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ELECTROWEAK BARYOGENESIS

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Abstract

From observations of the universe and fundamental assumptions about its composition at early times, the conclusion of the existence of a mechanism producing a net baryon number is drawn. In this thesis the concept of electroweak baryogenesis is presented, and a computation of the fundamental separation of particles and antiparticles due to bubble formation for the case of charginos is described, calculated and evaluated.

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1 Introduction

Cosmology is an interesting field of science with many obvious difficulties - it is impossible to start up an experimental universe to test what happens if some of the initial conditions are changed, it is impossible to make observations on our only available universe stemming from its earliest times and even the fact that we are *living in* the object of interest, and not viewing it from outside, may give us trouble. We therefore need to make some seriously strong assumptions about the universe:

- the laws of physics are the same everywhere and every time in the history of the universe
- the complete universe looks the same even in the regions outside the part which is visible to us, i.e. it is homogeneous and isotropic on large scales.

These are clearly strong philosophical constraints, but they need to be made to be able to say *anything at all* about the universe we are living in and its history. Also there is the problem with our limited wisdom - at present we do not have the physical knowledge to be able to describe the history of the universe all the way back to the big bang, and therefore assumptions have to be made about how the universe looked like at the time from which we can *hope* to be able to describe the evolution - the Planck time. These assumptions of course play a great role in the way we view the continuing cosmological history. Despite its difficulties cosmology is a living science which, of course, is strongly connected to other parts of the physical community - all kinds of physics appear in one place or another in the history of the universe. But cosmology is not only gaining from other parts of physics, but also putting constraints on them. The example always mentioned is the number of neutrinos which from cosmological reasons was restricted to less than four before the number was precisely measured to three at LEP. Finally, one can perhaps ask oneself of the meaning and purpose of exploring how the universe was created and how it looks in galaxies millions of lightyears away, but this urge for exploring and understanding is something that humanity has always had. Judging from the history of mankind and my personal experience, it seems to me that one of the most natural questions to ask, and one of the hardest to answer, is where we come from and where we are going. At least some of the long-term answers will lay within the realm of cosmology.

2 A brief history of the early universe

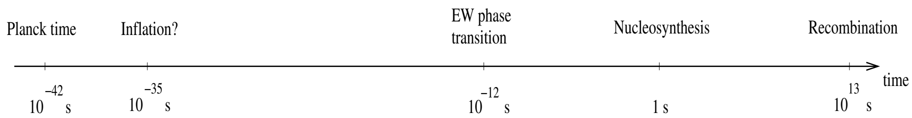
In 1922 Friedmann noticed that the solutions to the equations of general relativity in an isotropic and homogeneous space are not static, and hence proposed the idea that the universe must either be expanding or contracting[1]. That it was expanding was indeed observed in 1929 by Hubble, when he discovered his famous law $v = H_0 d$, stating that the

[1] To avoid this and keep the world static Einstein introduced the cosmological constant, which ever since has been going in and out of style for cosmologists.

velocity with which a distant galaxy is moving away from us is proportional to the distance to it. Tracing this movement backwards in time gives that the universe must have been both smaller and denser earlier in its history, and therefore, since the energy is conserved, it must also have been much hotter. One imagines that the universe was created in a big bang where both space, time and all the energy in the universe was created. As a consequence of the large energy concentration in the early universe the matter could not initially form any structures without immediately being torn apart, but as the space was expanding the energy density was diluted and the primordial soup, initially made up from elementary particles, could form more and more complex structures - first hadrons, then atoms, molecules, clouds, stars, planets etc.

Another aspect of this temperature change is that the primordial soup went through a series of phase transitions just like water will exist in the phases of gas, liquid and solid when cooled down from a high temperature.

The history of the early universe as we think of it is represented in the (log-scale) picture below.



Since we do not know anything about how and why the actual big bang took place, we have to make assumptions about the composition of the particle soup when it had cooled enough for us to apply our theories to it, and as usual the easiest and most intuitive assumptions are used. One that is crucial to this thesis is the assumption that there was created an equal number of particles and antiparticles in the big bang. This assumption is not entirely taken out of thin air because of its simplicity, there are some indications of that this indeed is a reasonable assumption [1]:

- Baryon (B) and lepton (L) number conservation are accidental symmetries of the minimal standard model for particle physics (SM), for example a dimension nine operator consisting of six right-handed quark fields in a colour singlet state,

$$\Lambda^{-5}(udd)^2$$

  is allowed and would lead to neutron-antineutron oscillations only suppressed by the large mass scale Λ. In the early universe when the temperature was high enough the effect of operators like this would not have been suppressed.

- Sphalerons, which are electroweak processes active at high temperatures, violate B and L (but not $B - L$). This will be explored later in this thesis.

- The X-boson vertices from several candidate Grand Unified Theories (GUT:s) also violate B and L.

Because of these multiple sources of B and L violation, any initial asymmetry would have been wiped out almost instantly producing net numbers of 0. Also there is the concept of inflation that is diluting every inhomogeneity in the initial universe, making them unmeasurable. This of course also applies specifically to the baryon asymmetry, which means that the net baryon number of the universe must have been produced *after* the phase of inflation. There are today clear evidence pointing towards inflation as a probable scenario in the peaks detected in the cosmic microwave background radiation, see for example [2].

However, looking at the part of the universe we can see, there does not seem to be any large amounts of antimatter - the complete universe seems to be made up by matter! Either there must be some mechanism dividing the matter and antimatter far apart, and we happen to live in a part of the universe where matter dominate, or there simply *is* an asymmetry between the numbers of particles and antiparticles, which implies that there must have been a mechanism producing a net baryon number in the universe.

Even a small net baryon number is enough, because when the universe had cooled enough the antimatter and matter annihilated leaving a little matter, which is all that is left in the universe today!

2.1 The Sakharov conditions

Sakharov pointed out in 1967 that three conditions had to be fulfilled to produce a net baryon number in the universe [1]:

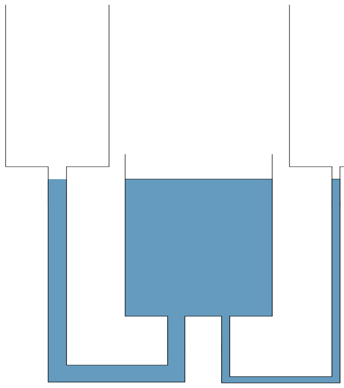
1. Interactions not conserving baryon number have to exist. This condition is obvious and leads in particular to proton decay, which has never been observed.

2. Charge conjugation symmetry (C) and the combined action of charge conjugation and parity change (CP) cannot be exact symmetries. This condition is forced upon us because otherwise the rate of reactions with antiparticles would be the same as the rate of reactions with particles, and no asymmetry could develop.

3. The universe must have been out of thermal equilibrium. This comes about because in thermal equilibrium the number densities of particles are described by Fermi-Dirac statistics, and since the particles and antiparticles, due to the invariance of the combined actions of CP and time reversal (CPT), have the same masses they will have the same number densities. This means that in an universe in thermal equilibrium a zero initial baryon number would always be zero.

2.2 The B-genesis machine

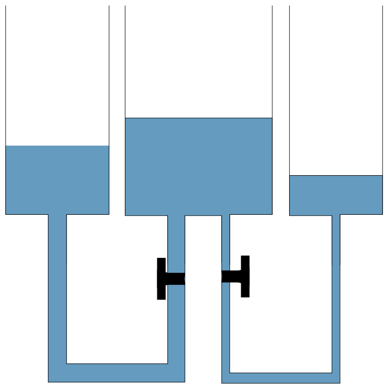
Rummukainen has developed a way to illustrate the need for the third Sakharov condition; imagine three connected vessels filled with water and connected in the way depicted below.



