A polarized version of the CCFM equation for gluons

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A derivation for a polarized CCFM evolution equation which is suitable to describe the scaling behavior of the the unintegrated polarized gluon density is given. We discuss the properties of this polarized CCFM equation and compare it to the standard CCFM equation in the unpolarized case.

I. INTRODUCTION

An understanding of the dynamics of the gluon inside the nucleon is one of the key issues in Quantum Chromodynamics. Especially the behavior of the unpolarized gluon distribution function at small momentum fractions has been intensively discussed over the years. One suggested mechanism for the gluon dynamics is given by the CCFM equation [1–4], where the dominance of factorized diagrams in strongly ordered soft gluon emission is used. Recently, it has been shown by using a Monte Carlo implementation that the CCFM equation gives a good description of a variety of processes in deep inelastic scattering at HERA ranging from small to large momentum fraction x such as forward jet cross sections, high $p_\perp$ particle spectra, charm and bottom production [5].

A completely different field where the gluon dynamics at small x should give rise to many interesting features is polarized deep inelastic scattering. Here one is interested in the way the spin is distributed among quarks and gluons in the nucleon. Small-x effects in polarized deep inelastic scattering raise considerable interest [6,7] due to the possibility that one may access the region of $x < 10^{-3}$ in future projects such at THERA [7]. Small-x contributions to polarized structure functions have been regarded in terms of limits of the standard DGLAP evolution equation [8,9]. Beyond this the double logarithmic contributions [10–13] and their resummation [14–16] have been investigated. A complete formalism incorporating DGLAP and $\ln^2(1/x)$ resummation has been given in [17,18].

In this contribution I will follow the principles discussed in [3] to derive a polarized version of the CCFM equation for gluons. Here the unintegrated polarized gluon distribution enters, which is discussed in Sec. II. After a presentation of the kinematical variables in Sec. III a discussion of the principles of the soft gluon factorization applicable in the polarized case is given in Sec. IV. The derivation of the polarized CCFM equation for gluons can be found in Sec. V, which is followed by a comparison to the unpolarized CCFM equation in Sec. VI. In order to be able to discuss finally the properties of this equation we consider a single iterative step in the solution of this equation which can be physically identified with dressed single gluon emission in Sec. VII and VIII.

II. THE PHYSICAL INTERPRETATION OF THE POLARIZED UNINTEGRATED GLUON DENSITY

In traditional spin-physics polarized gluons have been mostly considered as on-shell partons. As an on shell parton the gluon carries only two polarization states which lie parallel or antiparallel to the spin of the nucleon where the gluon is sitting.

In CCFM one deals, however, with off-shell gluons which can have three polarization states. We can decompose them again in terms of the spin states of the underlying nucleon. The kinematic situation in longitudinally polarized deep inelastic scattering (DIS) is shown in Fig. 1. The total hadronic cross section $\sigma_{h\ h_p h_e}$ depends in the high energy limit on the helicity state of the electron h_e and the helicity state of the proton h_p. In terms of the $k_\perp$ factorization it is a convolution of the unintegrated gluon density $g_{h_p h_g}$ and the 'partonic' off-shell cross section $\sigma_{p\ h_g h_e}$, where the spin state h_g of the gluon entering the box-graph comes in. Then we get for the cross sections of the two experiments where the spin-vectors of proton and electron one time lie parallel and one time anti-parallel to each other:

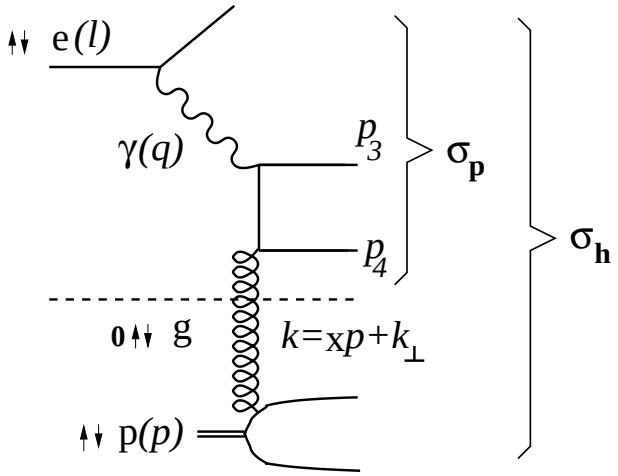

FIG. 1. $k_\perp$ factorization in case of polarized deep inelastic scattering (DIS) in terms of the unintegrated gluon density. The total hadronic cross section σ_h factorizes into the unintegrated gluon density and a partonic cross section σ_p.

$$\begin{aligned}\sigma_{h\,\uparrow\uparrow} &= g_{\uparrow\uparrow} \otimes \sigma_{p\,\uparrow\uparrow} + g_{\uparrow 0} \otimes \sigma_{p\,0\uparrow} + g_{\uparrow\downarrow} \otimes \sigma_{p\,\downarrow\uparrow} \\ \sigma_{h\,\downarrow\uparrow} &= g_{\downarrow\uparrow} \otimes \sigma_{p\,\uparrow\uparrow} + g_{\downarrow 0} \otimes \sigma_{p\,0\uparrow} + g_{\downarrow\downarrow} \otimes \sigma_{p\,\downarrow\uparrow} \,. \end{aligned} \tag{1}$$

As only relative orientations matter we can identify:

$$\uparrow\downarrow=\downarrow\uparrow= \times, \qquad \uparrow\uparrow=\downarrow\downarrow=\|, \qquad 0\uparrow= 0\downarrow \,. \tag{2}$$

So we obtain for the difference of the two hadronic cross sections:

$$\begin{aligned}\Delta\sigma_h &= \sigma_{h\,\times} - \sigma_{h\,\|} = \sigma_{h\,\downarrow\uparrow} - \sigma_{h\,\uparrow\uparrow} \\ &= (g_\| - g_\times) \otimes (\sigma_{p\,\times} - \sigma_{p\,\|}) = \Delta g \otimes \Delta\sigma_p \,. \end{aligned} \tag{3}$$

One should note that the polarization state 0 does not enter into the polarized cross section $\Delta\sigma_h$. Therefore, also in the unintegrated case, the polarized gluon distribution can be defined as the difference of the probability to find a gluon inside the proton with spin state aligned parallel to the proton spin minus the probability to find a gluon with the corresponding anti-parallel spin alignment. Between the unintegrated polarized gluon density $\Delta g(x, Q^2, k_\perp^2)$ and the integrated polarized gluon density $\Delta g(x, Q^2)$ one has the relation:

$$\Delta g(x, Q^2) = \int^{Q^2} k_\perp dk_\perp \, \Delta g(x, Q^2, k_\perp^2) \,. \tag{4}$$

The remarkable point is that on the right hand side we have a quantity derived from off-shell gluons while on the left hand side we have the standard gluon parton distribution where one is thinking in terms of on-shell gluons that have only two polarization states. It turns out indeed that the unintegrated polarized gluon distribution fits into the decomposition scheme of the integrated one. A systematic analysis of this topic has been performed in Ref. [19]. The CCFM equation I want to derive here in the polarized case is an evolution equation of the polarized off shell unintegrated gluon density. A systematic study of the evolution of unintegrated structure functions in terms of the DGLAP formalism can be found in [20].

