Effective Field Theoretical Approach to Black Hole Production

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A field theoretical description of mini black hole production at TeV energies is given taking into account the quantization of black holes in discrete resonances. The unknown quantum gravitational effects are absorbed in effective couplings, black hole masses and the Hawking temperature. The evaporation is described in terms of thermal field theory.

Recently the possibility to produce mini black holes at TeV energies in the extra dimension scenario [1] has been proposed. Up to now it remains controversial whether the semi-classical production cross section is exponentially suppressed [2] or not [3], but even if there is an exponential suppression the production of mini black holes at LHC should be still sizably large [4]. A couple of semi-classical calculations have been performed to check the sensitivity of hadron colliders and neutrino telescopes to mini-black hole production [5]. In this paper we would like to address the black hole production from an effective field theoretical ansatz, constructing an effective interaction Lagrangian and absorbing the unknown quantum physical effects in effective coupling, black hole mass and the Hawking temperature, hereby reproducing the semi-classical results in their proper limit. To make things more definite we would like to study the process: $e^-(l) + e^-(l') \to \mathrm{bh} \to e^-(k) + e^-(k')$.

The crucial point is that while the production of the black hole happens in the vacuum, its decay is a thermal evaporation. Production and decay are governed by the same coupling constant, but in the decay one electron is evaporated thermally and the other is left as a remnant. The e^-e^- mode is a possible scenario for future next linear colliders like NLC at DESY or CLIC at CERN. We have chosen this process because the calculations are quite simple due to the reduced background (no strong interactions), but in principle the method is applicable to other processes e. g. involving hadron colliders as well.

This technique allows to handle the mini black hole as a particle, taking into account quantum physical effects like interference processes. The absorption of the unknown quantum gravity into couplings, masses etc. could be seen in parallel to what is done in e.g. effective meson field theory where the unknown collective effects of strong interaction are absorbed in form factors, effective couplings and masses as well.

The starting point of our consideration is the classical black hole thermodynamics because it yields an explicit expression for the Hawking temperature we are using at the end. In analogy to standard thermodynamics one can formulate three basic laws, see e. g. [7]:

- (zeroth law) The surface gravity γ of a black hole is constant on the horizon.
- (first law) Given M_{bh} the mass, A the area, L the angular momentum, Ω the horizon angular velocity, Q the charge and ν the electrostatic potential (being zero at infinity) of a black hole one has the energy relation [8]:

$$\delta M_{\mathrm{bh}} = \frac{\gamma}{8\pi}\delta A + \Omega\delta L - \nu\delta Q = T_H\delta S + \Omega\delta L - \nu\delta Q \ . \quad (1)$$

- (second law) the area of a black hole is nondecreasing $\delta A > 0$ [9].

These laws suggest an identification between the horizon area of a black hole and its entropy [10], which is given by $S_{\mathrm{bh}} = A/4$ [11]. Taking into account the fact that a black hole can evaporate due to the Hawking radiation [12] one has to complete the area law by the entropy added by the particle ejected out of the black hole:

$$\delta S_{\mathrm{tot}} = \delta S_{\mathrm{outside}} + \frac{1}{4}\delta A \geq 0 \ . \quad (2)$$

The different quantities of a black hole can be expressed solely by the angular momentum L, its charge Q and its mass M_{bh}. Introducing the rationalized area $\alpha = A/(4\pi)$ and the orbital angular momentum parameter $\vec{a} = \vec{L}/M_{\mathrm{bh}}$ one obtains [10]:

$$\begin{aligned}
\alpha &= r_+^2 + a^2 = 2M_{\mathrm{bh}}r_+ - Q^2 \ , \qquad \vec{\Omega} = \frac{\vec{a}}{\alpha} \ , \\
r_\pm &= M_{\mathrm{bh}} \pm \sqrt{M_{\mathrm{bh}}^2 - Q^2 - a^2} \ , \qquad \nu = -\frac{Qr_+}{\alpha} \ , \\
T_{\mathrm{H}} &= \frac{\gamma}{2\pi} = \frac{(r_+ - r_-)}{4\pi\alpha} = \frac{2}{A}\sqrt{M_{\mathrm{bh}}^2 - Q^2 - a^2} \ . \quad (3)
\end{aligned}$$

The black hole is then interpreted as a black body radiator with Hawking temperature T_H.