The middle vessel is movable in the vertical direction causing water to flow into the other two vessels. Now imagine that the water flowing into the left vessel represents the particles and the water flowing into the right one antiparticles. CP-violation is represented by the different thickness of the tubes connecting the vessels to the one in the middle. The case of the thermal equilibrium is represented by a very slow movement of the middle vessel causing the water level in the right and left vessel to be the same at all times, and hence no net baryon number is produced.

If instead the middle vessel is moved fast upwards there will be a difference in the amount of water in the vessels, but as time goes by, the asymmetry will be wiped out.

This means that we must have a way of shutting the B-violating process off - there must be a way to stop the water from flowing forever into the left and right vessels. Then the situation will be as in the picture below after lifting the middle vessel fast and then turning the knobs to stop the water flow.



Now we have created an asymmetry in the amount of water in the left and right vessel - we have created an baryon-antibaryon asymmetry!

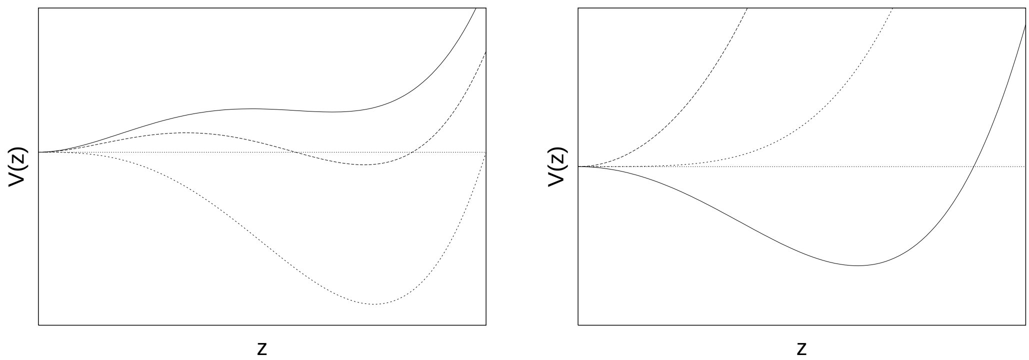
3 The electroweak phase transition - a way to fulfill the Sakharov conditions

3.1 Phase transitions

A phase transition is a concept from statistical physics describing a system changing its properties drastically as a consequence of for example a change in temperature or pressure. In our case it will be the temperature decrease in the expanding universe that triggers the phase transition. Phase transitions can be of different orders:

- A first order phase transition happen in a discontinuous way where the system abruptly changes its state from the symmetric to the broken phase.
- A second order phase transition happen in a smooth way, where the symmetric phase continuously changes into the broken phase.

The potentials for different temperatures in the different cases can be viewed in the diagrams below where the left one is describing a first order transition and the right one describes a second order transition.



As can be seen the in case of a second order phase transition the system can continuously fall into the broken minimum, whereas in the case of a first order phase transition, if the cooling rate is not very large, the system will have to tunnel the potential barrier to reach the broken minimum before the temperature is low enough to cause the minimum in $z = 0$ to become a maximum. This tunneling leads to the nucleation of bubbles containing the broken phase in a sea of the unbroken phase, and the transition happens through nucleation of more and more bubbles growing until the entire system is in the broken phase.

3.2 The Higgs mechanism and the electroweak phase transition

3.2.1 Within the SM

The Higgs mechanism is introduced to explain the fact that particles have mass. The idea is that the universe must be filled with a spin-zero field that is a doublet in the $SU(2)$

space and carries non-zero $U(1)$ hypercharge, but is a singlet in colour space. The particles can then interact with this field and therefore aquire mass. Because of states with one or more Higgs fields are not orthogonal to the vacuum even though they carry non-zero $SU(2)$ and $U(1)$ quantum numbers these symmetries are effectively broken. This means that the electroweak symmetries are valid for the Lagrangian of the SM, but not for the vacuum-state of the system, i.e. the symmetries are spontaneously broken, which is illustrated in the famous Mexican-hat potential.

Including finite temperature corrections leads to that the symmetry would not have been broken in the early high energy universe, and this spontaneous symmetry breaking occurring at some early stage in the universe could be viewed as a phase transition.

According to [3], the SM has such an electroweak phase transition of first order as long as the mass of the Higgs is smaller than 80 GeV, over which the transition instead becomes second order. Since 80 GeV definitely is less than the experimental bounds on the Higgs mass this means that the electroweak phase transition in the early SM-universe would have been second order.

3.2.2 Within the MSSM

It turns out that according to this theory one needs two Higgs fields pointing in different directions decided by their intermediate angle, β. These give rise to five Higgs particles; two of them electrically charged and three neutral, but also of course their supersymmetric partners (see Appendix B). Similar to the case in the SM there would have been a spontaneous breaking of this symmetry in the early universe. Which implies that there, even with today's experimental bounds of the Higgs mass, could have been an electroweak first order phase transition when the universe was approximately 10^{-12} s old and had a temperature of around 10^{12} K [1], [4].

3.3 The particle-antiparticle separation

A possible explanation of how to fulfill the Sakarhov conditions is that the baryon asymmetry came about during the breaking of the electroweak symmetry. The constraints from the Sakharov conditions on this model imply that the phase transition must have been of first order in order to fulfill the condition of non-equilibrium, which forces us to go into the realm of the MSSM. Also, as seen in the B-genesis machine, the B- and L-violating process has to be shut off in the broken phase.

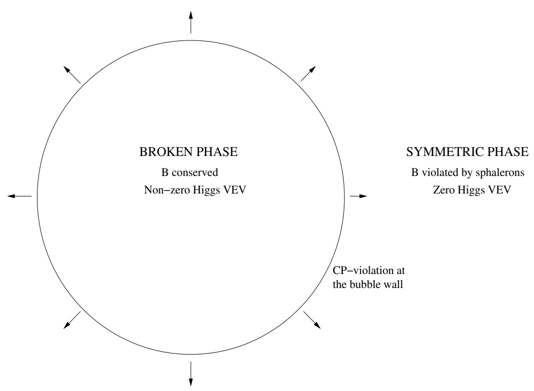
The transition went on by the nucleation of bubbles containing the broken phase with a non-zero Higgs vacuum expectation value (VEV), in a sea of the symmetric phase with a zero Higgs VEV. This means that the bubble walls behaved like quantum mechanical potentials against which particles either bounced or went trough, and also that CP will be violated at the bubble wall. As we will see later this means that the reflection and transmission can be different for left- and right-handed fermions as well as for particles and antiparticles, which in fact leads to a build-up of left-handed particles versus left-handed antiparticles in front of the expanding bubble wall. The net baryon number is however still zero at this point because it is balanced by an equal, but opposite, excess of right-handed particles to right-handed antiparticles.

3.4 Sphalerons - the baryon number wash-out and the actual baryogenesis

The concept of sphalerons is forced upon us in a non-Abelian gauge theory, causing the vacuum state to be degenerate, the different states being numbered by the integer Chern-Simons numbers, N_{CS} [4]. This enables transitions from one vacuum state to another, and a change of N_{CS} by one step causes the B and L to change by three steps, i.e. by the actions of sphalerons for example nine quarks can be turned into three leptons. Also, the sphalerons only affect the left-handed fermions and leave the right-handed ones unmodified.