III. THE KINEMATIC OF THE PROCESS

As in [3] we study the process of parton deep inelastic scattering, c. f. Fig. 2, where all lines, except the one with the momentum q, represent gluons. Kinematically we have:

$$p + q \to p' + q_1 + q_2 + \ldots + q_n \,. \tag{5}$$

Here q acts as the hard probe of the process:

$$q^2 = -Q^2 < 0, \qquad x = \frac{Q^2}{2p \cdot q} \,. \tag{6}$$

The transverse momenta of the outgoing soft gluons q_i are supposed to be smaller than the hard scale divided by x, i. e. $q_{i\perp}^2 < Q^2/x$. Kinematically one introduces two light-like vectors:

$$p = E(1,0,0,1), \qquad \bar{p} = E(1,0,0,-1), \qquad 2p\bar{p} = 4E^2 \; , \tag{7}$$

and decomposes the other momenta through:

$$q = -xp + \frac{Q^2}{x}\frac{\bar{p}}{2p\cdot\bar{p}}, \qquad q_i = y_i p + p\cdot q_i \frac{\bar{p}}{p\cdot\bar{p}} + q_{i\perp} \; . \tag{8}$$

Then, one has the relations:

$$2p\cdot q_i = \frac{q_{i\perp}^2}{y_i} \; . \tag{9}$$

So that the emitted soft gluons with momenta q_i are on-shell partons. If one assumes the hard scale Q^2 to be large one finds furthermore:

$$x \approx x_n = \left(1 - \sum_{i=1}^{n} y_i\right), \qquad p' \sim \bar{p} \; . \tag{10}$$

In the following a strong energy ordering is assumed:

$$y_1 \ll y_2 \ll \ldots \ll y_n \; . \tag{11}$$

Finally, the polarization vectors of the gluons involved have in the unpolarized case the simple form:

$$\epsilon_\mu^{(\lambda)}(q) = g_\mu^\lambda - \frac{q_\mu \eta^\lambda}{q\cdot\eta} \; , \tag{12}$$

where the gauge vector η is chosen to be $\eta = \bar{p} \sim p'$.

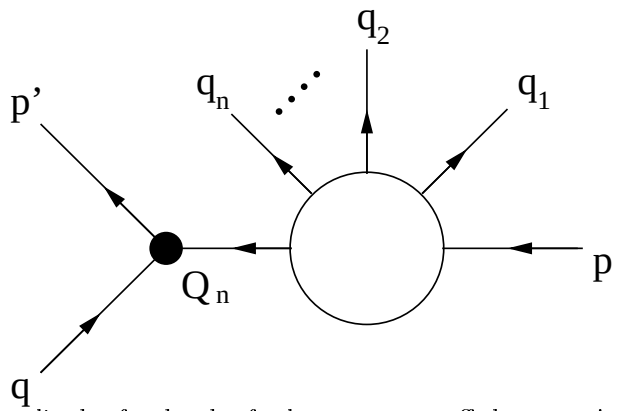

FIG. 2. Multi gluon emission amplitude of ordered soft gluons $q_1 \ldots q_n$ off the partonic gluon p. The hard scale is set by $Q^2 = -q^2$, while p' describes the outgoing final gluon.

IV. THE FACTORIZATION OF SOFT GLUON EMISSION IN THE CCFM APPROACH

Lets now consider the amplitude for the soft n-gluon emission in Fig. 2 in terms of $\langle acb_1 \ldots b_n | M_n(p,p',q_1,\ldots,q_n)\rangle$. The basic principle of the CCFM equation is the resummation and factorization of soft gluon emission. The main ingredient here is the radiation of soft real gluons off quasi-real partons, where except for the color matrices the emission vertex is reduced to an effective scalar coupling [21]:

$$\begin{aligned} q(p) \to q(p-q) + g(q) \quad &: \quad \frac{(\not{p}-\not{q})\gamma_\mu \not{p}}{(p-q)^2 + i\epsilon} \to -\frac{\not{p}}{pq - i\epsilon}\,(p_\mu + \mathcal{O}(|\vec{q}|)) \\ g(p) \to g(p-q) + g(q) \quad &: \quad d^{\lambda\lambda'}(p-q)\Gamma_{\lambda'\mu\nu'}(p-q,q,p)d^{\nu\nu'}(p) \to 2p_\mu d^{\lambda\nu}(p)\,(1+\mathcal{O}(|\vec{q}|)) \; . \end{aligned} \tag{13}$$

Here $d_{\mu\mu'}(q) = -\epsilon_{\mu(\lambda)}(q)\epsilon^{(\lambda)}_{\mu'}(q)$ and Γ denotes the three-gluon vertex. The factorization of the multi-gluon emission amplitude can be written in terms of a scalar current $\mathbf{J}^{(n-1)}_{\rm tot}$ which consists in the unpolarized case for small x of an eikonal part and a non-eikonal part. The eikonal part is inherited from the situation in QED where the soft photon emission factorizes exactly using the eikonal identity (c.f. Ref. [21]):

$$\sum_{\rm perm} \frac{1}{a_1}\frac{1}{a_1+a_2}\cdots\frac{1}{a_1+a_2+\ldots a_n} = \prod_{i=1}^{n} a_i^{-1}\ . \tag{14}$$

Using these principles one finds in the limit $x \to 0$ the following iterative factorization of the soft gluon emission of the softest gluon q_n from the amplitude M_n:

$$\begin{aligned}
M_n &= \frac{2(Q_n - x_n p)\cdot\epsilon^{(\lambda')}(p')}{x_n Q_n^2}\langle acb_1\ldots b_n|\mathbf{h_n}(pp'q_1\cdots q_n)\rangle \\
\langle acb_1\ldots b_n|\mathbf{h_n}(pp'q_1\ldots q_n)\rangle &\approx g_s\langle acb_1\ldots b_{n-1}|\mathbf{J}^{(n-1)}_{\rm tot}(q_n)|\mathbf{h_{n-1}}(pp'q_1\ldots q_{n-1})\rangle \\
\\
\mathbf{J}^{(n-1)}_{\rm tot}(q_n) &= \mathbf{J}^{(n-1)}_{\rm eik}(q_n) + \mathbf{J}_{\rm ne}(Q_n, q_n) \\
\mathbf{J}^{(n-1)}_{\rm eik}(q_n) &= -\hat{T}_p\frac{p^{(\lambda)}}{p\cdot q} + \hat{T}_{p'}\frac{p'^{(\lambda)}}{p'\cdot q} + \sum_{l=1}^{n-1}\hat{T}_l\frac{q_l^{(\lambda)}}{q_l\cdot q} \\
\mathbf{J}_{\rm ne}(Q_n, q_n) &= \frac{2(Q_{n-1} - x_{n-1}p)\cdot\epsilon^{(\lambda)}(q_n)}{Q^2_{n-1}}\hat{T}_{p'} \\
Q_{n-1} &= Q_n + q_n, \quad x_{n-1} = x_n + y_n\ .
\end{aligned} \tag{15}$$

Here $\hat{T}_q$ denotes the color charge of the gluon with the momentum q. In the eikonal current a polarization component is picked. For $x \to 1$ the factorization formula holds for the full amplitude and no non-eikonal contributions occur:

$$\langle acb_1\ldots b_n|\mathbf{M_n}(pp'q_1\ldots q_n)\rangle \approx g_s\langle acb_1\ldots b_{n-1}|\mathbf{J}^{(n-1)}_{\rm eik}(q_n)|\mathbf{M_{n-1}}(pp'q_1\ldots q_{n-1})\rangle\ . \tag{16}$$

In the polarized case we will see that the non-eikonal contribution is also absent in the case $x \to 0$. The essential point is that for the polarized contribution we need a spin correlation between the incoming gluon with momentum p and the outgoing gluon with momentum p'. This means that the polarization flow is in no case allowed to go through the soft emission. We illustrate the situation in Fig. 3. Here the ordered emission for small x is shown.