In this paper we wish to treat the black hole as a particle. As such it should be quantized in mass. The mass quantization in the conventional four dimensions has been given in Ref. [13]:

$$\begin{aligned}
M^2_{\mathrm{bh}\ n_b q j_z} &= g M_P^2 \left[n_b \left(1 + \alpha_{\mathrm{em}}\frac{q^2}{2n_b}\right)^2 + \frac{j_z^2}{n_b} \right] \ , \\
j_z^2 + \frac{1}{4}\alpha_{\mathrm{em}} q^2 &\leq n_b^2 \ , \quad (4)
\end{aligned}$$

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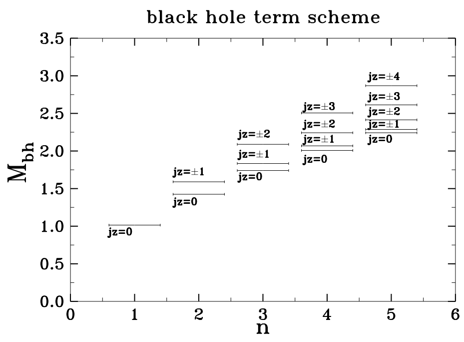

FIG. 1: Term scheme for a doubly charged black hole bh^{--}. The black hole mass $M_{\rm bh}$ is given in units of $M_P\sqrt{g}$, where M_P is the Planck mass or its equivalent in the extra dimension scenario, e.g. $M_P \sim 1$ TeV.

where M_P is the Planck mass, q the charge quantum number j_z the angular momentum quantum number. The quantum number n_b takes the quantization of the black hole horizon surface into account and should not be confused with the number of n extra dimensions used in later formulas. The pre-factor g is controversial. In Ref. [13] it was chosen to be $g = 1/2$ as the smallest possible quantum unity. The area quantization has been treated in the framework of loop quantum gravity (LQG) for the spheric symmetrical problem, see e.g. [14]. For a special choice of the quantum numbers for the edges of the surface geometry one obtains $g = \ln 2/(2\pi)$ [6] consistent with the result derived in [15] in the framework of a Chern Simons field theory. Recently, requiring that the entropy of the black hole should be maximal a value of $g = 0.614/\pi$ has been derived making use of the LQG result [6].

In the extra dimension scenario the fundamental Planck scale M_P should be at TeV range [16], which correspondingly should result in a discrete spectrum for the black holes spaced in the order of TeV distances. In $e^-e^- \to bh \to e^-e^-$, where we should have a doubly charged black hole ($q = -2$) we would arrive at the following term scheme, see Fig. 1.

The black hole quantization predicts that there is an isolated scalar black hole resonance at $M_P\sqrt{g}$. In the following we concentrate on this first scalar excitation and develop an effective field theory for it. In the calculation for the term scheme above we have set $\alpha_{\rm em} = 1/137$. The running of the electro magnetic coupling in the conventional standard model as included e.g. in PYTHIA [17] up to 10 TeV only makes a difference up to 12% which we have neglected here for simplicity, as the effect is not visible here. It would be a completely different story if also the GUT scale would be at considerable lower values, but as to a lack of a proper determination of such a scale we will not pursue this idea further here. It is our aim to develop a workable formalism which allows practical analysis for high-energy collider reactions. Interferences between the black hole production and decay and background processes described by conventional field theory are important. Yet the precise quantum gravitational production and evaporation process is not known. We therefore want to set aside these problems by factorizing the unknown quantum gravity physics in an effective coupling and regarding only the asymptotic initial and final states.

As an example we consider the production of doubly charged scalar black holes with angular momentum j=0 by two fermions e.g. electrons. For the interaction part of the Lagrangian one can set:

$$\begin{aligned}\mathcal{L}_{\rm int} &= i\kappa_{\rm eff} M_{\rm bh} \phi \bar{\Psi}_f \hat{C} \Psi_f + h.c.\,,\\ V_{\rm eff}(k_1,k_2) &= i\kappa_{\rm eff} M_{\rm bh} \hat{C}\,,\end{aligned} \tag{5}$$

where $\hat{C} = i\gamma_2\mathcal{K}$ is the usual charge conjugation operator with $\mathcal{K}$ being the complex conjugation. One should note that $\kappa_{\rm eff}$ may be different for different n_b, so that one has different couplings to different black hole micro states. The mass scale involved has been chosen to be the black hole mass $M_{\rm bh}$ and not the fermion mass m_f so that the coupling does not vanish in the limit of vanishing fermion rest mass m_f. Such a type of coupling can be compared to the coupling of two fermions to a doubly charged Higgs [18].