In fact it is more complicated than this since the particles thermalize, causing the free energy to increase and the sphaleron transitions will go primarily in the direction towards *destroying* the existing baryon number. The rates of these reactions are proportional to the Boltzmann-factor and the temperature to the fourth power, $T^4 e^{-\frac{E_{Sph}}{T}}$, but the sphaleron energy is in turn proportional to the Higgs VEV, v, giving that the rates are exponentially suppressed in the broken phase. In the symmetric phase, where $v = 0$, however, the rates are proportional to T^4 [3]. This means that sphaleron transitions will be fully functioning at high rates outside the bubbles, but efficiently shut off inside them. In short, this will lead to a situation where sphalerons, tending to minimize the energy outside the bubble walls by destroying some of the particle asymmetry, are converting some of the excess of left-handed particles to other left-handed quarks and leptons, leading to that an excess of baryons is created in front of the bubble wall [4].

This asymmetry sooner or later will fall inside the bubble as it expands, in which the sphalerons are not able to continue the conversion, leading to a baryon asymmetry inside the bubbles. More and more bubbles will nucleate, combining to larger bubbles and so on, until the entire universe is filled up with the new, broken phase. After this transition there was a little excess of particles versus antiparticles, and hence matter dominates over antimatter in the universe today. The situation is described and summarized in the picture below.



4 Two introductory examples

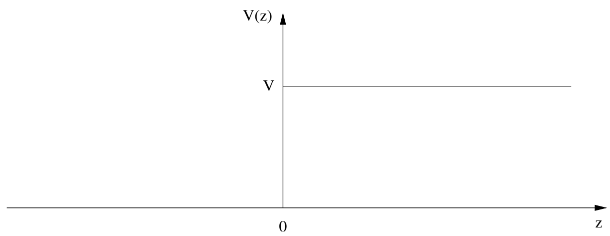
Because of the Yukawa coupling the Lagrangian in the SM and in the MSSM contains terms like

$$y_t \left(\bar{\psi}_L \phi \psi_R \right) + h.c.$$

where y_t is the Yukawa coupling and ϕ is the Higgs field. When the Higgs field has a value different from zero this has the structure of a mass-term, and we are led to a varying mass problem - the particles will have mass inside the bubbles but will be massless outside them. As the bubble is expanding in space it hits the surrounding particles causing them to either enter the bubble or bounce off its surface. To describe the situation mathematically we can of course transform our coordinate system so that we are in the inertial system of the wall, which means that the situation can be described by particles moving in an one-dimensional potential. In this section we will present two simplified examples of calculating the different reflection and transmission coefficients for particles and antiparticles in two different potentials - first the familiar Klein paradox and then the simplest example for asymmetric reflection, the complex two-step mass.

4.1 The Klein paradox

In this subsection we will, in principle, follow the presentation in [5], with only a few differences.



Let an incident plane wave, entirely made up from spin-up particles, with mass m and energy E come in from the negative z-axis and introduce a real, step potential with the value V at $z = 0$. According to the calculations in Appendix A, we can, ignoring the normalization, take the incoming wave to be:

$$\psi_{inc}(z) = \begin{pmatrix} 1 \\ 0 \\ \frac{k}{E+m} \\ 0 \end{pmatrix} e^{ikz}$$

where

$$k = \sqrt{E^2 - m^2}$$

The reflected wave can similarly be written as

$$\psi_{ref}(z) = a \left(\begin{array}{c} 1 \\ 0 \\ -\frac{k}{E+m} \\ 0 \end{array} \right) e^{-ikz} + b \left(\begin{array}{c} 0 \\ 1 \\ 0 \\ \frac{k}{E+m} \end{array} \right) e^{-ikz}$$

where the possibility of spin-change has been included. Similarly, the transmitted wave can be written

$$\psi_{trans}(z) = c \left(\begin{array}{c} 1 \\ 0 \\ \frac{q}{E-V+m} \\ 0 \end{array} \right) e^{iqz} + d \left(\begin{array}{c} 0 \\ 1 \\ 0 \\ -\frac{q}{E-V+m} \end{array} \right) e^{iqz}$$

Where we substituted E against $E-V$, giving an effective momentum of

$$q = \sqrt{(E-V)^2 - m^2}$$

The continuity condition for the wave function in $z = 0$ gives the demands

$$\left\{ \begin{array}{c} 1 + a = c \\ 1 - a = rc \end{array} \right.$$

where

$$r \equiv \frac{q}{k} \frac{E+m}{E-V+m}$$

all together resulting in

$$\psi_{ref}(z) = \frac{1-r}{1+r} \left(\begin{array}{c} 1 \\ 0 \\ -\frac{k}{E+m} \\ 0 \end{array} \right) e^{-ikz}$$

$$\psi_{trans}(z) = \frac{2}{1+r} \left(\begin{array}{c} 1 \\ 0 \\ \frac{q}{E-V+m} \\ 0 \end{array} \right) e^{iqz}$$

And from this we get

$$j_{inc} = \frac{2k}{E+m}$$

$$j_{ref} = -\frac{2k}{E+m} \frac{(1+r)^2}{(1-r)^2}$$

$$j_{trans} = \frac{2k}{E+m} \frac{4r}{(1+r)^2}$$

which means that

$$j_{inc} = j_{trans} + j_{ref}$$

just as it should. However, increasing the potential step causing $V > E + m$ means that $r < 0$ and

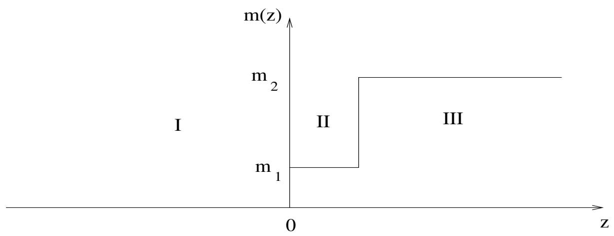
$$j_{ref} = \left(\frac{1-r}{1+r}\right)^2 j_{inc} > j_{inc}$$

which is the Klein paradox. The paradox as such has however nothing at all to do with this thesis, it is just included as an easy, straight forward example of calculation of transmission and reflection coefficients. The reason for the paradox appearing is because we introduce a classical potential which is varying over widths smaller than the de Broglie wavelength of the particle. The solution would be to treat the whole situation with the quantum field machinery.

4.2 A complex two-step varying mass

In this subsection we will study a simple example of a varying complex mass, relevant for the bubble wall problem.

Let particles move along the negative z-axis and encounter a two-step complex potential where the first step is m_1 at $z = 0$, and the second at $z = L$ has the height m_2. See picture below.