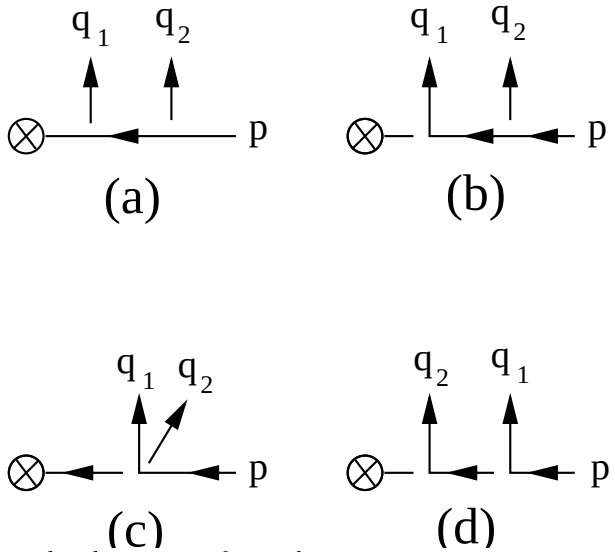

FIG. 3. Polarization flow for the ordered emission of two gluons $x \ll y_1 \ll y_2 \approx 1$. Diagram (a) shows the contribution in the polarized case where no polarization flow is to enter the soft gluon emission. Diagrams (b),(c) and (d) show the leading contribution in the unpolarized case for small x. Diagrams (b) and (c) give rise to the eikonal emission while diagram (d) is the one that accounts for the non-eikonal emission.

In the unpolarized case the leading diagrams are given by (b),(c) and (d). In the cases (b) and (c) the soft gluon emission contains no polarization flow and can therefore simply be factorized using the eikonal emission. In diagram (d) an additional contribution arises which gives rise to a non-eikonal contribution. In the polarized case the only contributing spin-configuration is the diagram (a), where the polarization vectors of the incoming gluon with momentum p and the outgoing gluon with momentum p' are directly correlated. The first result is therefore that in the polarized case the non-eikonal contribution is absent and we can use the eikonal factorization in the same way for large and for small x. Now we can just perform the steps leading to the CCFM equation which were presented in [3]. First, we single out the color amplitudes:

$$\langle acb_1 \dots b_n | \Delta \mathbf{M}_n \rangle = \sum_{\pi_{n+1}} \Delta M_n(pq_{l_0} \dots q_{l_n}) 2\mathrm{Tr}(\lambda^a \lambda^{b^{l_0}} \dots \lambda^{b^{l_n}}) \,, \tag{17}$$

where the sum is over permutations $l_0, \dots l_n$ with $q_0 = p'$, $b_0 = c$. Neglecting all non-leading collinear and non-planar terms, the color algebra yields ($\sigma_0 = N_c^2 - 1$):

$$|\Delta \mathbf{M}_n|^2 = \sigma_0 \left(\frac{C_A}{2}\right)^n \sum_{\pi_{n+1}} |\Delta M_n(pq_{l_0} \dots q_{l_n})|^2 \,. \tag{18}$$

Here $C_A = N_c$ is the number of colors. The factorization of the soft gluon emission in terms of eikonal currents leads to the following recurrence relation:

$$|\Delta M_n(\dots q_l q_n q_{l'} \dots)|^2 \approx -g_s^2 |\Delta M_{n-1}(\dots q_l q_{l'} \dots)|^2 (j_l(q_n) - j_{l'}(q_n))^2 \,, \quad j_l(q_n) = \frac{q_l}{q_l \cdot q_n} \,. \tag{19}$$

The important thing is now the initial condition because this is the place where the polarization enters. The hard splitting kernels in the CCFM equation should match the DGLAP splitting kernels in the limits $z \to 0, 1$. To see this relation in an explicit way we will work for the last step that generates the initial conditions with the Altarelli Parisi method itself. For this purpose we consider Fig. 4:

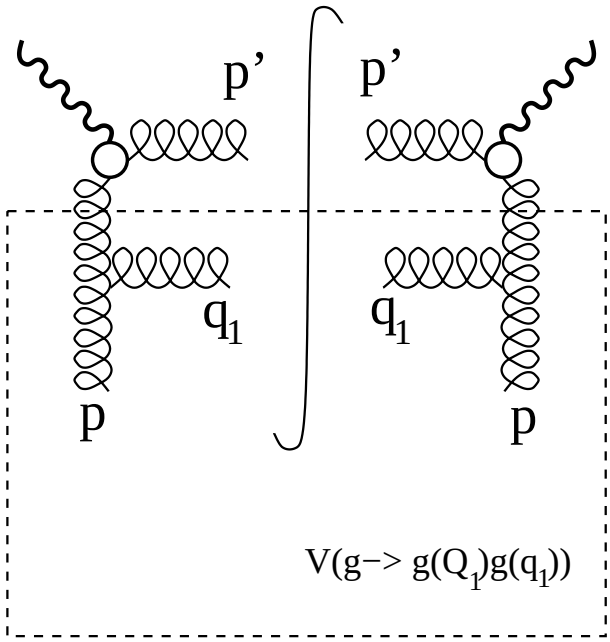

FIG. 4. Decomposition of the initial amplitude M_1 by the Altarelly Parisi method.

Here the initial amplitude is decomposed in the emission of the gluon with momentum q_1 and the coupling to the hard virtuality:

$$|M_1(pp'q_1; \lambda\lambda')|^2 \approx \sum_{\lambda'' \in \pm} |V_{G \to GG}(p, Q_1, q_1; \lambda, \lambda'')||H_{G \to G}(Q_1, p'; \lambda'', \lambda')|^2 \,. \tag{20}$$

In the Altarelli Parisi method we consider the virtuality of Q_1^2 to be comparatively small, so that all gluons in the proton can be treated as partons approximately. We first calculate the two gluon effective amplitude:

$$
\begin{aligned}
|H_{G\to G}(Q_1,p';\lambda'',\lambda')|^2 &= V_{\mathrm{eff}\,\mu\nu}(Q_1,p')V_{\mathrm{eff}\,\mu'\nu'},(Q_1,p')\Pi^{\mu\mu'}(x_1p,\eta,\lambda'')\Pi^{\nu\nu'}(p',\eta,\lambda') \\
&= \frac{1}{Q_1^4}\delta_{\lambda''\lambda'} \\
V_{\mathrm{eff}\,\mu\nu}(Q_1,p') &= \frac{1}{Q_1^2}\left[-g_{\mu\nu}+\frac{p'_\mu Q_{1\,\nu}}{x_1pp'}\right] \\
\Pi^{\mu\mu'}(x_1p,\eta,\lambda'') &= \frac{1}{2}\left(-g^{\mu\mu'}+\frac{\eta^\mu p^{\mu'}+\eta^{\mu'}p^\mu}{\eta\cdot p}-\lambda\frac{i}{\eta\cdot p}\epsilon^{\mu\mu'\eta p}\right) \\
\Pi^{\nu\nu'}(p',\eta,\lambda') &= \frac{1}{2}\left(-g^{\nu\nu'}+\frac{\eta^\nu p'^{\nu'}+\eta^{\nu'}p'^\nu}{\eta\cdot p'}-\lambda'\frac{i}{\eta\cdot p'}\epsilon^{\nu\nu'\eta p'}\right)\ . \qquad (21)
\end{aligned}
$$

