathrm{tr}\left(\mathrm{QUQU}^\dagger\right) \tag{4}$$

with $U=u^2=\exp(\lambda^a\pi^a/F_0)$ and $u_\mu = iu^\dagger(D_\mu U)u^\dagger$. $\lambda^a$ are the Gell-Mann matrices and the $\pi^a$ are the pseudoscalar-mesons $\pi$, $K$, and $\eta$. $Q=\mathrm{diag}(2/3,-1/3,-1/3)$

is the light-quark-charge matrix and $\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u$ and $\chi = 2B_0\,\mathrm{diag}(m_u, m_d, m_s)$ collects the light-quark masses. To this order, $F_\pi = F_0 = 87$ MeV is the pion decay coupling constant. Introductions to CHPT can be found in Refs. [13, 14].

To the same order in CHPT, the chiral Lagrangian describing $|\Delta S| = 1$ is

$$\begin{aligned}
\mathcal{L}^{(2)}_{|\Delta S|=1} &= C F_0^6 e^2 G_E \,\mathrm{tr}\left(\Delta_{32}\tilde{\mathrm{Q}}\right) \\
&\quad + C F_0^4 \Big[ G_8 \mathrm{tr}\left(\Delta_{32} \mathrm{u}_\mu \mathrm{u}^\mu\right) + \mathrm{G}_8' \mathrm{tr}\left(\Delta_{32}\chi_+\right) \\
&\quad + G_{27} t^{ij,kl} \mathrm{tr}\left(\Delta_{\mathrm{ij}} \mathrm{U}_\mu\right) \mathrm{tr}\left(\Delta_{\mathrm{kl}} \mathrm{u}^\mu\right) \Big] + \mathrm{h.c.}
\end{aligned} \tag{5}$$

with $\Delta_{ij} = u \lambda_{ij} u^\dagger$, $(\lambda_{ij})_{ab} = \delta_{ia}\delta_{jb}$, $\tilde{Q} = u^\dagger Q u$;

$$C = -\frac{3}{5}\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \approx -1.06 \cdot 10^{-6}\,\mathrm{GeV}^{-2}\,. \tag{6}$$

The SU(3) $\times$ SU(3) tensor $t^{ij,kl}$ can be found in [15]. Using this Lagrangian,

$$\begin{aligned}
a_2 &= \frac{\sqrt{3}}{9} C F_0 \left[ 10 G_{27} \left(m_K^2 - m_\pi^2\right) - 6 e^2 G_E F_0^2 \right] \\
a_0 &= \frac{\sqrt{6}}{9} C F_0 \left[ \left(9 G_8 + G_{27}\right)\left(m_K^2 - m_\pi^2\right) \right. \\
&\quad - \left. 6 e^2 G_E F_0^2 \right]
\end{aligned} \tag{7}$$

and $\delta_0 = \delta_2 = 0$. In the presence of CP-violation, the couplings $G_8$, $G_{27}$, and $G_E$ get an imaginary part. In the Standard Model, $\mathrm{Im}\, G_{27}$ vanishes and $\mathrm{Im}\, G_8$ and $\mathrm{Im}\, G_E$ are proportional to $\mathrm{Im}\,\tau$ with $\tau \equiv -\lambda_t/\lambda_u$ and $\lambda_i \equiv V_{id} V_{is}^*$ and where $V_{ij}$ are Cabibbo-Kobayashi-Maskawa (CKM) matrix elements.

## 2 Short-Distance Scheme and Scale Dependence

Strangeness changing transitions happen in the Standard Model by the exchange of one $W$-boson. This fact implies that *all* physics between 0 and $\infty$ has to be taken into account when calculating Kaon to pions amplitudes. In particular, this implies the intervention of strong interactions at all energies, i.e. from the perturbative region to the non-perturbative one.

The two very different scales involved in $K \to \pi\pi$, i.e. the $W$-mass and the Kaon mass make effective field theory methods very useful. This is standard and we outline the steps needed. Below the $W$-mass, the effective action $\Gamma_{\Delta S=1}$ is obtained by integrating out the heavy particles, top, $Z$, and $W$-bosons and using the operator product expansion (OPE). The leading contributions consist of four-quark operators and higher dimensional operators are suppressed by $M_W^2$. Using

the renormalization group equations, $\Gamma_{\Delta S=1}$ is brought down to some perturbative scale below the charm quark mass. Perturbative matching and OPE is used at thresholds of the successive heavy particles which are integrated out. This full process implies several choices of short-distance schemes, regulators, and operator basis. Of course, physical matrix elements cannot depend on these choices.