The relevant Dirac equation is

$$(i\partial_\mu \gamma^\mu - m(z)P_R - m^*(z)P_L)\,\psi = 0$$

We make the ansatz for ψ_1 in the different regions (see Appendix A for definitions)

$$\text{Region I: } \psi_{inc}(z) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ikz} \ , \ \psi_{ref}(z) = R \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ikz}$$

$$\text{Region II: } \psi_{\rightarrow}(z) = A \begin{pmatrix} c_p \\ 1 \end{pmatrix} e^{ipz} \ , \ \psi_{\leftarrow}(z) = B \begin{pmatrix} 1 \\ c_p^* \end{pmatrix} e^{-ipz}$$

$$\text{Region III: } \psi_{trans}(z) = T \begin{pmatrix} c_q \\ 1 \end{pmatrix} e^{iqz}$$

$$p = \sqrt{E^2 - m_1} \ , \ q = \sqrt{E^2 - m_2}$$

where

$$c_p = \frac{m_1}{E+p} \; , \; c_q = \frac{m_2}{E+q}$$

leading to

$$|R_1| = \frac{\left|1 + c_p K e^{-2ipL}\right|}{\left|c_p^* + K e^{-2ipL}\right|}$$

where

$$K = \frac{c_q c_p^* - 1}{c_p - c_q}$$

The same calculations for ψ_2 gives

$$|R_2| = \frac{\left|1 + c_p^* K^* e^{-2ipL}\right|}{\left|c_p + K^* e^{-2ipL}\right|}$$

meaning that the amount of reflected particles can be different for ψ_1 and ψ_2, just like we want, and in this case, it turns out, there will not be any Klein paradox.

Also it becomes obvious why we used two steps instead of just one - one is not enough. If $m_1 = m_2$ we would have $c_p = c_q$ and $|R_1| = |R_2|$ and no asymmetry could develop. The obvious generalization to do after this is to exchange the two steps with an infinite number of steps - to make the potential continuous. This has been done in [10] and the results produced are qualitatively similar to the ones in this section.

5 The chargino case

Even though there is asymmetry already in the situation described above, it turns out [6] that the largest effects for the asymmetry are expected for charginos, the supersymmetric partners of the W-bosons and the charged Higgs-bosons (for further description of supersymmetry see Appendix B). In this case we have two fermions with a position dependent mass-matrix as introduced in Appendix A, and this mass-matrix is the source of the strong CP-violation making the asymmetry so large.

Charginos are strongly coupled to third generation quarks, which means that any asymmetry created in the amount of anticharginos and charginos would, via their subsequent decays, create an B-asymmetry.

The mass-matrix can in this case be taken to [6]

$$M = \begin{pmatrix} m & gH_2(z) \\ gH_1(z) & \mu \end{pmatrix} \equiv \begin{pmatrix} m & u_2 \\ u_1 & \mu \end{pmatrix}$$

where

$$H_1(z) = H(z) \sin\beta$$

$$H_2(z) = H(z) \cos\beta$$

and

$$H(z) = \frac{v}{2\sqrt{2}} \left(1 + \tanh\left(\frac{z}{w}\right)\right)$$

β is here the angle between the Higgs fields, v is the Higgs VEV in the broken phase and w is the width of the bubble wall.

If we choose the matrices U and V so that the off-diagonal 2×2-matrices in

$$\partial_3 \widetilde{\psi}_{s=+1} = i \begin{pmatrix} -E & UM^\dagger V^\dagger \\ -VMU^\dagger & E \end{pmatrix} \widetilde{\psi}_{s=+1}$$

$$\partial_3 \widetilde{\psi}_{s=-1} = i \begin{pmatrix} -E & VMU^\dagger \\ -UM^\dagger V^\dagger & E \end{pmatrix} \widetilde{\psi}_{s=-1}$$

are diagonal matrices with the properties

$$UM^\dagger V^\dagger = \begin{pmatrix} m_1^* & 0 \\ 0 & m_2^* \end{pmatrix}, \; VMU^\dagger = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

we get

$$UM^\dagger MU^\dagger = \begin{pmatrix} |m_1|^2 & 0 \\ 0 & |m_2|^2 \end{pmatrix}, \; VMM^\dagger V^\dagger = \begin{pmatrix} |m_1|^2 & 0 \\ 0 & |m_2|^2 \end{pmatrix}$$

This results in an ordinary eigenvector problem giving the U- and V-matrices to be

$$V = diag\left(e^{i\phi_i}\right) \sqrt{\tfrac{2}{\Lambda(\Lambda+\Delta)}} \begin{pmatrix} \frac{1}{2}(\Delta + \Lambda) & a \\ -a^* & \frac{1}{2}(\Delta + \Lambda) \end{pmatrix}$$

$$U = diag\left(e^{i\overline{\phi}_i}\right) \sqrt{\tfrac{2}{\overline{\Lambda}(\overline{\Lambda}+\overline{\Delta})}} \begin{pmatrix} \frac{1}{2}\left(\overline{\Delta} + \overline{\Lambda}\right) & b \\ -b^* & \frac{1}{2}\left(\overline{\Delta} + \overline{\Lambda}\right) \end{pmatrix}$$

Where we have defined

$$a = mu_1^* + \mu^* u_2$$

$$\Delta = |m|^2 - |\mu|^2 - |u_1|^2 + |u_2|^2$$

$$\Lambda = \sqrt{\Delta^2 + 4\,|a|^2}$$

$$b = m^* u_2 + \mu u_1^*$$

$$\overline{\Delta} = |m|^2 - |\mu|^2 + |u_1|^2 - |u_2|^2$$

$$\overline{\Lambda} = \sqrt{\overline{\Delta}^2 + 4\,|b|^2}$$

Now we see that in the symmetric phase, where $H(z) = 0$ and the mass-matrix already is diagonal, $U = V = 1$ and the mass eigenvalues are m and μ.

5.1 Outline of the method used

When $|z| = L >> 0$ the solutions to the Dirac equation will be of the form described in Appendix A, and we can, deep in the broken phase, initiate wavefunctions to be

$$\tilde{\psi}_{1,s=+1}(L) = \begin{pmatrix} m_1^* \\ 0 \\ E + p_1 \\ 0 \end{pmatrix}, \tilde{\psi}_{2,s=+1}(L) = \begin{pmatrix} 0 \\ m_2^* \\ 0 \\ E + p_2 \end{pmatrix}$$

$$\tilde{\psi}_{1,s=-1}(L) = \begin{pmatrix} m_1 \\ 0 \\ E + p_1 \\ 0 \end{pmatrix}, \tilde{\psi}_{2,s=-1}(L) = \begin{pmatrix} 0 \\ m_2 \\ 0 \\ E + p_2 \end{pmatrix}$$

Where we do not care about the phases of the functions, because we will anyway average over the different phases of the incoming particles. Then we transform these solutions according to

$$\psi_{s=\pm 1}(L) = \begin{pmatrix} U^\dagger & 0 \\ 0 & V^\dagger \end{pmatrix} \tilde{\psi}_{s=\pm 1}(L)$$

and evolve them with the equations below using the fourth order Runge-Kutta method implemented in a C++ code.

$$\partial_3 \psi_{s=+1} = i \begin{pmatrix} -E & M^\dagger \\ -M & E \end{pmatrix} \psi_{s=+1}$$

$$\partial_3 \psi_{s=-1} = i \begin{pmatrix} -E & M \\ -M^\dagger & E \end{pmatrix} \psi_{s=-1}$$

Finally at $z = -L$ we can transform them back according to

$$\tilde{\psi}_{s=\pm 1}(-L) = \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix} \psi_{s=\pm 1}(-L)$$

but this is an identity if we go deep enough into the symmetric phase.