$H\sim\delta_{\lambda\lambda'}$ reflects the fact that the effective vertex cannot flip the helicity of the spin-1 gluon. To do such a thing one would require a spin-2 particle which does not exist in the process. We should remember that in the end of the calculation we can take the gauge vector $\eta\sim p'$. As a next step we turn to the calculation of the splitting amplitude $V_{G\to GG}$. Here we apply the Altarelli Parisi method where all three gluons are approximately parton like. To to this we choose the following parameterization [22]:

$$
\begin{aligned}
p &= (P,P,\mathbf{0}) \\
Q_1 &= (x_1P+\frac{p_\perp^2}{2Px_1},x_1P,\mathbf{p}_\perp) \\
q_1 &= ((1-x_1)P+\frac{p_\perp^2}{2P(1-x_1)},(1-x_1)P,-\mathbf{p}_\perp) \\
n &= (P,-P,\mathbf{0})\ . \qquad (22)
\end{aligned}
$$

Here n acts as a gauge vector to remove unphysical decrees of freedom. Then one obtains for the splitting amplitude leaving out the color factors which will be provided later:

$$
\begin{aligned}
|V_{G\to GG}(p,Q_1,q_1;\lambda,\lambda'')|^2 &= \sum_{\lambda'''\in\pm}|V_{G\to GG}(p,Q_1,q_1;\lambda,\lambda'',\lambda''')|^2 \\
&= 2g_s^2p_\perp^2\left[\frac{(1-x_1+x_1^2)^2}{(1-x_1)^2x_1^2}+\lambda\lambda''\frac{2-3x_1+2x_1^2}{(1-x_1)^2x_1}\right] \\
V_{G\to GG}(p,Q_1,q_1;\lambda,\lambda'',\lambda''') &= -g_s\left\{-\left[(p+q_1)\epsilon^{*b}_{Q_1,\lambda''}\right]\left(\epsilon^a_{p,\lambda}\cdot\epsilon^{*c}_{q_1,\lambda'''}\right)\right. \\
&\quad +\left[(q_1-Q_1)\epsilon^a_{p,\lambda}\right]\left(\epsilon^{*b}_{Q_1,\lambda''}\cdot\epsilon^{*c}_{q_1,\lambda'''}\right) \\
&\quad \left.+\left[(p+Q_1)\epsilon^{*c}_{q_1,\lambda'''}\right]\left(\epsilon^a_{p,\lambda}\cdot\epsilon^{*b}_{Q_1,\lambda''}\right)\right\} \\
\left(\epsilon^a_{p,\lambda}\right)_\mu\left(\epsilon^{*a'}_{p,\lambda}\right)_{\mu'} &= \frac{1}{2}\left(-g_{\mu\mu'}+\frac{n_\mu p_{\mu'}+n_{\mu'}p_\mu}{n\cdot p}+\lambda\frac{i}{n\cdot p}\epsilon_{\mu\mu'np}\right)\ . \qquad (23)
\end{aligned}
$$

The next step is to put the two amplitudes together:

$$
|\Delta M_1(pp'q_1)|^2=|M_1(pp'q_1;++)|^2-|M_1(pp'q_1;+-)|^2=4g_s^2\frac{p_\perp^2}{Q_1^4}\frac{2-3x_1+2x_1^2}{(1-x_1)^2x_1}\ . \qquad (24)
$$

Next we have to make use of the fact, that according to Eq. (9):

$$
p_\perp^2=q_{1\perp}^2=-2p\cdot q_1y_1\approx Q_1^2(1-x_1)\ , \qquad (25)
$$

which in turn results in:

$$
\begin{aligned}
|\Delta M_1(pp'q_1)|^2 &= 4g_s^2\frac{1}{Q_1^2}\frac{2-3x_1+2x_1^2}{(1-x_1)x_1} \\
&\approx\ = 2g_s^2\left(\frac{2p\cdot q_1}{Q_1^2}\right)\frac{p\cdot p'}{(p'\cdot q_1)(p\cdot q_1)}\frac{2-3x_1+2x_1^2}{x_1} \\
&\approx g_s^2\frac{2-3x_1+2x_1^2}{x_1}\left(j_p(q_1)-j_{p'}(q_1)\right)^2 \\
&= \begin{cases} g_s^2\frac{2}{x_1}\left(j_p(q_1)-j_{p'}(q_1)\right)^2 & \text{for}\quad (x_1\to 0) \\ g_s^2\left(j_p(q_1)-j_{p'}(q_1)\right)^2 & \text{for}\quad (x_1\to 1)\end{cases}\ . \qquad (26)
\end{aligned}
$$

Here we used that $p' \cdot q_1 \approx (1-x_1)pp'$ and that for parton-like soft gluons one can set $-2p \cdot q_1 \approx Q_1^2$. We can interpolate the two limits using the effective expression:

$$|\Delta M_1(pp'q_1)|^2 \equiv g_s^2 \frac{2-x_1}{x_1} \left(j_p(q_1) - j_{p'}(q_1)\right)^2 \ . \tag{27}$$

One should note that in the unpolarized case this method just reproduces the results in [3]. Finally, we arrive for the amplitude at tree level to:

$$|\Delta M_n^{(\mathrm{tree})}|^2 \approx \sigma_0 \left(g_s^2 C_A\right)^n \frac{2-x_n}{x_n} \sum_{\pi_{n+1}} W_n(pq_{l_0} \dots q_{l_n}), \quad W_n(pq_{l_0} \dots q_{l_n}) = \frac{(p \cdot p')^2}{(p \cdot q_{l_0}) \dots (p \cdot q_{l_n})} \ , \tag{28}$$

where W_n is the multi-eikonal distribution. The result is similar to the one obtained in the unpolarized case in Ref. [3] Eq. (4.16) except for the factor $(2-x_n)/x_n$. We will see that this naturally leads to the polarized DGLAP splitting kernel in the limit $z \to 1, 0$ noting that our result is valid for small as well as for large z.

V. THE DERIVATION OF THE POLARIZED CCFM EQUATION

The result of the previous section allows a simple and straightforward derivation of the polarized CCFM equation. To do this we we consider the contribution to the polarized structure function:

$$\sigma_0 \Delta F(Q,x) = \Delta F_0(x) + \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} (dq_i) \Theta(Q - q_{i\perp}) |\Delta M_n|^2 \delta\left(1 - \frac{x}{x_n}\right), \quad (dq_i) = \frac{d^2 q_i}{2\omega_i (2\pi)^3} \ . \tag{29}$$

Now inserting the tree-level amplitude one obtains:

$$\begin{aligned}
\sigma_0 \Delta F^{(\mathrm{tree})}(Q,x) &= \Delta F_0^{(\mathrm{tree})}(x) + \sum_{n=1}^{\infty} \int \prod_{i=1}^{n} \frac{d\xi_i}{\xi_i} \frac{dy_i}{y_i} \bar{\alpha}_s \Theta(Q - q_{i\perp}) \frac{2-x_n}{x_n} \delta\left(1 - \frac{x}{x_n}\right) \Theta^{\xi}_{n,\dots,2,1} \\
&\equiv \Delta F_0^{(\mathrm{tree})}(x) + \sum_{n=1}^{\infty} \int \prod_{i=1}^{n} \frac{d\xi_i}{\xi_i} dz_i \frac{\alpha_s}{2\pi} \Delta P(z_i) \Theta(Q - q_{i\perp}) \delta(x - z_1 \dots z_n) \Theta^{\xi}_{n,\dots,2,1} \\
\bar{\alpha}_s &= C_A \alpha_s / \pi, \qquad x_n = 1 - y_1 - \dots - y_n \\
x_n &= z_1 z_2 \dots z_n, \qquad y_l = x_{l-1}(1 - z_l) \\
\xi_i &= \frac{p \cdot q_i}{E \omega_i}, \qquad \Theta^{\xi}_{l_k \dots l_1} = \prod_{i=1}^{k-l} \Theta(\xi_{l_{i+1}} - \xi_{l_i}) \ .
\end{aligned} \tag{30}$$