Explicit calculations have been performed including gluonic and electromagnetic Penguin-diagrams to one-loop in [16] and to two-loops in [17] in two schemes. These are the naive dimensional regularization (NDR) scheme and the 't Hooft-Veltman (HV) scheme. The choice of the operator basis and other choices like infrared regulators can be found in those references.

The Standard Model $\Gamma_{\Delta S=1}$ effective action at scales $\nu$ below the charm quark mass, is [18]

$$\Gamma_{\Delta S=1} \quad \sim \quad \sum_{i=1}^{10} C_i(\nu) \int \mathrm{d}^4x \, Q_i(x) + \mathrm{h.c.} \tag{8}$$

where $C_i = z_i + \tau y_i$ are Wilson coefficients and $Q_i(x)$ are four-quark operators. At this level, we have resummed the large logarithms $[\alpha_s(\nu)\log(\nu/M_W)]^n$ and $\alpha_s[\alpha_s(\nu)\log(\nu/M_W)]^n$ to all orders in $n$.

At low energies, it is more convenient to describe the $\Delta S = 1$ transitions with an effective action $\Gamma^{LD}_{\Delta S=1}$ which uses hadrons, constituent quarks, or other objects to describe the relevant degrees of freedom. A regularization scheme like an Euclidean cut-off which separates long-distance physics from the short-distance physics which is integrated out and working in four dimensions is also more practical, as well as another operator basis, for instance the color singlet Fierzed one. The effective action $\Gamma^{LD}_{\Delta S=1}$ depends on all these choices and in particular on the scale $\mu_c$ introduced to regulate the divergences generated analogously as (8) depends on the scale $\nu$. It also depends on effective couplings $g_i$ analogously to the Wilson coefficients in (8). As usual, matching conditions have to be set between the two effective field theories. This is done by requiring that $S$-matrix elements of asymptotic states are the same at some perturbative scale

$$\langle 2|\Gamma^{LD}_{\Delta S=1}|1\rangle \quad = \quad \langle 2|\Gamma_{\Delta S=1}|1\rangle \, . \tag{9}$$

Both sides are separately scale and scheme independent and therefore the short-distance scale and scheme dependences are consistently treated. This matching is done at the perturbative level using the OPE in QCD. Notice that even if the regulator chosen is the same in both sides, there can be finite terms appearing in the matching. The matching conditions fix *analytically* the short-distance behavior of the couplings $g_i$

$$g_i(\mu_c, \cdots) \quad = \quad \mathcal{F}(C_i(\nu), \alpha_s(\nu), \cdots) \, . \tag{10}$$

This was done explicitly in [19] for $\Delta S = 2$ transitions and used in [12] for $\Delta S = 1$ transitions. To the best of our knowledge, these are the first places were

the short-distance scheme dependence is analytically cancelled in the final matrix element at next-to-leading order in the $1/N_c$ expansion. Notice that in the whole process we never required the matching of scales in two different regularization schemes as is sometimes stated in the literature for this type of $1/N_c$ calculations.

### 2.1 The heavy $X$-bosons Method

We find it convenient to use an effective field theory of heavy color-singlet $X$-bosons coupled to QCD currents and densitities as argued in [19, 20, 21, 22]. One advantage of this is that two-quark currents are unambiguously identified and that QCD densities are much easier to match than four-quark operators. E. g., the operator

$$Q_1(x) \;=\; [\overline{s}\gamma_\mu(1-\gamma_5)d]\,(x)\;[\overline{u}\gamma_\mu(1-\gamma_5)u]\,(x)$$

is reproduced by the effective action

$$\begin{aligned}\Gamma_X \;&\equiv\; g_1(\mu_c,\cdots)\int \mathrm{d}^4y\, X_1^\mu\,\{[\overline{s}\gamma_\mu(1-\gamma_5)d]\,(x)\\ &+\;[\overline{u}\gamma_\mu(1-\gamma_5)u]\,(x)\}\end{aligned} \tag{11}$$

We use an Euclidean cut-off at $\mu_c$ in four dimensions as regulator of the UV divergences, with this the $X$-boson effective action is completely specificied. This regulator is very convenient to separate long- from short-distance physics. In (11), the higher than $\mu_c$ degrees of freedom of quarks and gluons have been integrated out.

