ALTERNATIVE SCENARIOS FOR THE FRAGMENTATION OF A GLUONIC LUND STRING

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The assumptions in the Lund model suffice to prescribe a unique stochastic process for the fragmentation of a string into a set of hadrons, so long as the string is "flat", ie as long as the state described by the string consists only of a quark and an antiquark stretching a constant force field between them. Emission of gluons causes the string to trace more complicated surfaces in Minkowski space, and some form of generalization of the 1+1 dimensional model is required. One such generalizaiotn has been developed and implemented as a Monte Carlo routine "JETSET" by Torbjörn Sjöstrand, which has been hightly successful in describing experimental data. But there are theoretical reasons to believe that the fragmentation scheme employed in JETSET is not entirely satisfactory; most notably, non-adherance to the Lund Area law, and certain problems in handling transverse momenta. A few alternative scenarios, which we have examined in detail and implemented in separate computer programs, will be presented here, with comparisons to JETSET in certain simple cases. Our effort has been to preserve the area law for the fragmentation of a gluonic string, while we explored the possibility of allowing the fragmentation process to reshape the string surface slightly.

1 Introduction

The Lund model uses the decay of a massless relativistic string as a model for hadronization. Breaking of a string into a set of hadrons is thought of as a stochastic process with a constraint that the hadrons must be on the mass shell, a condition of saturation and an assumption of symmetry between the quark and the antiquark ends of the string. These assumptions lead [1,2] to a unique solution for the probabilities involved in string breaking. The probability for a certain hadronic state to emerge by the fragmentation of a string turns out to be proportional to the hadronic phase space and the negative exponential of the area spanned by the string in space-time before it decays.

2 Gluonic strings

2.1 Directrix

The principle of least action applied to a string gives a minimal surface as the world surface of the string, described by a boundary curve called directrix. The

[a] talk presented by Sandipan Mohanty

assumption that the string doesn't have any longitudinal degrees of freedom, and that the string tension has the same value in the rest frame of a local piece of string, lead to the conclusion that the directrix is a curve which has a light like tangent every where.

The directrix is determined in terms of the initial conditions on the string. Normally we are interested in a string state that was produced from a few quarks and gluons originating at a point or from a very small volume where perturbative effects dominate. So, we can think of the initial conditions on the string to be a few colour connected lumps of energy with different energy momentum vectors, which were created at the same point in space at some time. This initial condition leads to a very simple directrix : Partonic energy momenta arranged one after another in colour order.

2.2 Fragmentation as a process along the directrix

Since the string is completely described in terms of the directrix, it should be possible to describe all its properties, including how it fragments, as processes along the directrix. Without going into the details, let's just note that the following algorithm implements fragmentation of a string as a process along the directrix and is completely equivalent, for gluonless strings, to the iterative process used for instance in JETSET.

Let's define: $x_n = \sum_{i=1}^{n} p_i$, where p_i is the momentum of the i'th rank particle, and an associated vector q_n for the n'th breakup vertex. (for the gluonless case, this is just the vector position of the breakup point).

- Initialize $q_0 = k_q$, and x=0.
- pick a random number z from the distribution $f(z) = \frac{(1-z)^a}{z} e^{-\frac{bm^2}{z}}$.
- starting from the tip of the vector $(x_i + q_i)$, which is a point on the directrix, find a segment k along the directrix, such that $kq_i = \frac{m^2}{z}$, and k is lightlike.
- find particle momentum p, and next q from : $p_{i+1} = \frac{1}{2}l + \frac{1}{2}k$, and $q_{i+1} = q_i - \frac{1}{2}l + \frac{1}{2}k$ where, $l = 2z(q_i - \frac{q_i^2}{2q_i k}k)$
- update $x_{i+1} = x_i + p_i$, and the point on the directrix marked by x+q
- repeat all steps (except initialization!)

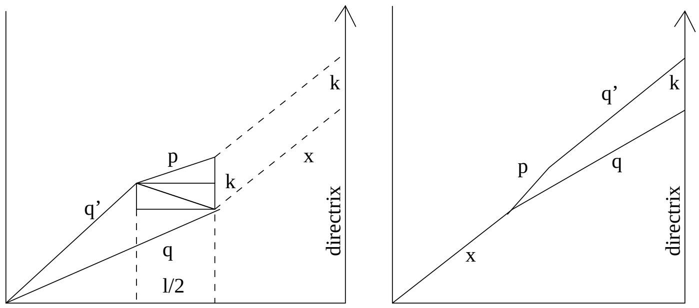

Figure 1: A fragmentation step between vertices at q and q'. Particle momentum p is the sum of a fraction z of the light-like vector $q - \frac{q^2}{2qk}k$ (written as $\frac{l}{2}$ in the text) and $\frac{k}{2}$. The figure on right shows the same step with the vector x drawn from the origin instead of the vectors q_i.