The equivalence symbol ($\equiv$) means that the two expressions become equal in the limit $z_i \to 1, 0$, and one obtains as hard splitting kernel for the polarized CCFM equation:

$$\Delta P(z) = \frac{2C_A(2-z)}{1-z} = 2C_A \left(\frac{1}{1-z} + 1\right) \ . \tag{31}$$

In the limit $z \to 1$ this result is identical to the pole contribution of the corresponding polarized DGLAP splitting function [22]:

$$\Delta P_{gg\ \mathrm{pole}}(z) = 2C_A \left[\frac{2 - 3z + 2z^2}{(1-z)} + \left(\frac{11}{12} - \frac{1}{3}\frac{T_R}{C_A}\right) \delta(1-z)\right]\Bigg|_{\mathrm{pole}} = \frac{2C_A}{1-z} \ . \tag{32}$$

Here $T_R = N_f/2$ is the number of flavors. In the limit $z \to 0$ one obtains $4C_A$ in accordance with the corresponding limit in the DGLAP splitting kernel. The result Eq. (30) is exactly identical to the one derived in [3] Eq. (4.29). The only difference is that we have to exchange $\Delta P(z)$ by $P(z)$ which is given by:

$$P(z) = 2C_A \left(\frac{1}{z} + \frac{1}{1-z}\right) \ . \tag{33}$$

In the unpolarized case $P(z)$ corresponds to the pole structure of the corresponding DGLAP splitting function for gluons $P_{gg}(z)$ [22]:

$$P_{gg\ \mathrm{pole}}(z) = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \ + \left(\frac{11}{12} - \frac{1}{3}\frac{T_R}{C_A} \right) \delta(1-z) \right] \Bigg|_{\mathrm{pole}} = 2C_A \left(\frac{1}{z} + \frac{1}{1-z} \right) . \tag{34}$$

In polarized and unpolarized case the $1/(1-z)$ pole is the same, while in the polarized case the $1/z$ pole is missing. These findings explain now simply how the polarized CCFM equation should be constructed:

- The hard splitting function $\Delta P(z)$ in the polarized CCFM equation should have the form $P(z) = 2C_A(2-z)/(1-z)$. The form is valid both for small and for large z because the factorization in the polarized case is valid in both limits as shown in the previous section. For $z \to 0, 1$ this splitting function becomes identical to the polarized DGLAP splitting function.
- Adding virtual corrections to the tree level results means to multiply with the corresponding form factors as demonstrated in [3]. The requirement that the polarization flow does not enter into the soft emission means that the non-eikonal form factor is absent. So we have to amend only the eikonal form factor to the tree level result. The eikonal form factor is the same as in the unpolarized case because the polarization flow does not enter into the soft emission. The statement means that on the average the soft emission does not know anything of the initial spin state of the gluon.

With these principles in mind we can write down the integral form for the polarized CCFM equation taking into account only gluons:

$$\begin{aligned} \Delta g(x, \vec{k}_\perp^{\,2}, Q^2) &= \Delta g_0(x, \vec{k}_\perp^{\,2}, Q^2) \\ &+ \int_x^1 \frac{dz}{z} \int_0^{2\pi} \frac{d\theta_{q'_\perp}}{2\pi} \int_{Q_0^2}^{\infty} \frac{d\vec{q}_\perp^{\,\prime\,2}}{\vec{q}_\perp^{\,\prime\,2}} \Theta(Q - z|\vec{q}_\perp^{\,\prime}|) \Delta_{\mathrm{e}}^{(g)}(\bar{Q}^2, (z\vec{q}_\perp^{\,\prime},)^2) \Delta \mathcal{P}_{gg}(z, \vec{q}_\perp^{\,\prime\,2}, \vec{k}_\perp^{\,2}) \Delta g(x/z, \vec{k}_\perp^{\,\prime\,2}, \vec{q}_\perp^{\,\prime\,2}) \\ \vec{k}'_\perp &= \vec{k}_\perp + (1-z)\vec{q}_\perp^{\,\prime} . \end{aligned} \tag{35}$$

Here the CCFM-kernel has the structure:

$$\Delta \mathcal{P}_{\mathrm{gg}}(z, q^2, k_\perp^2) = \frac{\alpha_s(q^2(1-z)^2)}{2\pi} \Delta P_{gg}(z) . \tag{36}$$

The CCFM kernel in the polarized case is $k_\perp$ independent due to the absence of the non-eikonal form factor and the way the scale in α_s is chosen. But in general it is a function of $k_\perp$, for example in the unpolarized case. The eikonal form factor $\Delta_{\mathrm{e}}^{(g)}$ is taken from Ref. [3]:

$$\Delta_{\mathrm{e}}^{(g)}(\bar{q}^2, (zq)^2) = \exp\left(- \int_{(zq)^2}^{\bar{q}^2} \frac{dq'^2}{q'^2} \int_0^{1-Q_0/q'} \frac{dz}{1-z} \frac{\alpha_s(q'^2(1-z)^2)}{\pi} C_A \right) , \tag{37}$$

and the hard splitting kernel reads:

$$\Delta P_{gg}(z) = \frac{2C_A(2-z)}{1-z} . \tag{38}$$

VI. COMPARISON BETWEEN THE POLARIZED AND UNPOLARIZED CCFM EQUATION

In order to understand the physics of the result obtained in the previous chapter we compare it to the unpolarized CCFM equation. To make the comparison as instructive as possible we show the corresponding equations here together: In integral form the polarized and unpolarized CCFM equation reads:

$$\begin{aligned} \Delta g(x, \vec{k}_\perp^{\,2}, Q^2) &= \Delta g_0(x, \vec{k}_\perp^{\,2}, Q^2) \\ &+ \int_x^1 \frac{dz}{z} \int_0^{2\pi} \frac{d\theta_{q'_\perp}}{2\pi} \int_{Q_0^2}^{\infty} \frac{d\vec{q}_\perp^{\,\prime\,2}}{\vec{q}_\perp^{\,\prime\,2}} \Theta(Q - z|\vec{q}_\perp^{\,\prime}|) \Delta_{\mathrm{e}}^{(g)}(\bar{Q}^2, (z\vec{q}_\perp^{\,\prime},)^2) \Delta \mathcal{P}_{gg}(z, \vec{q}_\perp^{\,\prime\,2}, \vec{k}_\perp^{\,2}) \Delta g(x/z, \vec{k}_\perp^{\,\prime\,2}, \vec{q}_\perp^{\,\prime\,2}) \\ g(x, \vec{k}_\perp^{\,2}, Q^2) &= g_0(x, \vec{k}_\perp^{\,2}, Q^2) \\ &+ \int_x^1 \frac{dz}{z} \int_0^{2\pi} \frac{d\theta_{q'_\perp}}{2\pi} \int_{Q_0^2}^{\infty} \frac{d\vec{q}_\perp^{\,\prime\,2}}{\vec{q}_\perp^{\,\prime\,2}} \Theta(Q - z|\vec{q}_\perp^{\,\prime}|) \Delta_{\mathrm{e}}^{(g)}(\bar{Q}^2, (z\vec{q}_\perp^{\,\prime},)^2) \mathcal{P}_{gg}(z, \vec{q}_\perp^{\,\prime\,2}, \vec{k}_\perp^{\,2}) g(x/z, \vec{k}_\perp^{\,\prime\,2}, \vec{q}_\perp^{\,\prime\,2}) . \end{aligned} \tag{39}$$