The complete $X$-boson effective action for the Standard Model $\Delta S = 1$ transitions can be found in [12]. We are now ready to calculate consistently $\Delta S = 1$ Green functions with the $X$-boson effective theory.

## 3 Long-Distance–Short-Distance Matching

We study the two-point $\Delta S = 1$ Green function

$$\Pi(q^2) \equiv i\int \mathrm{d}^4x\, e^{iq\cdot x}\langle 0|T(P_i^\dagger(0)P_j(x)e^{i\Gamma_X}|0\rangle$$

where $P_i$ are pseudoscalar sources with the quantum numbers to describe $K\to\pi$ amplitudes.

After reducing the external legs one gets $K\to\pi$ off-shell amplitudes. Taylor expanding them in external momentum and $\pi$, and $K$ masses one can obtain the couplings of the CHPT Lagrangian [15, 23]. Then one predicts $K\to\pi\pi$ at a given order. This procedure is unambiguous. One can use also $K\to\pi$ and $K\to$ vacuum transitions form lattice simulations to predict the lowest order CHPT couplings [15, 23, 24].

Up to know we have not used any $1/N_c$ argument. We count the $g_i$ couplings as effectively order 1 since they contain the large logarithms of the running between $M_W$ and $\mu_c$ even though some of them start at NLO order in $1/N_c$. At leading order in the $1/N_c$ expansion the contribution to the Green's function (3) is factorizable. It only involves two-point functions. At this order the result is model independent and the scale and scheme dependence is exactly taken into account. However, we know that the factorizable contribution fails to reproduce $\mathrm{Re}\, a_0$ by a factor around 6 and $\mathrm{Re}\, a_2$ by a factor around 1/2.

At the same order, $Q_6$ contributes to $G_8$ proportional to

$$|g_6|^2 \sim C_6(\nu)\langle 0|\overline{q}q|0\rangle^2(\nu)\, L_5(\hat{\mu}) \tag{12}$$

and $Q_8$ to $e^2 G_E$ proportional to

$$|g_8|^2 \sim C_8(\nu)\langle 0|\overline{q}q|0\rangle^2(\nu)\,. \tag{13}$$

To leading order in $1/N_c$, the scale dependence of the Wilson coefficients cancels exactly the one from the quark condensate [25, 26]. The scale $\hat{\mu}$ dependence of $L_5$ requires NLO in $1/N_c$ contributions to cancel. In fact, at this order there also appears an IR divergence in the factorizable contribution of $Q_6$ to $G_8$ that necessarily requires non-factorizable contributions to cancel [22].

The non-factorizable contribution is NLO in the $1/N_c$ expansion and involves the integration of four-point functions $\Pi_{P_i P_j J_a J_b}$ over the $X$-boson momentum $r_E$ that flows through the currents/densities $J_a$ and $J_b$. It can be schematically written as

$$\Pi(q^2) \sim \int \frac{\mathrm{d}^4 r_E}{(2\pi)^4}\, \Pi_{P_i P_j J_a J_b}(q_E, r_E)\,. \tag{14}$$

We separate long- from short-distance physics with an Euclidean cut-off $\mu$ in $r_E$. The short distance part of the integral from $\mu$ up to $\infty$ can be treated at NLO within OPE QCD using a cut-off regulator if $\mu$ is high enough.

It has been emphasized that dimension eight operators may be numerically important when the cut-off scale $\mu$ is of the order of 1 GeV [21, 27]. This issue can be treated in a straightforward way in our approach. We have to use the OPE in the short-distance part up to the required dimensions. In fact, those contributions are under control at NLO in $1/N_c$ using factorized matrix elements. We stopped at dimension six operators for the present results, we plan to check the effect of dimension eight operators elsewhere.

Up to here, there is no model dependence in our evaluation of $K \to \pi$ amplitudes at NLO in $1/N_c$ within QCD.