The sum of these wavefunctions can now be written as a sum of the incoming and reflected wavefunctions for $s = \pm 1$ respective

$$T_{1,s}\tilde{\psi}_{1,s}(-L) + T_{2,s}\tilde{\psi}_{2,s}(-L) = I_{1,s}\tilde{\psi}^{in}_{1,s} + I_{2,s}\tilde{\psi}^{in}_{2,s} + R_{1,s}\tilde{\psi}^{ref}_{1,s} + R_{2,s}\tilde{\psi}^{ref}_{2,s}$$

where $T_{i,s}$, $I_{i,s}$ and $R_{i,s}$ are complex numbers. These equations can now be solved for $|T_{i,s}|$ and $|I_{i,s}|$ independently for $s = \pm 1$ by normalizing $|I_{i,s}| = 1$ and averaging over all possible incoming wave function phase angles, meaning that the reflected current becomes

$$j_R = |R|^2 \overline{\tilde{\psi}}^{ref} \gamma_3 \tilde{\psi}^{ref}$$

One problem using this method occurs when $E < |m_2|$ because then this solution should be exponentially suppressed in the broken phase and tend to blow up as the integration proceeds. However, knowing the problem's existence and nature, there is no problem in caring for it by not going to far from the wall in the integrations.

5.2 The values for parameters used

Some of the numerical values used are of course varied as seen in the plots below. The temperature is taken to be 150 GeV, and the Higgs-fields to

$$H_1(z) = H(z)\sin\beta$$

$$H_2(z) = H(z)\cos\beta$$

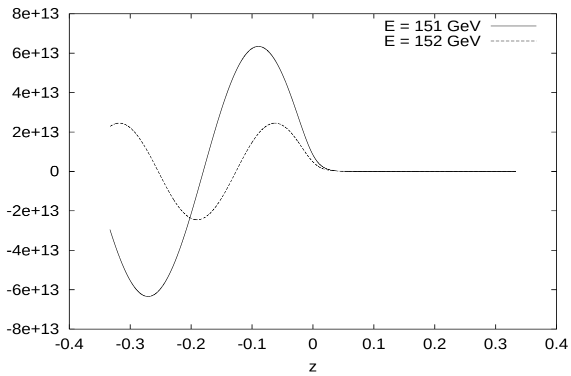
where

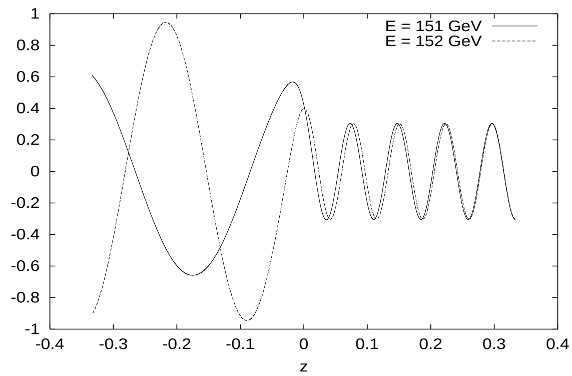
$$H(z) = \frac{v}{2\sqrt{2}}\left(1 + \tanh\left(\frac{z}{w}\right)\right)$$

The parameter v is the Higgs VEV in the broken phase and it is set to 100 GeV, the w-parameter is the wall-width and takes values around 0.01 GeV^{-1} and the β is the angle between the Higgs fields, which we specified by taking $tan\beta = 10$. As for the mass parameters m will take values around 150 GeV and $\mu = |\mu|\, e^{0.05i}$ where $|\mu|$ too will take values around 150 GeV. The gauge coupling strength, g, will be fixed to 2/3.

5.3 The results

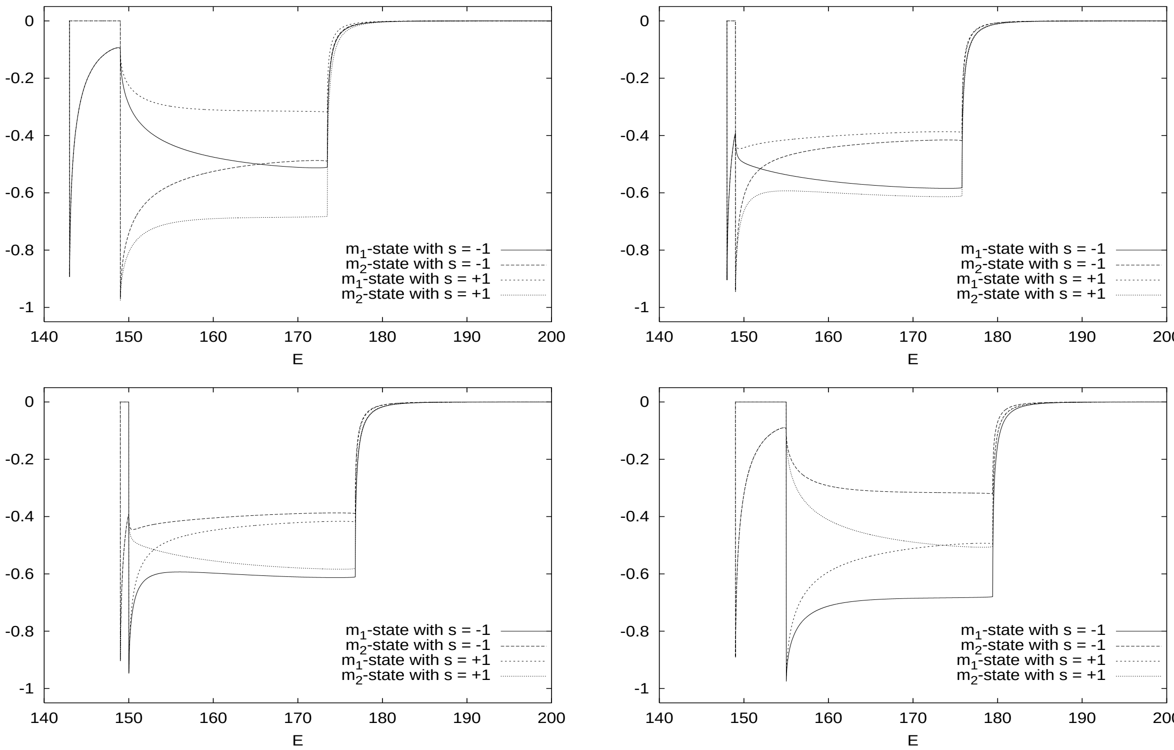
For illustration and test of the program we first made some plots showing how the wave function varies along the z-axis. The left plot below shows the real part of the first component of the wave function for the m_1-state with spin +1 as a function of z in GeV^{-1} for two different values of E, when we used $|\mu| = 149$ GeV and $m = 150$ GeV. The right plot shows the same thing for the m_2-state.



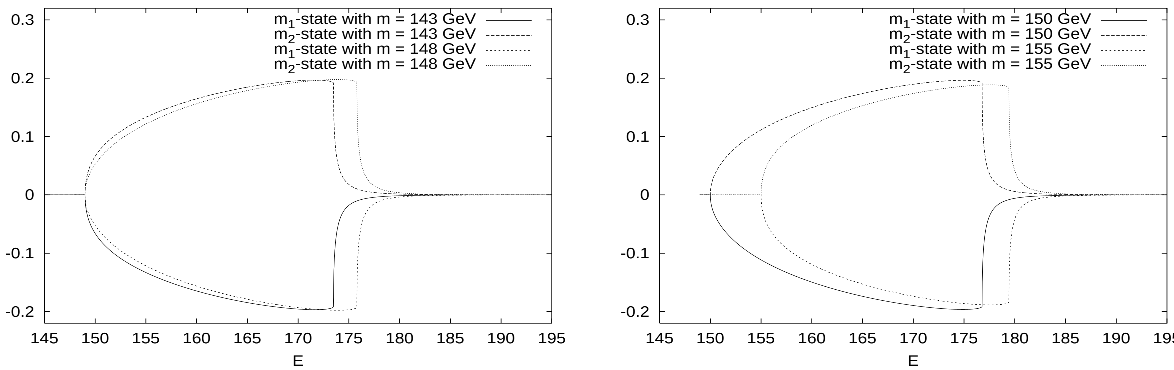


In both cases we have $m_2 < E < m_1$ and the plots behave just as expected; the m_1-state will not exist inside the bubbles whereas the m_2-state will.