For the CCFM splitting kernels one has:

$$\Delta\mathcal{P}_{\rm gg}(z,q^2,k_\perp^2) = \frac{\alpha_s(q^2(1-z)^2)}{2\pi}\Delta P_{gg}(z)$$
$$\mathcal{P}_{\rm gg}(z,q^2,k_\perp^2) = \frac{\alpha_s(q^2(1-z)^2)}{2\pi}\Delta_{\rm ne}(z,q,k_\perp)P_{gg\ {\rm pole}}(z)\ , \tag{40}$$

with the eikonal and non-eikonal form factors and hard splitting kernels given by:

$$\Delta_{\rm e}^{(g)}(\bar{q}^2,(zq)^2) = \exp\left(-\int_{(zq)^2}^{\bar{q}^2}\frac{dq'^2}{q'^2}\int_0^{1-Q_0/q'}\frac{dz}{1-z}\frac{\alpha_s(q'^2(1-z)^2)}{\pi}C_A\right)$$
$$\Delta_{\rm ne}(z,q,k_\perp) = \exp\left(-\frac{\alpha_s(q^2(1-z)^2)}{\pi}C_A\int_z^1\frac{dz'}{z'}\int_{(z'q)^2}^{k_\perp^2}\frac{dq^2}{q^2}\right)$$
$$\Delta P_{gg}(z) = 2C_A\frac{2-z}{1-z}$$
$$P_{gg\ {\rm pole}}(z) = 2C_A\left(\frac{1}{z}+\frac{1}{1-z}\right)\ . \tag{41}$$

Comparing the two equations one can say something of the underlying physics:

- In the limit of large x when the non-eikonal form factor becomes equal to unity, the CCFM equation in the unpolarized as well as in the polarized case becomes identical to the corresponding DGLAP evolution equation, except for the eikonal form factor.
- There is no $1/z$ pole in the polarized hard splitting kernel $\Delta P(z)$ in the polarized CCFM equation in parallel to the fact that there is no 1/z pole in the polarized DGLAP splitting function either.
- In the polarized case we need to see spin correlations all along over the way of the soft emission. For this correlation only diagrams contribute that have no polarization flow into the soft gluons. For this reason there are no non-eikonal contributions to the polarized CCFM equation. This is very natural because the non-eikonal contributions in the unpolarized CCFM equation couple actually only to the 1/z pole, which is absent in the polarized case.
- In the polarized case the hard splitting function has the form $(2-z)/(1-z)$ which means that for small z the polarization is not enhanced by a pole as opposite to the unpolarized case.

To resum, these findings say that soft gluon emission destroys to a large extent the definite polarization state of the incoming gluon and that the soft emission knows on the average not very much of the initial polarization of the gluon. This result is quite remarkable because it justifies a long termed used practice in polarized MC event generators like PEPSI [23], where the unpolarized parton showering formalism has been used to simulate the soft gluon emission in polarized events [24].

VII. SINGLE DRESSED GLUON EMISSION

In principle it would be now desirable to take some input distribution Δg_0, evolve it and compare it with data. Such a project is however beyond the scope of this article because there are a couple of severe problems to be solved first. Among those are:

- There is no simple analytic formalism to solve the CCFM equation like in DGLAP with the conversion into Mellin moments. Indeed a genuine solution of the CCFM equation seems to be possible only by means of Monte Carlo technique.
- For small z the hard splitting kernel for gluons goes to a constant, as also the quark splitting functions at least in the DGLAP formalism do. So the influence of the quarks in the polarized case is in principle of the same order as the gluons and cannot be neglected.

Instead of this we want to investigate a bit more closely how the equation works in the polarized case. In principle a possible solution could be obtained iteratively by:

$$g_{n+1}(x,\vec{k}_\perp^2,Q^2) = g_0(x,\vec{k}_\perp^2,Q^2) + \Delta_e(Q^2,Q_0^2)\int_{Q_0^2}^{Q^2}\frac{dq^2}{q^2}\int_x^1\frac{dz}{z}\int_0^{2\pi}\frac{d\theta_{q'_\perp}}{2\pi}\frac{\Delta\mathcal{P}_{gg}(z,q^2/z^2,\vec{k}_\perp^2)}{\Delta_e(q^2,Q_0^2)}g_n(x/z,\vec{k'_\perp}^2,q^2/z^2)$$
$$g(x,\vec{k}_\perp^2,Q^2) = \lim_{n\to\infty} g_n(x,\vec{k}_\perp^2,Q^2)\ . \tag{42}$$

To restrict the numerical effort as much as possible we restrict ourself to the first order of this iteration and calculate Δg_1, and for comparison also g_1. From a physical point of view this corresponds to a single dressed gluon emission. It is a dressed emission because the eikonal form factor includes virtual corrections to the single soft gluon emission. In this case all necessary formulas become quite simple. As the starting distributions we use the usual ansatz where the $k_\perp$ dependence is determined by a Gaussian, c.f. [5]:

$$\Delta g_0(x,\vec{k}_\perp^2,Q^2) = Nx^\alpha(1-x)^\beta\frac{1}{k_0^2}\exp\left(-\frac{\vec{k}_\perp^2}{2k_0^2}\right)\Delta_e(Q^2,Q_0^2)$$
$$g_0(x,\vec{k}_\perp^2,Q^2) = N'x^{\alpha'}(1-x)^{\beta'}\frac{1}{k_0^2}\exp\left(-\frac{\vec{k}_\perp^2}{2k_0^2}\right)\Delta_e(Q^2,Q_0^2)\ . \tag{43}$$

Using this ansatz one can perform for the single dressed gluon emission the angle integration analytically:

$$\Delta g_1(x,\vec{k}_\perp^2,Q^2) = \Delta g_0(x,\vec{k}_\perp^2,Q^2) + \Delta_e(Q^2,Q_0^2)\int_{Q_0^2}^{Q^2}\frac{dq^2}{q^2}\int_x^1\frac{dz}{z}\frac{\Delta\mathcal{P}_{gg}(z,q^2/z^2,\vec{k}_\perp^2)}{\Delta_e(q^2,Q_0^2)}\Delta g_0(x/z,Q_0^2)$$
$$\times\frac{1}{k_0^2}\exp\left[-\frac{\left(\frac{1-z}{z}q\right)^2+k_\perp^2}{2k_0^2}\right]I_0\left(\frac{(1-z)qk_\perp}{zk_0^2}\right)\ . \tag{44}$$

Here I_0 is the modified Bessel function of zero order as obtained from:

$$I_0(\alpha) = \int_0^{2\pi}\frac{d\theta}{2\pi}\exp(\alpha\cos\theta)\ . \tag{45}$$

One should notice that this approximate solution holds for the unpolarized case as well, so we can summarize our result for g_1 and Δg_1 to be:

$$\Delta g_1(x,\vec{k}_\perp^2,Q^2) = \Delta g_0(x,\vec{k}_\perp^2,Q^2) + \Delta_e(Q^2,Q_0^2)\int_{Q_0^2}^{Q^2}\frac{dq^2}{q^2}\int_x^1\frac{dz}{z}\Delta\mathcal{P}'_{gg}(z,q^2/z^2,\vec{k}_\perp^2)\Delta g_0(x/z,Q_0^2)$$
$$g_1(x,\vec{k}_\perp^2,Q^2) = g_0(x,\vec{k}_\perp^2,Q^2) + \Delta_e(Q^2,Q_0^2)\int_{Q_0^2}^{Q^2}\frac{dq^2}{q^2}\int_x^1\frac{dz}{z}\mathcal{P}'_{gg}(z,q^2/z^2,\vec{k}_\perp^2)g_0(x/z,Q_0^2)\ , \tag{46}$$

using

$$\Delta\mathcal{P}'_{gg}(z,q^2/z^2,k_\perp^2) = \frac{\alpha_s\left(q^2\frac{(1-z)^2}{z^2}\right)}{2\pi}\Delta_{\rm e}(Q_0^2,q^2)\Delta P_{\rm gg}(z)$$
$$\times\frac{1}{k_0^2}\exp\left[-\frac{\left(\frac{1-z}{z}q\right)^2+k_\perp^2}{2k_0^2}\right]I_0\left(\frac{(1-z)qk_\perp}{zk_0^2}\right)$$
$$\mathcal{P}'_{gg}(z,q^2/z^2,k_\perp^2) = \frac{\alpha_s\left(q^2\frac{(1-z)^2}{z^2}\right)}{2\pi}\Delta_{\rm ne}(z,q/z,k_\perp)\Delta_{\rm e}(Q_0^2,q^2)P_{\rm gg\ pole}(z)$$
$$\times\frac{1}{k_0^2}\exp\left[-\frac{\left(\frac{1-z}{z}q\right)^2+k_\perp^2}{2k_0^2}\right]I_0\left(\frac{(1-z)qk_\perp}{zk_0^2}\right)\ . \tag{47}$$