What remains is the long distance part from 0 up to $\mu$. For very small values of $\mu$ one can use CHPT and still the result is model independent. However, CHPT at order $p^4$ starts not to be enough already at relatively small values of $\mu$, i.e. $\sim$ 400 MeV. To match with the short-distance part, we need clearly

to go beyond this energy. The first step is to use a good hadronic model for intermediate energies. The model we used is the ENJL model described in [28]. It has several good features -it includes CHPT to order $p^4$, for instance- and also some drawbacks as explained in [19]. Work is in progress to implement the large $N_c$ constraints along the lines of [29]. There is very interesting work on a different approach to the calculation of weak matrix elements within the $1/N_c$ expansion in [21, 30]. It has started to produce the first results for three- and four-point Green's functions. In particular the determination of the $B_K$ parameter in the chiral limit is clean [31]. Their result agrees well with the chiral limit calculation in [19, 21] done within our present approach.

# 4 $\varepsilon'_K$ in the Chiral Limit

To a very good approximation [14, 18],

$$|\varepsilon'_K| \simeq \frac{1}{\sqrt{2}}\frac{\mathrm{Re}\, a_2}{\mathrm{Re}\, a_0}\left\{-\frac{\mathrm{Im}\, a_0}{\mathrm{Re}\, a_0}+\frac{\mathrm{Im}\, a_2}{\mathrm{Re}\, a_2}\right\}. \tag{15}$$

The isospin amplitudes $a_I$ are defined in (3). The lowest order CHPT values for $\mathrm{Re}\, a_0$ and $\mathrm{Re}\, a_2$ can be obtained from a fit to $K\to\pi\pi$ and $K\to\pi\pi\pi$ amplitudes up to order $p^4$ in [32].

Our results in [22] reproduce the value for $\mathrm{Re}\, a_0$ finding a nice matching of short-and long-distances. For the coupling $G_{27}$, which dominates $\mathrm{Re}\, a_2$, we don't find such good stability though the behaviour is much better than the quadratic divergence found in [9, 33]. In Figure 1 we give $G_{27}$ and in Figure 2 we present $\mathrm{Re}\, G_8$.

The $\Delta I = 1/2$ enhancement is reproduced within 40 %. However, due to the present lack of a complete understanding of $\mathrm{Re}\, a_2$, we prefer to use the *experimental* values for the lowest order CHPT amplitudes $\mathrm{Re}\, a_I$ [32] to predict $\varepsilon'_K$.

At lowest order in CHPT, the imaginary part of $G_8$ is almost all from the $Q_6$ operator with very small contributions from $Q_3$, $Q_4$, and $Q_5$. Then to a very good approximation

$$\begin{aligned}\mathrm{Im}\, G_8 &\simeq -\frac{80}{3}\,\mathrm{Im}\,\tau\, y_6(\nu)\\ &\times \frac{\langle 0|\overline{q}q|0\rangle^2(\nu)}{F_0^6}\, L_5(\hat{\mu})\, B_6(\hat{\mu},\nu).\end{aligned} \tag{16}$$

At large $N_c$, $B_6(\hat{\mu},\nu) = 1$. At the same order, the imaginary part of $e^2 G_E$ is almost all from the $Q_8$ operator with a very small contribution from $Q_7$. Then to a very good approximation

$$\mathrm{Im}\,(e^2 G_E) \simeq -5\,\mathrm{Im}\,\tau\, y_8(\nu)$$

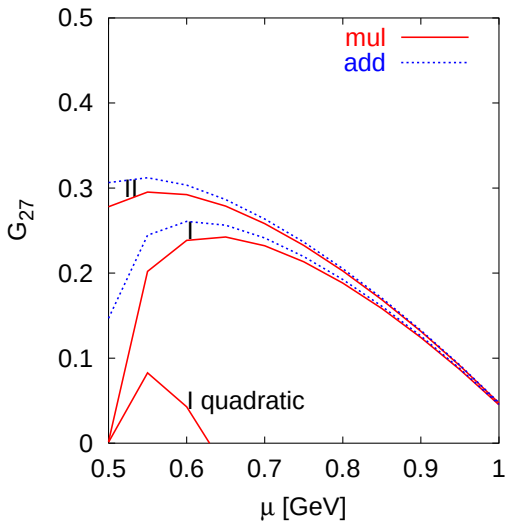


Figure 1: We plot the result for $G_{27}$. Curves I and II are different ways of doing the two-loop $\alpha_s$ running and *mul* and *add* are the resummation on the Wilson coefficients is done either multiplicatively or additively. The curve *quadratic* is the result of references [9, 33].