This algorithm implements area law in the following way.. The area of the quadilaterlal formed by q_i, k, q_{i+1} and p, an area associated with production of one particle, is the same as the area of the flat triangular region between q_i and q_{i+1} (translated to the origin) plus a half the hadron mass. So the area between the curve traced by the particle momenta arranged in a chain and the directrix is equal to the area spanned by the string before it decays. Building up the area traced by the string in this way, was used in making analytical calculations in the Lund model[3].

The above expressions for q_i, q_{i+1} and p are completely symmetric with respect to forward and backward steps along the directrix. One can define $\bar{z}$ such that,

$k = 2\bar{z}(q_{i+1} - \frac{q_{i+1}^2}{2q_{i+1}l}l)$, and $lq_{i+1} = \frac{m^2}{\bar{z}}$.

The relation between z and $\bar{z}$ can be found from the above: $\bar{z} = \frac{\frac{m^2}{z}}{q_1^2 + \frac{m^2}{z}}$, which inverts to, $z = \frac{\frac{m^2}{\bar{z}}}{q_2^2 + \frac{m^2}{\bar{z}}}$.

The square of the vectors q_i evolve as, $q_{i+1} = (1-z)(q^2 + \frac{m^2}{z})$

Eventually the vector $x+q$ will reach close to a gluon cornor, a place where the directrix changes direction. The remaining straight segment (which we will call 'c'), might be such that $cq < \frac{m^2}{z}$. It is then impossible to select a k that is a fraction of c and fullfills $kq = \frac{m^2}{z}$.

3 Fragmentation of a gluonic string

For a string with gluons, one can't assume that any region of the string is equivalent to any other region except for scaling and Lorentz transformations. The energy momenta of the decay products are not simple linear combinations of two light like vectors. The assumptions mentioned at the beginning of this article, therefore, do not lead to any definite scheme for fragmentation of a gluonic string.

3.1 Method used in JETSET

To solve the problem the following additional inputs were made into the Monte Carlo program JETSET :

- The string state as determined by perturbation theory (e.g. the dipole cascade model), is considered to be frozen during the fragmentation process. That is, the process of hadronization doesn't interfere with the structure of the string.

- z is to be reinterpreted as a connection between q_{i+1}^2 and q_i^2 through the relation $q_{i+1} = (1-z)(q_i^2 + \frac{m^2}{z})$, where q_is are the vector positions of the fragmentation vertices, on the string surface.

This last relation and $p^2 = m^2 = -(\Delta q)^2$ always uniquely determine a point q_{i+1} on the string surface. This means that the vertices obtained for the flat string by area law fragmentation, are projected on to the surface of the multigluon state. Unfortunately the area below the vertices is not preserved by this projection. That is, the probability for decay of the string into a cluster of hadrons is no longer proportional to the area spanned by the string before it decays.

3.2 Alternatives: Modifying the directrix

We start by recalling that a string state is a record of the quarks and gluons present in a hadronization environment. In particular, we note that large flat areas on the string surface correspond to large squared mass between two partons, and small areas correspond to low mass between two partons. At the point where our process along the directrix stopped, we were left with a mass cq_i which was of the order of a hadron mass. It was impossible to find a k that was along the directrix, was light-like, and had the required product with q, because the available lightlike segment of the directrix was too short.

But the directrix is not calculated to infinite precision. Parton shower in the dipole cascade model for instance, is continued down to a certain scale in gluonic transverse momentum. The smaller this scale, the more gluons we would have. And at about the energy scale of hadron masses, a new physical phenomenon takes place: hadrons form. So, the detailed structure of the string resolved to the hadronization scale, might perhaps be affected by the process by which hadrons form.

In the following, this idea that the dipole cascade model determines a string state approximately, and the small scale details are fixed dynamically by the process of hadronization, has been used to break the deadlock in the fragmentation algorithm at gluon cornors. We think of the segments k in each step of the fragmentation process outlined earlier, as the segments of the real directrix.

As long as these can be chosen along the perturbatively calculated directrix, there is no problem. This can't be done when the directrix changes direction, as mentioned earlier. Since k can not be chosen as a fraction of c, we take c and another segment of the directrix, K (not necessarily lightlike), and express their sum as two different lightlike vectors. That is, we write, $c+K = k+c'$, such that k and c' are lightlike. We demand also that $kq = \frac{m^2}{z}$, so that we can make one more fragmentation step using this k.

These conditions do not determine the vectors k and c' uniquely, since the length of the vector K is also unknown. By adding one more condition these vectors can be fixed. We tested several choices for this condition...

- eg2: Choose k as the unique light-like vector in the plane spanned by q and Q which is closer to Q than q.
- eg3: Choose k such that c and c' have the same lengths in the rest frame of q

We note that for both these choices, it may happen that the modification of the directrix will contain sharp "conrors". Such situations will break the coherence conditions in a perturbative QCD cascade, in particular, the requirement of strong angular ordering.

- eg4: Same as eg3 except for the provision that 3-vector part of c' would not be allowed have an angle in excess of 90 degrees with the next local part of the old directrix.

It is possible to introduce further "smoothness conditions" to get rid of these problems but we have found that the following choice solves the problem everywhere.