As a next step we take an example of determining the effective coupling in a crude approximation from the production cross section of a black hole by two colliding electrons. Here and in the following we will neglect the electron mass m_e throughout, because it is more than 6 orders of magnitude smaller than the TeV scale which sets the black hole mass involved. Taking the amplitude for black hole production by electrons:

$$\begin{aligned} &M(p,p') = i\kappa_{\rm eff} M_{\rm bh} \bar{u}(p)\hat{C}u(p')\,, \quad k = (M_{\rm bh},0,0,0)\,,\\ &p = \left(\frac{1}{2}\sqrt{s},0,0,\frac{1}{2}\sqrt{s}\right)\,, \quad p' = \left(\frac{1}{2}\sqrt{s},0,0,-\frac{1}{2}\sqrt{s}\right)\,, \end{aligned} \tag{6}$$

one can make a connection with the geometrical production cross section via:

$$\begin{aligned}\sigma_{\rm bh} &= \frac{1}{4}\frac{1}{4pp'}|M|^2 \int \frac{d^4k}{(2\pi)^3}(2\pi)^4\delta(p+p'-k)\,\delta\left(k^2 - M_{\rm bh}^2\right)\\ &= \frac{\pi}{4}\kappa_{\rm eff}^2 M_{\rm bh}\,\delta\left(\sqrt{s}-M_{\rm bh}\right) \to \pi\frac{\kappa_{\rm eff}^2}{4}\theta\left(\frac{M_{\rm bh}}{2} - |\sqrt{s}-M_{\rm bh}|\right) = \pi R_S^2\,\theta\left(\frac{M_{\rm bh}}{2} - |\sqrt{s}-M_{\rm bh}|\right) \equiv \sigma_{\rm geom}\,. \end{aligned} \tag{7}$$

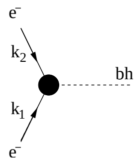

FIG. 2: Effective vertex for the production process $e^- e^- \to$ bh.

In the arrow we transform according to local duality the δ function into a finite size step function of width $M_{\rm bh}$ in order to make contact with the geometrical cross section. $R_{\rm S}$ is the Schwarzschild radius which in $n+3$ dimensions is [19]:

$$R_{\rm S} = \frac{1}{\sqrt{\pi} M_{\rm P}} \left[\frac{M_{\rm bh}}{M_{\rm P}} \left(\frac{8\Gamma\left(\frac{n+3}{2}\right)}{n+2} \right) \right]^{1/(n+1)} . \quad (8)$$

Here we have suggested $M_{\rm bh}$ as the scale for the effective width of the black hole production in vacuum to compare the field theoretical cross section to the geometrical one. This comparison then suggests to set $\kappa_{\rm eff}/2 = R_{\rm S}$. Of course such a contact between geometrical and field theoretical cross section is a crude approximation to reality. Practically, $\kappa_{\rm eff}$ will be an effective coupling constant absorbing the unknown quantum gravitational physics. It is then simply a constant that has to be determined by experiment.

As already discussed the evaporation of the black hole is thermal. Therefore it should be possible to describe it in the framework of thermal field theory which has been developed in [20]. In this connection we obtain for the partial width bh $\to e^- e^-$ using again $k = (M_{\rm bh}, 0, 0, 0)$:

$$\begin{aligned} \Gamma_{{\rm bh}\to e^- e^-} &= \frac{\kappa_{\rm eff}^2 M_{\rm bh}^2}{2 M_{\rm bh}} \int \frac{d^4 p}{(2\pi)^3} \int \frac{d^4 p'}{(2\pi)^3} \frac{1}{e^{\beta_{\rm H} |p_0|} + 1} \\ &\times \delta\left(p^2\right) \delta\left({p'}^2\right) (2\pi)^4 \delta\left(p + p' - k\right) {\rm Tr}\left[p_\mu \gamma^\mu \hat{C} p'_\nu \gamma^\nu \hat{C}^\dagger \right] \\ &= \frac{\kappa_{\rm eff}^2 M_{\rm bh}^3}{8\pi} \frac{1}{e^{\beta_{\rm H} M_{\rm bh}/2} + 1} . \end{aligned} \quad (9)$$

Here $T_{\rm H} = 1/\beta_{\rm H}$ is the Hawking temperature which is in $n+3$ dimensions given by [19]:

$$T_{\rm H} = M_{\rm P} \left(\frac{M_{\rm P}}{M_{\rm bh}} \frac{n+2}{8\Gamma\left(\frac{n+3}{2}\right)} \right)^{1/(n+1)} \frac{n+1}{4\sqrt{\pi}} . \quad (10)$$

One should note that the factor $1/(\exp(\beta_{\rm H} M_{\rm bh}/2) + 1)$ does not belong to the coupling but to the outgoing electrons. The principle should be that each evaporated particle goes with the corresponding thermal occupation number due to the proper statistic it belongs to. For the final evaporation it is actually only one particle that evaporates while the other one is just the remaining remnant where the energy is fixed by energy momentum conservation. As one can not distinguish in our case which of the electrons has evaporated and which is the remnant we end up with a single factor $1/(\exp(\beta_{\rm H} M_{\rm bh}/2) + 1)$. If we had a different black hole evaporating in an electron and a photon we had to multiply with a factor $\left(1/(e^{\beta_{\rm H} M_{\rm bh}/2} - 1) + 1/(e^{\beta_{\rm H} M_{\rm bh}/2} + 1)\right)$ to allow one time for a photon and one time for an electron evaporation whereas the other particle is just the remnant.