$$\times \quad \frac{\langle 0|\overline{q}q|0\rangle^2(\nu)}{F_0^6}\, B_8(\nu) \tag{17}$$

and $B_8(\nu) = 1$ at large $N_c$. With the present values of the CKM matrix elements

$$\mathrm{Im}\,\tau = -6.72 \cdot 10^{-4}\,. \tag{18}$$

We use the chiral limit $\overline{MS}$ quark-condensate [34],

$$\begin{aligned} B_0(1\mathrm{GeV}) &\equiv -\frac{\langle 0|\overline{q}q|0\rangle(1\mathrm{GeV})}{F_0^2} \\ &= (1.75 \pm 0.40)\,\mathrm{GeV}\,. \end{aligned} \tag{19}$$

Its leading scale dependence is analytically canceled by the one in the Wilson coefficients $C_6$ and $C_8$ [25, 26]. The uncertainty induced in $\varepsilon'_K$ by $B_0$ is around 40 % and often not included in error estimates. Incidentally, higher order corrections to the contribution of $Q_6$ to $G_8$ do not change the chiral limit condensate into the strange quark condensate as is commonly used [22].

There is no NLO in $1/N_c$ correction to $e^2 G_E$ from $Q_8$ in nonet symmetry. This is a model independent result. In the real world, the $\mathrm{U}(1)_A$ anomaly gives a mass to the pseudoscalar singlet field and octet symmetry is a good symmetry. Therefore, there are $1/N_c^2$ corrections to this result which we intend to study within our $1/N_c$ approach elsewhere [35] using data to obtain $B_8$ [10, 29, 36] as

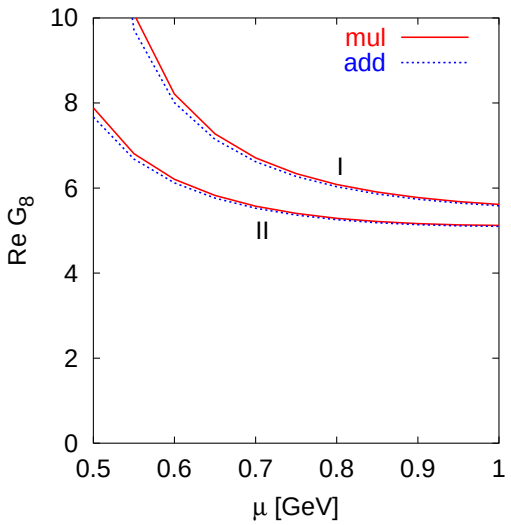


Figure 2: We plot the result for $\mathrm{Re}\, G_8$. The labels are like in Figure 1.

well as a lowest meson dominance model. They will give the size of the $\mathrm{U}(1)_A$ anomaly corrections to our NLO in $1/N_c$ results. The results in [10, 36] studied the $B_8$ parameter with the additional assumption that dimension six operators dominate the OPE.

Our values of the relevant bag parameters in the NDR scheme at 2 GeV are in Table 1.

Table 1: Results for $B_6(\nu)^{(1/2)NDR}$, $B_7(\nu)^{(3/2)NDR}$ and $B_8(\nu)^{(3/2)NDR}$ at $\nu = 2$ GeV. The $\overline{MS}$ value $\overline{m}_s(2\mathrm{GeV}) = (119 \pm 12)$ MeV was used in [10] and for rescaling the results in [36, 37].

| Method | $B_6^{(1/2)NDR}(2\mathrm{GeV})$ | $B_7^{(3/2)NDR}(2\mathrm{GeV})$ | $B_8^{(3/2)NDR}(2\mathrm{GeV})$ |
|---|---|---|---|
| NLO $1/N_c$ [$\chi$ limit] [12] | $2.5 \pm 0.4$ | $0.8 \pm 0.1$ | $1.35 \pm 0.20$ |
| LMD [37] | – | 0.94 | – |
| Dispersive [36] | – | $0.78 \pm 0.17$ | $1.6 \pm 0.4$ |
| QCD Sum Rules [10] | $1.0 \pm 0.5$ | $0.7 \pm 0.2$ | $1.70 \pm 0.39$ |
| CHPT NLO $1/N_c$ [9] | $1.5 \sim 1.7$ | $-0.1 \sim 0.09$ | $0.4 \sim 0.7$ |
| Lattice [38] | – | $0.5 \sim 0.8$ | $0.7 \sim 1.1$ |
| Chiral Quark Model [8] | $1.2 \sim 1.7$ | $\sim 0.9$ | $\sim 0.9$ |
| FSI Omnès [11] | 1.55 | – | 0.93 |