VIII. NUMERICAL ANALYSIS OF THE SINGLE DRESSED GLUON EMISSION

As the CCFM equation is a leading order equation we have to compare it to the leading order DGLAP evolution. Consequently, we also have to use for the running coupling α_s the leading order formula:

$$\begin{aligned}\alpha_s(\mu^2) &= \frac{4\pi}{\beta_0 \ln\left(\mu^2/\Lambda^2\right)}, \qquad \beta_0 = 11 - \frac{2}{3}N_f \\ \Lambda_{\overline{\rm MS}}^{(3,4,5,6)} &= 204, 175, 132, 66.5 \text{ MeV} \\ m_{c,b,t} &= 1.4, 4.5, 175 \text{ GeV} .\end{aligned} \tag{48}$$

The values for $\Lambda_{\overline{\rm MS}}$ and the threshold quark masses are taken from [25]. As a next step we have to choose the input distribution. With the simplifications done we only need the x-dependent distributions at the input scale Q_0^2. We choose for the unpolarized gluon distribution GRV(98) LO [25] and in the polarized case GRSV(00) standard scenario [26]. The input scale is in both cases $Q_0^2 = 0.26$ GeV^2:

$$\begin{aligned}x g'_{\rm GRV}(x, Q_0^2) &= 17.47 x^{1.6}(1-x)^{3.8} \\ x\Delta g'_{\rm GRSV}(x, Q_0^2) &= 1.669 x^{1.79}(1-x)^{0.15}(x g'_{\rm GRV}(x, Q_0^2)) .\end{aligned} \tag{49}$$

For practical implications it is now also advantageous to transform the eikonal form factor into a single integral:

$$\begin{aligned}-\frac{\ln \Delta_{\rm e}^{(g/q)}(q_2^2, q_1^2)}{C_A} &= \int_{q_1^2}^{q_2^2} \frac{dq^2}{q^2} \int_0^{1-Q_0/q} \frac{dz}{1-z} \frac{\alpha_s(q^2(1-z)^2)}{\pi} \\ &= \int_0^{1-\frac{Q_0}{q_1}} \frac{dz}{1-z} \int_{q_1^2}^{q_2^2} \frac{dq^2}{q^2} \frac{\alpha_s\left(q^2(1-z)^2\right)}{\pi} + \int_{1-\frac{Q_0}{q_1}}^{1-\frac{Q_0}{q_2}} \frac{dz}{1-z} \int_{Q_0^2/(1-z)^2}^{q_2^2} \frac{dq^2}{q^2} \frac{\alpha_s\left(q^2(1-z)^2\right)}{\pi} \\ &= \int_0^{1-\frac{Q_0}{q_1}} \frac{dz}{1-z} \int_{q_1^2(1-z)^2}^{q_2^2(1-z)^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s\left(\mu^2\right)}{\pi} + \int_{1-\frac{Q_0}{q_1}}^{1-\frac{Q_0}{q_2}} \frac{dz}{1-z} \int_{Q_0^2}^{q_2^2(1-z)^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s\left(\mu^2\right)}{\pi} \\ &= \int_{Q_0^2}^{q_2^2} \frac{d\mu^2}{\mu} \frac{\alpha_s(\mu^2)}{\pi} \int_{\max\left(0, 1-\frac{\mu}{q_1}\right)}^{\min\left(1-\frac{Q_0}{q_1}, 1-\frac{\mu}{q_2}\right)} \frac{dz}{1-z} + \int_{Q_0^2}^{Q_0^2 \frac{q_2^2}{q_1^2}} \frac{d\mu^2}{\mu} \frac{\alpha_s(\mu^2)}{\pi} \int_{1-\frac{Q_0}{q_1}}^{\min\left(1-\frac{Q_0}{q_2}, 1-\frac{\mu}{q_2}\right)} \frac{dz}{1-z} \\ &= \int_{Q_0^2}^{q_1^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{q_1^2}\right) - \int_{Q_0^2}^{q_2^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{q_2^2}\right)\end{aligned} \tag{50}$$

Also the non-eikonal kernel, which is important for the unpolarized case, can be given an analytical structure:

$$\Delta_{\rm ne}(z, q, k_\perp) = \exp\left[-\frac{\alpha_s(Q^2(1-z)^2)}{\pi} C_A \ln(z) \left(\ln z - \ln \frac{k_\perp^2}{q^2}\right)\right] . \tag{51}$$

Figs. 5 and 6 show the $k_\perp$ dependence of the functions xg_1 and $x\Delta g_1$ for $Q^2 = 2$ GeV^2 and $Q^2 = 5$ GeV^2 for various values of x. We have chosen relatively small values of Q^2 because we cannot expect to describe large Q^2 values well by a single dressed gluon emission. As our input scale Q_0^2 is quite low, many dressed gluon emissions are necessary to get on to a stage where a realistic gluon distribution is obtained. Here we only want to see what effect the single dressed gluon emission has on the $k_\perp$ distribution. It is seen that for small values of $k_\perp$ there is a difference between the polarized and the unpolarized case due to the fact that in the polarized case the non-eikonal form factor is absent. In both cases one sees that the dressed single soft gluon emission leads to a strong broadening of the $k_\perp$ dependence. To a good approximation the $k_\perp$ distribution from the single dressed gluon emission is again of Gaussian type, especially in the polarized case where the non-eikonal form factor is absent. In general, for large $k_\perp$ the behavior is similar in the unpolarized case and in the polarized case.

IX. SUMMARY AND CONCLUSIONS

In this paper we have derived a polarized version for the pure gluon part of the CCFM equation. Comparing the polarized version with the unpolarized one the following remarkable features are seen:

- We have seen that for $z \to 1$ the hard splitting kernel in the polarized CCFM equation coincides with the $1/(1-z)$ pole in the corresponding polarized DGLAP splitting function. This is in parallel to the unpolarized case where the $1/(1-z)$ pole in the hard splitting function P(z) is identical to the $1/(1-z)$ pole in the corresponding unpolarized DGLAP splitting function.
- We have shown that due to spin correlation in the polarized case no non-eikonal contributions arise, and that for the same reason also the non-eikonal form factor does not enter into the polarized CCFM equation.
- This is consistent with the observation that there is no 1/z pole in the polarized case for the hard splitting function $\Delta P(z)$ because the non-eikonal form factor is coupled to that pole. The absence of such a 1/z pole in $\Delta P(z)$ is furthermore consistent with the fact that such a pole does not exist in the corresponding polarized DGLAP splitting function for the gluons.
- Finally, we have shown that considering single dressed gluon emission one finds for large $k_\perp$ a similar broadening of the $k_\perp$ distribution as in the unpolarized case. Differences occur only for small $k_\perp$, where the absence of the non-eikonal form factor becomes noticeable.