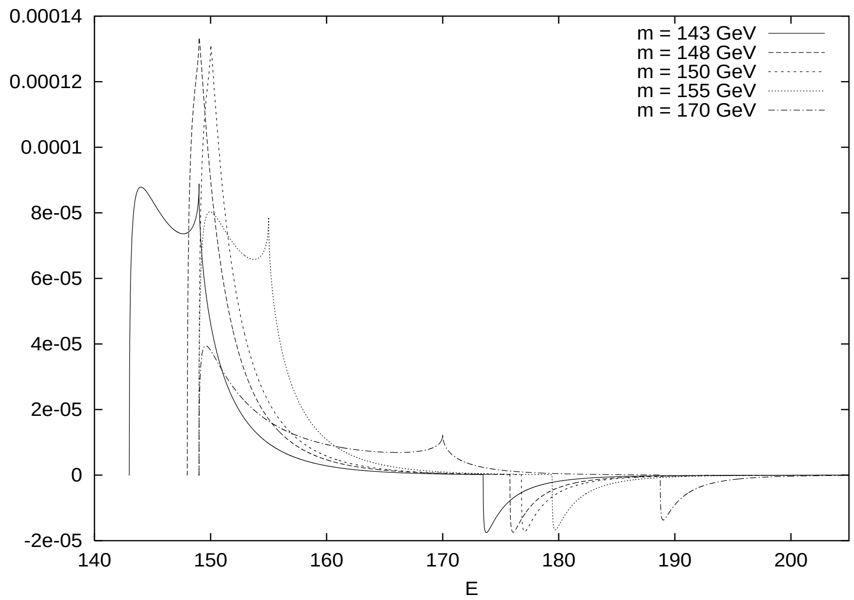
The interesting things, however, are the different currents for the different particles flowing into and out of the bubbles. Below we have made plots showing the different reflected currents as functions of the energy, E. We have here used $m = 143$ GeV and $|\mu| = 149$ GeV for the upper left plot and $m = 148$ GeV together with $|\mu| = 149$ GeV for the right one. For the lower plots we used $m = 150$ GeV and $|\mu| = 149$ GeV for the left one and $m = 155$ GeV together with $|\mu| = 149$ GeV for the right one.



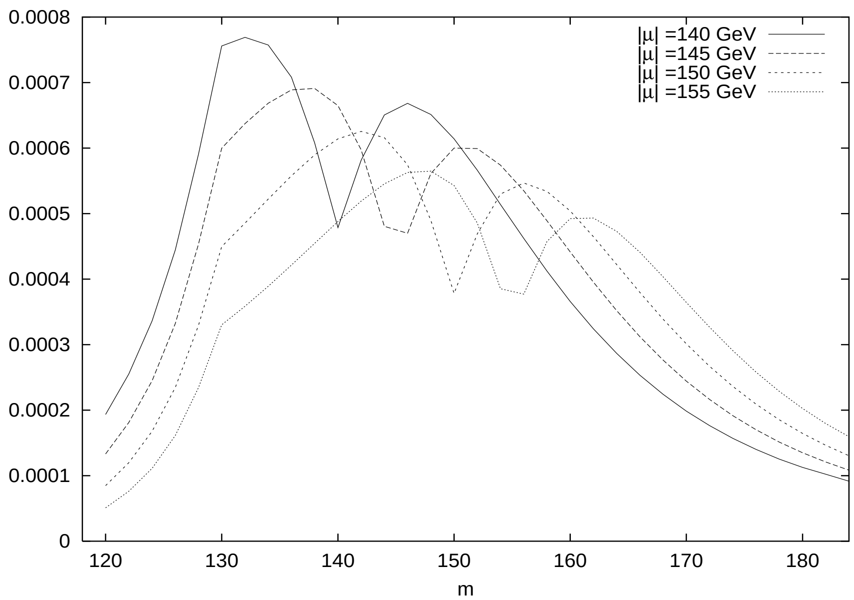
As can be seen the currents are clearly different for different mass and spin states, and the next few plots shows *how* they are different. Firstly we show the difference in reflected current for the negative spin state and the positive one for the m_1-state and the same thing for the m_2-state as functions of E. In the left plot we used $m = 143$ GeV and 148 GeV together with $|\mu| = 149$ GeV, and in the right plot we used $m = 150$ GeV and 155 GeV together with $|\mu| = 149$ GeV.



Secondly we plot the total difference for the reflected currents with different spin states for different energies when the mass parameters are fixed to $m = 143$ GeV, 148 GeV, 150 GeV, 155 GeV respective 170 GeV and $|\mu| = 149$ GeV.



And then we plot the total difference for the reflected currents with different spin states integrated over energies between 130 GeV and 220 GeV against the mass parameter m when the parameter $|\mu|$ is fixed to 140 GeV, 145 GeV, 150 GeV and 155 GeV.



6 Future calculations

We have now calculated the amount of separation of charginos and anticharginos in the vicinity of the bubble wall, but as mentioned before, we have not got to the stage of a net baryon number yet. First the asymmetry of charginos needs to be transported to quarks, and for a description of this calculation see [6]. This will will lead to a local asymmetry in baryons that sphaleron reactions will tend to eliminate, and as described earlier these reactions only happen outside the bubbles; the asymmetry inside the bubbles is conserved. For a description of the sphaleron rates see for example [9]. To get a realistic description of the situation one would also need to let the particles' energies be thermically distributed, and the wall as such is of course also important; what shape it has, what velocity it has and so on.

Not until all these things are included and cared for in the calculation we can finally calculate the baryon asymmetry this model can produce in the universe, and find out whether it is a possible scenario in comparing it to observations. If it is successful it will not only be another in the line of arguments for MSSM, but also putting constraints on the theory - one would most likely need to have μ, m and v within certain values to produce the observed baryon asymmetry.[2]

7 Summary

In this thesis we have described the concept of electroweak baryogenesis as a possible way to get from the assumed initial condition with equal amounts of matter and antimatter to today's situation with an excess of matter. We have also made a calculation of the separation of particles and antiparticles for charginos and roughly outlined what steps should be taken to, from this, calculate the actual baryon asymmetry this mechanism can produce.

8 Acknowledgments

I would of course like to thank my advisors; Johan Bijnens, Kimmo Kainulainen and Kari Rummukainen for their help, encouragement and small discussions, which were very enlightening and helpful for me. Also Torbjörn Sjöstrand has helped me with some small things during my thesis work. Thank you all!

[2] However, as described earlier, the entire calculation relies on the cosmological assumptions, and therefore it will not be a proof for MSSM but only pointing out MSSM as a probable theory.

Appendix A; The Dirac equation

Since the Schrödinger equation contains a first order derivative in space and a second order in time, it cannot be relativistic, and this forced Dirac to construct a new equation built on certain constraints [5]:

- It should have the same order derivative in space and time.
- Its solutions ψ must fulfill the Klein-Gordon equation based on the relativistic relation $E^2 = p^2 + m^2$.
- There must exist a conserved positive four-vector current density.
- The components of ψ are independent functions of the coordinates.