Details and the input parameters of the determination of $\varepsilon'_K/\varepsilon_K$ are in [12]. Here we only give the results and main conclusions. To lowest order in $1/N_c$ and

in the chiral limit, we get

$$\left|\frac{\varepsilon'_K}{\varepsilon_K}\right|_\chi = (33-16)\cdot 10^{-4} = (17\pm 7\pm 6)\cdot 10^{-4}$$
$$= (17\pm 9)\cdot 10^{-4} \qquad (20)$$

The first error is from the uncertainty in the value of the quark condensate and the second is a typical 1/3 on the factorizable contributions. We cannot give here an error estimate due to the non-inclusion of non-factorizable $1/N_c$ corrections since they are a new type of contributions.

Including our calculated NLO in $1/N_c$ non-factorizable contributions we get

$$\left|\frac{\varepsilon'_K}{\varepsilon_K}\right|_\chi = (83-23)\cdot 10^{-4} = (60\pm 24\pm 20)\cdot 10^{-4}$$
$$= (60\pm 30)\cdot 10^{-4} \qquad (21)$$

again in the chiral limit. The first error is once more due to the uncertainty in the quark condensate. The second one is an estimate of the model uncertainty, i.e. around 1/3 of the factorizable contribution plus 40 % of the non-factorizable one added quadratically. Notice that there is a factor larger than three between (20) and (21). This result is plotted in Figure 3 where one can appreciate the quality of the matching.

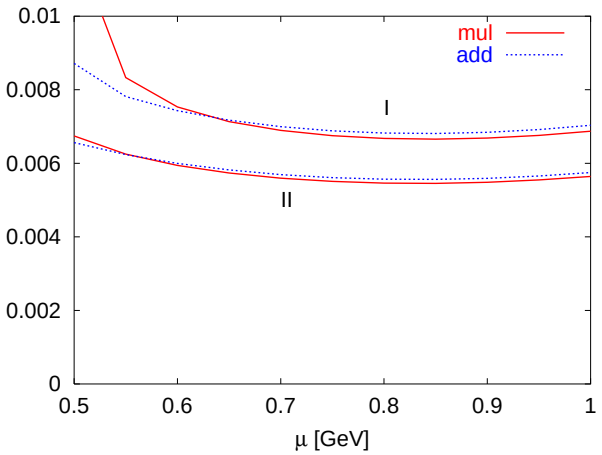


Figure 3: We plot our result for $\mathrm{Im}\, G_8$ at NLO in $1/N_c$. The labels of the curves are as for Figure 2.

To get the result in (21) we have used our chiral limit NLO in $1/N_c$ determinations of $\mathrm{Im}\, G_8$ which we show in Figure 4 and of $\mathrm{Im}\, G_E$ which is in Figure 5.

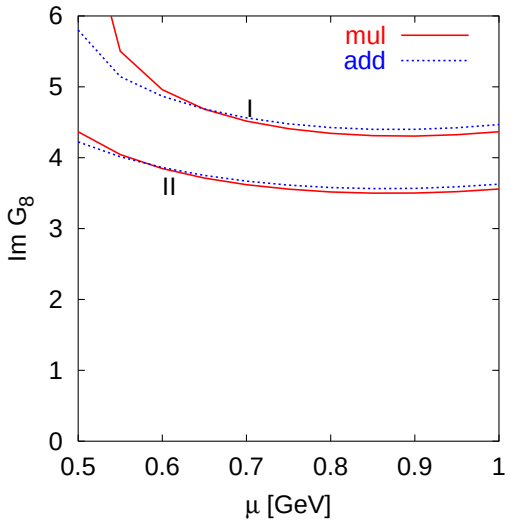


Figure 4: We plot our result for $\mathrm{Im}\, G_8$ at NLO in $1/N_c$. The labels of the curves are as for Figure 2.