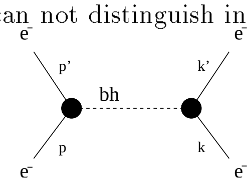

FIG. 3: Graph for the process $e^- e^- \to {\rm bh} \to e^- e^-$.

As a last application of the method we calculate the $e^- e^- \to e^- e^-$ cross section, for the first scalar black hole excitation using the method of effective thermal field theory discussed above. For the amplitude we find c.f. Fig. 3:

$$\mathcal{M} = M(p, p') \frac{1}{s - \left(M_{\rm bh} + \frac{i}{2}\Gamma_{\rm total}\right)^2} M(k, k') . \quad (11)$$

Then the corresponding cross section has the form:

$$\begin{aligned} \sigma(s) &= \frac{1}{4} \int \frac{d^4 k}{(2\pi)^3} \delta\left(k^2\right) \int \frac{d^4 k'}{(2\pi)^3} \delta\left(k'^2\right) \frac{|\mathcal{M}|^2}{4 p p'} \frac{1}{e^{\beta_{\rm H} k_0} + 1} (2\pi)^4 \delta\left(p + p' - k - k'\right) \\ &= \frac{\kappa_{\rm eff}^4 M_{\rm bh}^4 s}{16\pi} \frac{1}{\left(s - \left(M_{\rm bh}^2 - \frac{\Gamma_{\rm total}^2}{4}\right)\right)^2 + M_{\rm bh}^2 \Gamma_{\rm total}^2} \frac{1}{e^{\beta_{\rm H}\sqrt{s}/2} + 1} . \end{aligned} \quad (12)$$

In Fig. 4 we show the total cross section $ee \to {\rm bh} \to ee$ in the Khriplovich scenario Ref. [6], i.e. for a first scalar black hole mass of $M_{\rm bh} \approx 448.5$ GeV assuming $M_{\rm P} = 1$ TeV for different values of n (extra dimensions). One should keep in mind that at such comparatively small energies, i.e. smaller than a few times the Planck mass, it may be controversial whether such a resonance should be interpreted as a genuine black hole or

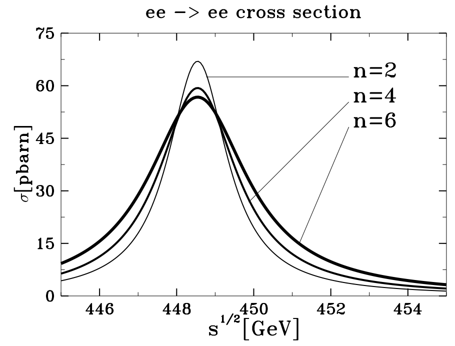

FIG. 4: Total cross section for the reaction $ee \to \mathrm{bh} \to ee$ in the Khriplovich scenario [6].

rather as a string excitation. We have set to leading order $\Gamma_{\rm total} = \Gamma_{{\rm bh}\to e^- e^-}$ assuming strict lepton number conservation, as current bounds from muon decay into three electrons otherwise require a Planck scale in the 100 TeV range. It is seen that the width of the resonance in first order is about 2-3 GeV and that the total cross section is in the range of pbarn.

In this paper we intend to give a description for the black hole in terms of a particle. Here the mini black holes should arise as discrete resonances in accordance with the idea of Beckenstein that the black hole area should be quantized. The in vacuum production of black holes can be described by means of standard field theory where the unknown quantum gravity is absorbed in an effective coupling constant $\kappa_{\rm eff}$. The thermal evaporation is correspondingly described in terms of the thermal field theory. Such a description allows to take into consideration quantum interference effects which are not accessible in a semi-classical description and will be helpful for the experimental analysis of events that may come from mini-black holes produced at high energy scattering experiments. The formalism shown here can be easily generalized to a variety of processes, for example also to mini black hole production at LHC. Within this framework and restricting to the well isolated first scalar black hole resonance, the analysis of the quantum gravitational effects boils down to the measurement of only three independent quantities: The effective coupling κ_{eff}, $M_{\rm bh}$ (the first scalar black hole mass) and the Hawking temperature $T_{\rm H} = 1/\beta_{\rm H}$. In this way studying definite subprocesses in possible black hole production in terms of an effective quantum field theory may allow to pin down the quantum gravitational content to actually three parameters. These parameters as soon as obtained experimentally can then be compared to various quantum gravitational scenarios to improve our understanding of the fundamental laws of gravitational physics.

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