These constraints and a little trial and error led Dirac to the equation nowadays wearing his name;

$$(i\partial_\mu \gamma^\mu - m)\,\psi = 0$$

Where it turns out that the γ-objects are 4×4-matrices, in which there is some freedom of choice. The convention used by us in this thesis is described in Appendix C. The enhancements of the Dirac equation described below are also found just by trial and error, and are used because of their resemblance with nature and are not derived from any other fundamental principles than the ones described above.

In the calculations in this thesis we will always restrict ourselves to the case in which the particles are incident at right angles to an one-dimensional potential, and so will the calculations in this Appendix be. This is obviously no serious restriction because we can always Lorentz-boost us into that frame of reference.

Free plane wave solutions

To find plane wave solutions to the free Dirac equation

$$(i\partial_\mu \gamma^\mu - m)\,\psi = 0$$

we can demand the solutions to be on the form

$$\begin{cases} \psi^{(+)}(z) = e^{-ikz}u(k) \\ \psi^{(-)}(z) = e^{ikz}v(k) \end{cases}$$

Where $\psi^{(+)}(z)$ has positive energies and $\psi^{(-)}(z)$ has negative energies. We also demand that

$$\begin{cases} k^0 = E \geq 0 \\ k^\mu k_\mu = m^2 \end{cases}$$

leading to

$$\begin{cases} (k_\mu\gamma^\mu - m)\, u(k) = 0 \\ (k_\mu\gamma^\mu + m)\, v(k) = 0 \end{cases}$$

If we now assume

$$\begin{cases} m \neq 0 \\ k^\mu = (m, \overline{0}) \end{cases}$$

the equations transform to

$$\begin{cases} (\gamma^0 - I)\, u(m,\overline{0}) = 0 \\ (\gamma^0 + I)\, v(m,\overline{0}) = 0 \end{cases}$$

Which means that there are four independent solutions - two u's and two v's - meaning we can take

$$u^{(1)}(m,\overline{0}) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},\ u^{(2)}(m,\overline{0}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$v^{(1)}(m,\overline{0}) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},\ v^{(2)}(m,\overline{0},) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Now we would like to Lorenz-boost these solutions to energy-momentum k, but it is easier using

$$(k_\mu\gamma^\mu - m)\,(k_\mu\gamma^\mu + m) = 0$$

giving

$$\begin{cases} (k_\mu\gamma^\mu - m)\, a\, (k_\mu\gamma^\mu + m)\, u^{(\alpha)}(m,\overline{0}) = 0 \\ (k_\mu\gamma^\mu + m)\, b\, (k_\mu\gamma^\mu - m)\, v^{(\alpha)}(m,\overline{0}) = 0 \end{cases}$$

and hence we can take

$$\begin{cases} u^{(\alpha)}(k) = a\,(k_\mu\gamma^\mu - m)\, u^{(\alpha)}(m,\overline{0}) \\ v^{(\alpha)}(k) = b\,(k_\mu\gamma^\mu - m)\, v^{(\alpha)}(m,\overline{0}) \end{cases}$$

If we define $\varphi^{(\alpha)}(m,\overline{0})$ and $\chi^{(\alpha)}(m,\overline{0})$ as the non-vanishing two-spinors of $u^{(\alpha)}(m,\overline{0})$ and $v^{(\alpha)}(m,\overline{0})$ respective, we find

$$u^{(\alpha)}(k) = a \begin{pmatrix} (E+m)\varphi^{(\alpha)}(m,\overline{0}) \\ \overline{k}\cdot\overline{\sigma}\varphi^{(\alpha)}(m,\overline{0}) \end{pmatrix}$$

$$v^{(\alpha)}(k) = -b \begin{pmatrix} \overline{k}\cdot\overline{\sigma}\chi^{(\alpha)}(m,\overline{0}) \\ (E+m)\chi^{(\alpha)}(m,\overline{0}) \end{pmatrix}$$

Normalizing the found solutions to

$$\overline{u}^{(\alpha)}(k)u^{(\beta)}(k) = \delta_{\alpha\beta}, \overline{v}^{(\alpha)}(k)v^{(\beta)}(k) = -\delta_{\alpha\beta}$$

$$\overline{u}^{(\alpha)}(k)v^{(\beta)}(k) = \overline{v}^{(\alpha)}(k)u^{(\beta)}(k) = 0$$

leads to the free, plane wave solutions

$$u^{(\alpha)}(k) = \begin{pmatrix} \sqrt{\frac{E+m}{2m}}\varphi^{(\alpha)}(m,\overline{0}) \\ \frac{\overline{k}\cdot\overline{\sigma}}{\sqrt{2m(E+m)}}\varphi^{(\alpha)}(m,\overline{0}) \end{pmatrix}$$

$$v^{(\alpha)}(k) = \begin{pmatrix} \frac{\overline{k}\cdot\overline{\sigma}}{\sqrt{2m(E+m)}}\chi^{(\alpha)}(m,\overline{0}) \\ \sqrt{\frac{E+m}{2m}}\chi^{(\alpha)}(m,\overline{0}) \end{pmatrix}$$

Solutions for a complex position dependent mass

If the mass is complex and z-dependent, the Dirac equation looks like

$$\left(i\partial_\mu\gamma^\mu - m(z)P_R - m^*(z)P_L\right)\psi = 0$$

which written out explicitly will be

$$i\partial_0 \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \\ \psi^{(4)} \end{pmatrix} + i\partial_3 \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \\ \psi^{(4)} \end{pmatrix} -$$

$$-m(z) \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \\ \psi^{(4)} \end{pmatrix} - m^*(z) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \\ \psi^{(4)} \end{pmatrix} = 0$$

and we can divide the equation into two as below, where we also assumed $\psi(t,z) = e^{-iEt}\psi(z)$.

$$\begin{cases} \left(-E\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + i\partial_3 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} m^*(z) & 0 \\ 0 & m(z) \end{pmatrix}\right)\begin{pmatrix} \psi^{(1)} \\ \psi^{(3)} \end{pmatrix} = 0 \\ \left(-E\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + i\partial_3 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} m^*(z) & 0 \\ 0 & m(z) \end{pmatrix}\right)\begin{pmatrix} \psi^{(2)} \\ \psi^{(4)} \end{pmatrix} = 0 \end{cases}$$

These can be transformed to

$$\begin{cases} \partial_3\psi_1 = i\begin{pmatrix} -E & m(z) \\ -m^*(z) & E \end{pmatrix}\psi_1 \\ \partial_3\psi_2 = i\begin{pmatrix} -E & m^*(z) \\ -m(z) & E \end{pmatrix}\psi_2 \end{cases}$$

where we have defined

$$\psi_1 = \begin{pmatrix} \psi^{(2)} \\ -\psi^{(4)} \end{pmatrix} , \psi_2 = \begin{pmatrix} \psi^{(3)} \\ -\psi^{(1)} \end{pmatrix}$$

Solutions for charginos

In the case of charginos in MSSM the Dirac equation in our Lorenz-boosted case is

$$\left(i\partial_\mu\gamma^\mu - MP_R - M^\dagger P_L\right)\psi = 0$$

which is an 8×8-equation with a 2×2-mass matrix

$$M = \begin{pmatrix} m & gH_2(z) \\ gH_1(z) & \mu \end{pmatrix}$$

The matrices can be written as direct products of the Pauli matrices like

$$\gamma^0 = (-\sigma^1 \otimes I) \ , \ \gamma^i = (i\sigma^2 \otimes \sigma^i) \ , \ \gamma^5 = (\sigma^3 \otimes I)$$
$$P_R = (p_R \otimes I) \ , \ P_L = (p_L \otimes I)$$

where

$$p_R = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} , p_L = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