# 5 Higher Order CHPT Corrections

The rôle of final state interactions in the standard [5, 6] predictions of $\varepsilon'_K/\varepsilon_K$ has been recently clarified in [39]. See also [40].

We have taken a different strategy to predict $\varepsilon'_K$. We start with Equation (15). The ratio $\mathrm{Im}\, a_I/\mathrm{Re}\, a_I$ has no final state interactions to all orders. One of the main problems at present of $K \to \pi\pi$ lattice calculations is that they cannot include FSI. There are very interesting recent developments towards a full calculation within lattice QCD including FSI, see [41] and references therein for previous work.

It would be very interesting to see how much of the ratios $\mathrm{Im}\, a_I/\mathrm{Re}\, a_I$ could still be obtained from lattice simulations *without* FSI. One could use CHPT to eliminate analytically the FSI contributions for both the real and imaginary parts.

The only place where FSI are present is in the normalization factor $\mathrm{Re}\, a_2/\mathrm{Re}\, a_0$, but one can take as a first step its experimentally known value. In fact, lattice calculations can provide us with an estimate of $\varepsilon'_K/\varepsilon_K$ to lowest order CHPT with $K \to \pi$ and $K \to$ vacuum results [12]. This would be very interesting. Of course, one also wants to predict the $\Delta I = 1/2$ enhancement ratio $\mathrm{Re}\, a_2/\mathrm{Re}\, a_0$ but $K \to \pi$ calculations look more straightforward in the lattice at present [24].

About isospin breaking due to $m_u \neq m_d$, only effects due to $\pi^0 - \eta$ mixing [25, 42] are under control. They have recently calculated to CHPT order $p^4$ [42]. Though $\pi^0 - \eta$ mixing is all from $m_u \neq m_d$ isospin breaking at lowest CHPT order, this is not true at higher orders. In fact, due to the smallness of the order $p^4$ term

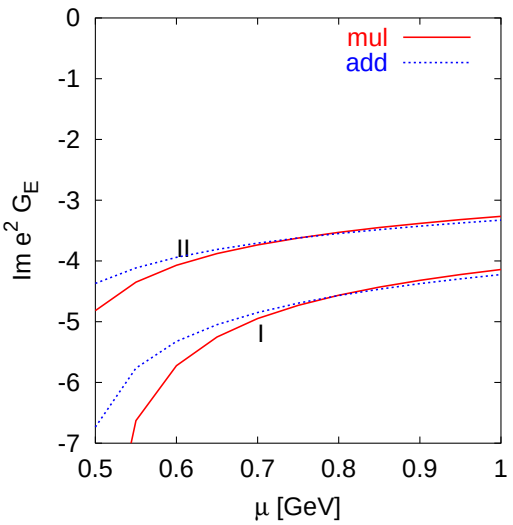


Figure 5: We plot our result for $\mathrm{Im}\, G_E$ at NLO in $1/N_c$. The labels of the curves are as for Figure 2.

in the $\pi^0-\eta$ mixing contribution, it could be that there are other $m_u-m_d$ effects equally important at the same order, so that a full $p^4$ calculation is mandatory [43, 42]. This effect adds a contribution to Im $a_2$ which is parameterized usually as

$$\frac{[\mathrm{Im}\ a_2]_{IB}}{\mathrm{Re}\ a_2} \;=\; \Omega_{IB}\,\frac{\mathrm{Im}\ a_0}{\mathrm{Re}\ a_0}\,. \tag{22}$$

We will use the value $\Omega_{IB}=0.16\pm 0.03$ [42]

Other purely real $p^4$ corrections even in the isospin limit are mostly unknown. They have been taken into account within the approach in [8] and partially in [9].

Electromagnetic corrections have been considered in [26, 44]. Very little is known at present of the size of the order $p^4$ contributions and beyond. Obviously there is more work to be done in this direction.

To lowest order in $1/N_c$ but including FSI and $\pi^0-\eta$ chiral corrections -notice that these two corrections are actually NLO in $1/N_c$- we get

$$\begin{aligned}\left|\frac{\varepsilon'_K}{\varepsilon_K}\right| &= (24-15.5)\cdot 10^{-4} = (8.5\pm 3.5\pm 3)\cdot 10^{-4}\\ &= (8.5\pm 4.6)\cdot 10^{-4}\,.\end{aligned} \tag{23}$$

The errors correspond to the same discussion of the chiral limit results (20) and in particular do not include the error of the non-included non-factorizable contributions. This result is model independent.