- eg5: Chose c' such that after the production of a particle using k, c' can be used for the production of another particle, so that after two steps the vector x+q is back on the directrix. That is, the required additional condition is $c'q' = \frac{m^2}{z'}$, where q' is the q vector obtained after the production step involving k.

We summarize in figures 2, the results of using these methods and compare them to a curve produced using JETSET in one simple situation: a perturbative string state consisting of a quark, an antiquark and a single gluon. We plot the rapidity distributions obtained from the different methods here. It is interesting to see that the fragmentation procedure produces too many particles over a large region (cf figure 2) although the modification of the directrix is actually only of the order of the hadron mass scale everywhere.

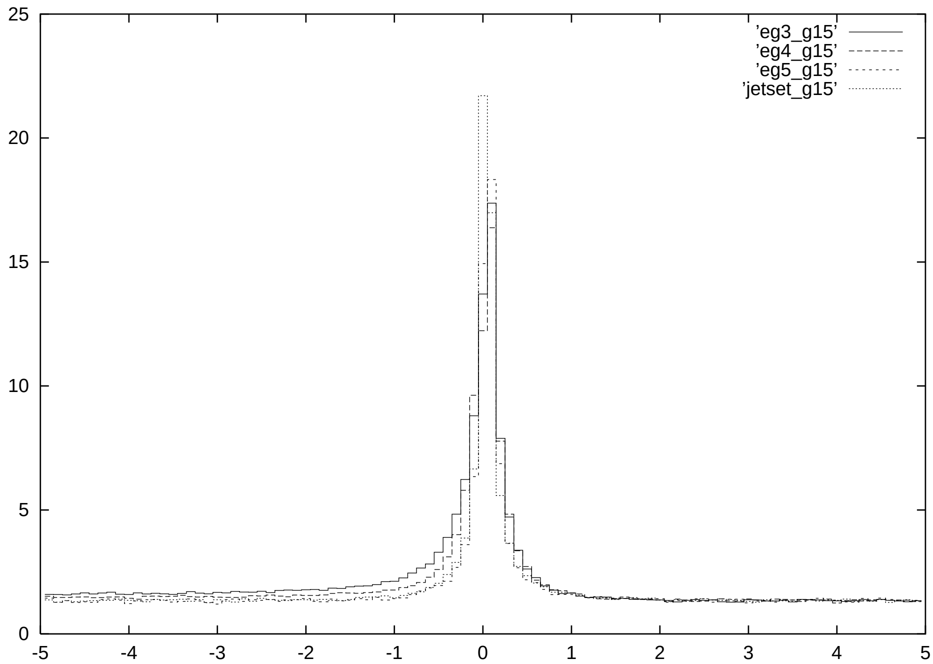

Figure 2: Rapidity distribution for a 3 jet event obtained from different fragmentation proceedures mentioned in the text.

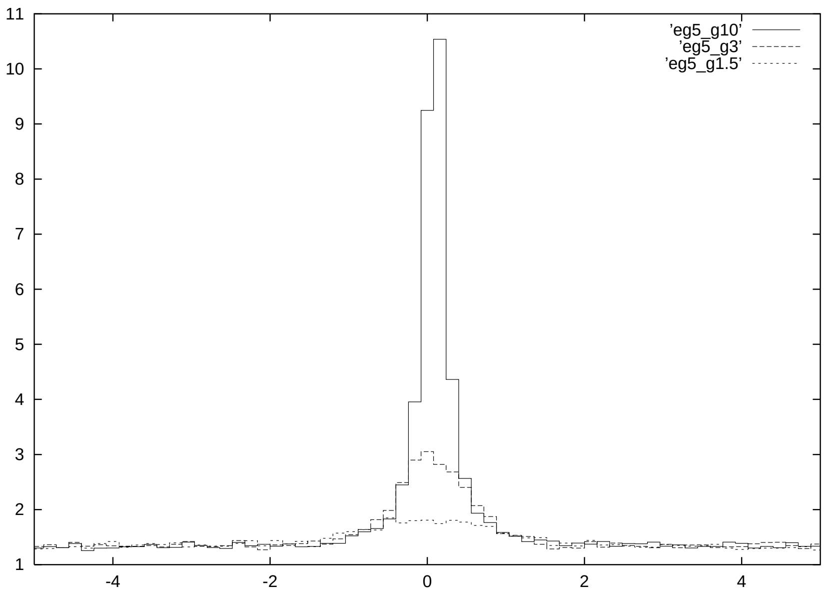

Figure 3: Rapidity distribution for a 3 jet event from eg5, for gluon transverse momentum values 10,3 and 1.5 GeV

4 Conclusions

We reported at the meeting that the additional soft gluons which we introduced to effectuate area law fragmentation have to have an angular ordering property. We have since the time of the meeting learned many more things (to be discussed in a forthcoming paper). But our major finding is that any modification of the directrix must be such that the bremstahlung coherence conditions of perturbative QCD must be fulfilled even if the modifications are chosen to be small and local. The method indicated at the end of the talk could be chosen as a general strategy for these modifications.

References

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