Next steps to be taken is a comparison to data. Here the first main difficulty will be to solve the polarized CCFM equation which seems to be possible only in terms of a Monte Carlo simulation of the soft emission. The second problem lies in the fact that the contribution from the quarks to the total polarized cross section is in principle of the same order as the one from the gluons. Therefore, it will be necessary to extend this formalism to the polarized contributions of the quarks as well. When there are quarks as initial states, a different type of vertex arises, where the polarization flow necessary enters into the soft emission of the final quark. Non-eikonal contributions become important and the simple reasoning of the polarization flow we discussed for the gluons cannot be applied. Future work will therefore have to deal predominantly with those two problems before a comprehensive comparison to data and a comparison to other calculations like the one in Ref. [12] can be done.

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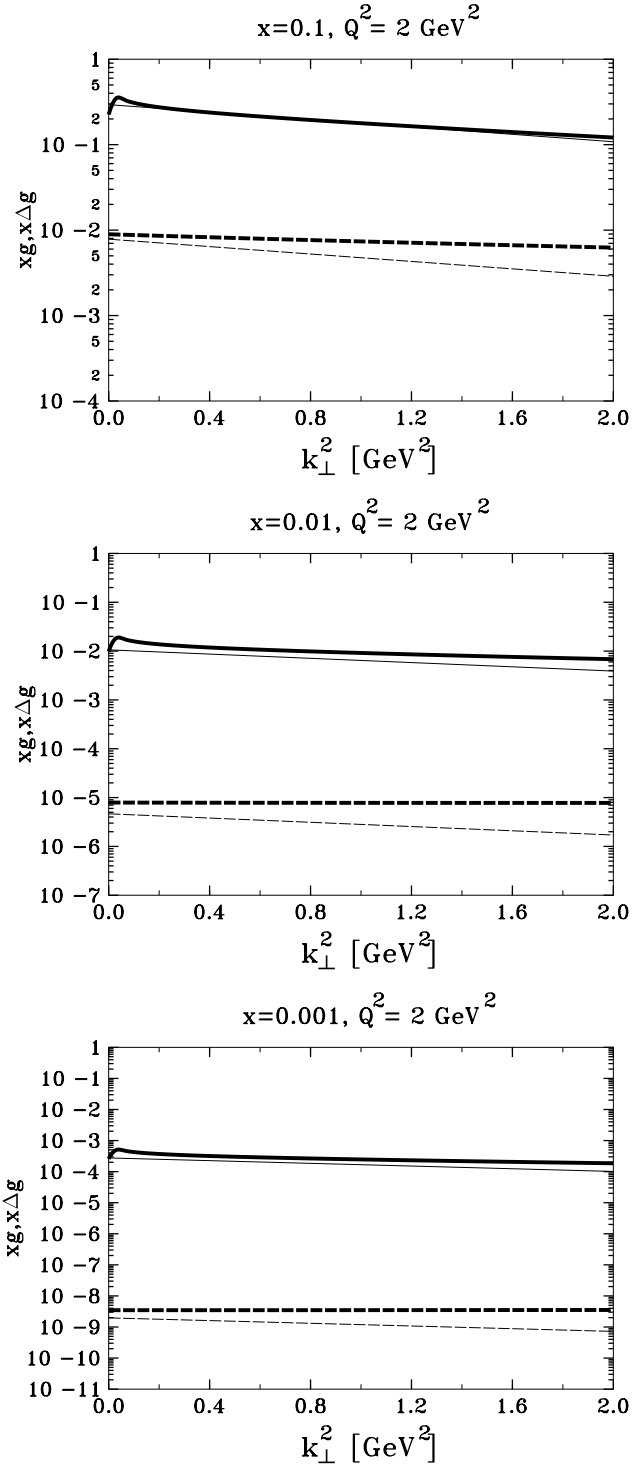

FIG. 5. $k_\perp$ dependence of the unintegrated gluon distribution for $Q^2 = 2$ GeV^2:
bold solid line $xg_1(x, k_\perp^2)$, thin solid line $xg_0(x, k_\perp^2)$,
bold dashed line $x\Delta g_1(x, k_\perp^2)$, thin dashed line $x\Delta g_0(x, k_\perp^2)$.

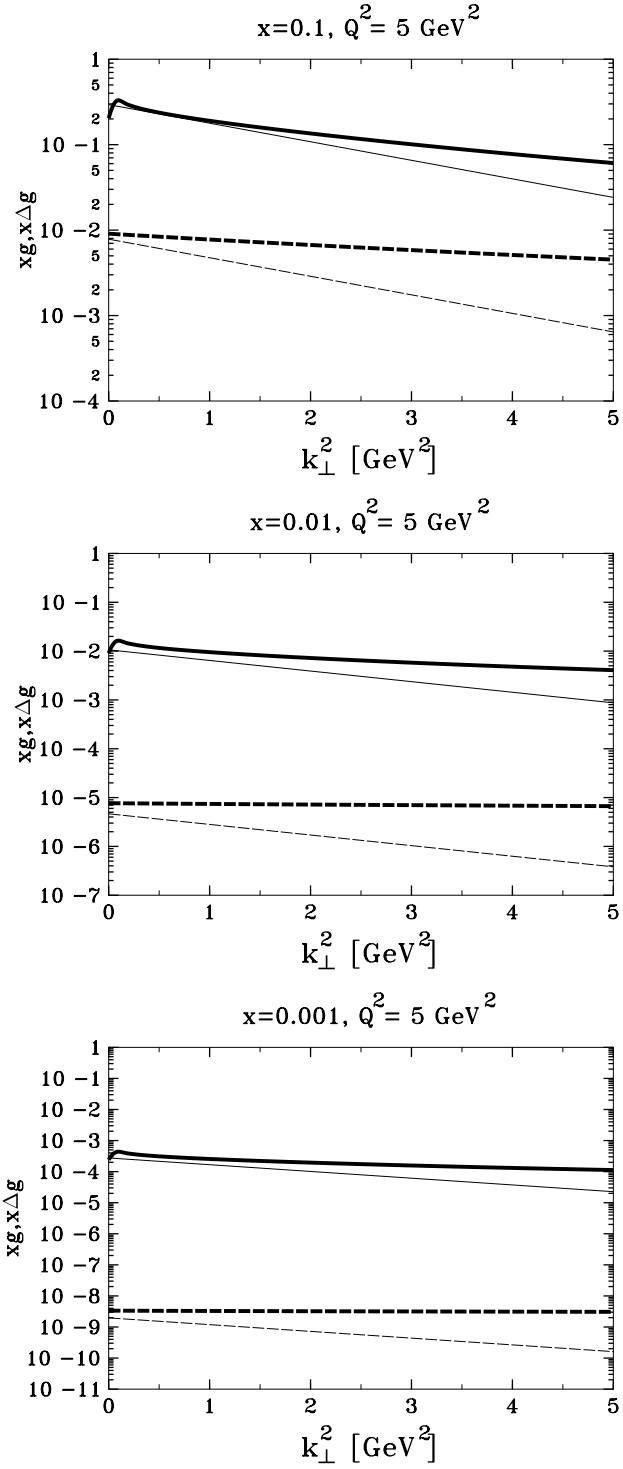

FIG. 6. $k_\perp$ dependence of the unintegrated gluon distribution $Q^2 = 5$ GeV^2:
bold solid line $xg_1(x, k_\perp^2)$, thin solid line $xg_0(x, k_\perp^2)$,
bold dashed line $x\Delta g_1(x, k_\perp^2)$, thin dashed line $x\Delta g_0(x, k_\perp^2)$.