Notice that small changes in the isospin zero or in the isospin two contributions are extremely amplified, so that a 20 % correction to both but in opposite directions can translate into a factor two in the value of $\varepsilon'_K$.

Including our calculated NLO in $1/N_c$ non-factorizable contributions, we get

$$\begin{aligned} \left|\frac{\varepsilon'_K}{\varepsilon_K}\right| &= (59.6 - 25.6)\cdot 10^{-4} = (34 \pm 14 \pm 11)\cdot 10^{-4} \\ &= (34 \pm 18)\cdot 10^{-4}\,. \end{aligned} \tag{24}$$

Notice that the two known chiral corrections reduce the value of $\varepsilon'_K$.

# 6 Summary

We have reported on a calculation of $\varepsilon'_K/\varepsilon_K$ at next-to-leading in a $1/N_c$ expansion and in the chiral limit [12]. Emphasis has been given to discuss the short-distance scheme and scale dependence matching in Section 2 and to the long-distance short-distance matching in Section 3.

We have also given the two model independent results at leading order in the $1/N_c$ expansion and in the chiral limit in (20) and including the two known chiral corrections, namely FSI and $\pi^0 - \eta$ mixing in (23).

We also have given the main result of our work which is the value of $\varepsilon'_K/\varepsilon_K$ in the chiral limit and at next-to-leading order in a $1/N_c$ expansion (21). Including again the two known higher order CHPT corrections to this result, we obtain our final number for $\varepsilon'_K/\varepsilon_K$ in (24). Notice however, as discussed in Section 5, that most of the higher order CHPT corrections are actually unknown and as remarked in several recent work they could affect numerically the Standard Model value of $\varepsilon'_K/\varepsilon_K$ significantly [12, 43, 44].

We would like to discuss the determinations of the two main bag parameters for $\varepsilon'_K$ in Table 1. About $B_6$, which is the most uncertain, at present, there are only two estimates of the non-factorizable corrections not proportional to quark masses to this important parameter. They are the results in [9] and in [12].

The Dortmund group [9] has included non-factorizable corrections to $B_6$ in the chiral limit. For that, they use CHPT to order $p^4$ and find a quadratic dependence in the cut-off scale. They have also included partially higher order CHPT corrections to that result.

To the best of our knowledge, the result in [12], is the first one that includes NLO in $1/N_c$ non-factorizable corrections to $B_6$ in the chiral limit and find a logarithmic matching with the Wilson coefficients see Figure 3 for $\varepsilon'_K/\varepsilon_K$ and Figure 4 for the imaginary part of $G_8$.

The Trieste group [8] uses $B_6 = 1$ in the chiral limit and all the difference between the factorizable result and their final number quoted in the Table 1 are higher order CHPT corrections. They have not calculated non-factorizable corrections in the chiral limit.

Notice that the calculation of $B_6$ in [10] only includes the factorizable contributions.

In [11], the leading order in $1/N_c$ values of $B_6 = 1$ and $B_8 = 1$ are corrected with $\pi\pi$ final state interactions using the Omnès solution. In our opinion, this is at present the best way of taking into account within CHPT those important corrections.

For $Q_8$ we go at NLO in $1/N_c$ but in the chiral limit, and we find that, as explained in Section 4 and [12],

$$B_8^{NDR}(2\text{GeV})_\chi = 1.35 \pm 0.20 \tag{25}$$

is a model independent result. There are however $1/N_c^2$ corrections induced by the $\text{U}(1)_A$ anomaly. These effects were estimated in [10, 36] with the additional assumption that dimension six operators dominate the OPE in this case. We plan to investigate this issue within our approach [35].

Again, only NLO corrections proportional to quark masses are included in the result of the Trieste group to $B_8$ [8]. The results from the lattice [38] do not include FSI but we don't known how much do they include of other higher order chiral corrections.

Let us conclude that a lot of advances have been done towards the prediction of $\varepsilon'_K/\varepsilon_K$ within the Standard Model. For us, there are two main questions: which is the value of $B_6$ in the chiral limit and which are the non-FSI chiral corrections to it. This work tries to answer the first of these questions. There is going on a lot of activity to clarify these issues by several groups and we foresee a very exciting nearby future.

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