If we then change our coordinate system so that all the movement is in the z-direction, we can rewrite the equation above according to

$$\left(i\left(-\sigma^1 \otimes I \otimes I\right)\partial_0 + i\left(i\sigma^2 \otimes \sigma^3 \otimes I\right)\partial_3 - (p_R \otimes I \otimes M) - \left(p_L \otimes I \otimes M^\dagger\right)\right)\psi = 0$$

Multiplication by γ^0 gives

$$\left(i\left(I \otimes I \otimes I\right)\partial_0 + i\left(\sigma^3 \otimes \sigma^3 \otimes I\right)\partial_3 - \left(-\sigma^1 p_R \otimes I \otimes M\right) - \left(-\sigma^1 p_L \otimes I \otimes M^\dagger\right)\right)\psi = 0$$

Now the middle component is diagonal in all terms, but differs in sign in the ∂_3-term, which means that we can split the 8×8-equation into two 4×4-equations, with $s = \pm 1$

$$\left(i\left(I \otimes I\right)\partial_0 - is\left(\sigma^3 \otimes I\right)\partial_3 - \left(-\sigma^1 p_R \otimes M\right) - \left(-\sigma^1 p_L \otimes M^\dagger\right)\right)\psi = 0$$

or, slightly rewritten

$$\left(i\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\partial_0 - is\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\partial_3 + \begin{pmatrix} 0 & M^\dagger \\ M & 0 \end{pmatrix}\right)\psi = 0$$

Introduction of two unitary matrices, U and V, and multiplication according to

$$\begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix}\left(i\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\partial_0 - is\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\partial_3 + \begin{pmatrix} 0 & M^\dagger \\ M & 0 \end{pmatrix}\right)\begin{pmatrix} U^\dagger & 0 \\ 0 & V^\dagger \end{pmatrix}\begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix}\psi = 0$$

leads to

$$\left(i\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\partial_0 - is\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\partial_3 + \begin{pmatrix} 0 & UM^\dagger V^\dagger \\ VMU^\dagger & 0 \end{pmatrix} - is\begin{pmatrix} UU^{\dagger'} & 0 \\ 0 & -VV^{\dagger'} \end{pmatrix}\right)\widetilde{\psi} = 0$$

where

$$\widetilde{\psi} = \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix}\psi$$

Simplification with help of the assumption $\psi(t,z) = e^{-iEt}\psi(z)$ gives

$$\left(is\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\partial_3 - \begin{pmatrix} E & UM^\dagger V^\dagger \\ VMU^\dagger & E \end{pmatrix} + is\begin{pmatrix} UU^{\dagger'} & 0 \\ 0 & -VV^{\dagger'} \end{pmatrix}\right)\widetilde{\psi} = 0$$

If the derivatives of the U- and V-matrices are zero, we get

$$\left(is\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\partial_3 - \begin{pmatrix} E & UM^\dagger V^\dagger \\ VMU^\dagger & E \end{pmatrix}\right)\widetilde{\psi} = 0$$

and the same transformations as in the case of a varying complex mass within the SM gives the relations for $s = \pm 1$ respective

$$\partial_3\widetilde{\psi}_{s=+1} = i\begin{pmatrix} -E & UM^\dagger V^\dagger \\ -VMU^\dagger & E \end{pmatrix}\widetilde{\psi}_{s=+1}$$

$$\partial_3\widetilde{\psi}_{s=-1} = i\begin{pmatrix} -E & VMU^\dagger \\ -UM^\dagger V^\dagger & E \end{pmatrix}\widetilde{\psi}_{s=-1}$$

If the U- and V-matrices are chosen so that the diagonal matrices in the equation above have the properties

$$UM^\dagger V^\dagger = \begin{pmatrix} m_1^* & 0 \\ 0 & m_2^* \end{pmatrix},\ VMU^\dagger = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

we will get solutions for $s = \pm 1$ respectively

$$\widetilde{\psi}_{s=+1}(z) = \begin{pmatrix} (E+p_1)\,e^{-ip_1z} \\ (E+p_2)\,e^{-ip_2z} \\ m_1e^{-ip_1z} \\ m_2e^{-ip_2z} \end{pmatrix},\ \widetilde{\psi}_{s=+1}(z) = \begin{pmatrix} m_1^*e^{ip_1z} \\ m_2^*e^{ip_2z} \\ (E+p_1)\,e^{ip_1z} \\ (E+p_2)\,e^{ip_2z} \end{pmatrix}$$

$$\widetilde{\psi}_{s=-1}(z) = \begin{pmatrix} (E+p_1)\,e^{-ip_1z} \\ (E+p_2)\,e^{-ip_2z} \\ m_1^*e^{-ip_1z} \\ m_2^*e^{-ip_2z} \end{pmatrix},\ \widetilde{\psi}_{s=-1}(z) = \begin{pmatrix} m_1e^{ip_1z} \\ m_2e^{ip_2z} \\ (E+p_1)\,e^{ip_1z} \\ (E+p_2)\,e^{ip_2z} \end{pmatrix}$$

Appendix B; The minimal supersymmetric standard model

For a more extensive review of MSSM see for example [7]. This appendix is mostly based on information gathered from this reference.

Supersymmetry started out as an imaginary extension of the SM, but is today widely believed in, even though no supersymmetric particles have been directly observed. The idea is that there is an operator able of transforming a fermion into a boson and vice versa, giving that every particle in the SM has a supersymmetric partner with different spin. The ones important to this thesis are the so called charginos; the fermionic partners to the W gauge bosons and the charged Higgs bosons.

The possible interactions among these new particles are given by taking the interactions from the SM and exchanging the particles in the vertices for their supersymmetric partners. The substitution must be made in pairs in order to correctly conserve the angular momentum in the reaction. This means that

- supersymmetric particles will always be produced in pairs from normal particles and vice versa
- in the decay products of a supersymmetric particle there will always be at least one supersymmetric particle
- the lightest supersymmetric particle will be stable since there are no particles into which it can decay

These rules can actually be viewed as consequences of the conservation of R-parity[3] - normal particles have $R = 0$, supersymmertic particles have $R = +1$ and supersymmetric antiparticles have $R = -1$. Specifically important for this work is that the rules lead to that any asymmetry created in the amount of anticharginos versus charginos would, due to their strong coupling to third generation quarks, via their subsequent decays lead to an asymmetry in the amount of quarks versus antiquarks.

As for the *minimal* supersymmetric model, this is just one of the several possible supersymmetric models, but it is the one with the lowest particle content and the slightest step away from SM, which makes it easiest to grasp and the most natural next step towards a complete theory.

As we mentioned there is no clear evidence for supersymmetry, but there are some experimental hints that point towards it. For example to unify the running coupling constants at grand unified energies supersymmetry is needed, the lightest supersymmetric particle is a good candidate for the dark matter in the universe and also the field of research described in this thesis needs supersymmetry to fulfill the Sakharov conditions described elsewhere in the work.

[3] This symmetry is true in MSSM, but it does not need to be true in other supersymmetric models.

Appendix C; The matrix convention used

The convention for the γ-matrices used in this thesis is the one referred to as the chiral representation in [5], i.e.

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

where I are the 2×2 unitary matrix and σ_i are the Pauli matrices, as usual given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The projection operators are given by

$$P_R = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_L = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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