BOSE EINSTEIN CORRELATIONS IN THE LUND MODEL FOR MULTIJET SYSTEMS

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The interference based analysis of Bose Einstein Correlations in the Lund Model has hitherto been limited to simple strings without gluonic excitations. A new fragmentation method based on the Area Law in the Lund Model allows such an analysis to be extended to multigluon strings.

1. Introduction

The Bose Einstein effect or the enhancement of the two particle correlation function for identical bosons with very similar energy momenta is well known in hadronic interactions. Since hadronisation is mostly described through phenomenological models and Monte Carlo simulations, which are based on classical probabilistic concepts, quantum mechanical effects such as the Bose Einstein Correlations (BEC) pose a problem.

In the event generator PYTHIA, where hadronisation is handled through the Lund string fragmentation model, this effect is mimicked by introducing an attractive interaction between identical bosons in the final state. The purpose behind this is to parametrise the effect, rather than to provide a physical model for it.

A physical model for describing the BEC effect within the string fragmentation scenario was developed by Andersson and Hofmann in [1] which was later extended by Andersson and Ringnér in [2]. They showed that associating an amplitude with the decay of a string into a set of hadrons in the Lund Model leads to interference effects which enhance the probability for identical bosons forming a shade closer in the phase space than what would be expected in a purely classical treatment, and identical fermions a shade farther appart.

But their formulation was limited to the simplest string configuration,

i.e., a string stretched between a quark and an antiquark with no gluonic excitations. Comparison with direct experimental data on BEC was not feasible, since a proper description of the properties of hadronic jets requires parton showers, and subsequent fragmentation of multigluon strings. Even though PYTHIA implements one approach towards multigluon string fragmentation, the interference based model for Bose Einstein effect of Andersson and Ringnér could not be extended to the multigluon string fragmentation scheme in PYTHIA.

Recently, an alternative way to fragment the multigluon string has been developed in [3]. Unlike the approach in PYTHIA, this method does not try to follow the complicated surface of a multigluon string. It is based on the observation that the string surface is a minimal area surface in space-time, and hence it is completely determined by its boundary. An attempt was made to reformulate string fragmentation as a process along this boundary, called the "directrix". The result was a new scheme for string fragmentation, with a simple generalisation to multigluon strings. This method of hadronisation has been implemented in an independent Monte Carlo routine called "ALFS" (for "Area Law Fragmentation of Strings")[a].

Particle distributions from ALFS are in agreement with those of PYTHIA on the average, but there are differences at an exclusive event to event basis, which may show up in higher moments of the distributions. It was also understood that the interference based model for the BEC effect can be extended to multigluon string fragmentation in ALFS.

In Sec. 2 this new fragmentation scheme will be summarised very briefly. A brief description of the basic physics of the interference based approach to the BEC appears in Sec. 3. In Sec. 4 the concept of coherence chains will be introduced which allows the extension of the analysis of BEC in the Lund Model to multigluon strings. Finally, some preliminary plots obtained by using this method to analyze two particle correlations will be presented in Sec. 5.

2. String Fragmentation as a process along the directrix

We recall that the probability for the formation of a set of hadrons from a given set of partons in the Lund Model, is given by what is known as the "Area Law". It states that this probability is the product of the final state phase space and the negative exponential of the area spanned by the string before it decays into the hadrons (cf. Figure 1):

[a] available on request from the author.

$$dP_n(\{p_j\}; P_{tot}) = \prod_{j=1}^{n} N_j d^2p_j \delta(p_j^2 - m_j^2) \delta(\sum_{j=1}^{n} p_j - P_{tot}) \exp(-bA) \quad (1)$$

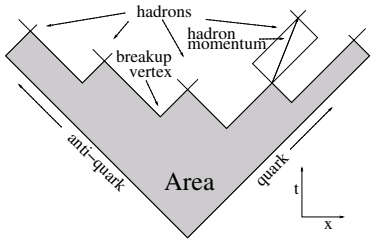

Figure 1. Fragmentation of a String without gluonic excitations in the Lund Model.

An iterative process based on the result in Eq. (1) is fairly straight forward to construct for systems without gluons. In the Lund Model, gluons are thought of as internal excitations on the string. A string with many such excitations traces complicated surfaces consisting of a large number of independent planar regions in space–time. One example can be seen in Figure 6 in Sec. 3, which illustrates the surface of a string with just one gluon. Calculating the energy momenta of the hadrons resulting from a decay of strings with many gluons is rather difficult.

But since the world surface of a string is a minimal area surface, it has many important symmetry properties which may be exploited while considering its decay into a set of hadrons. Minimal surfaces are completely specified by their boundaries. For a string in the Lund Model, this boundary, called the "directrix", is the trajectory of the quark or the antiquark (one of the end points). Since the directrix determines the string surface, it is possible to formulate string fragmentation as a process along the directrix, as shown in [3].

The directrix for a string, which can be thought to originate at a single point in space–time, is particularly simple and easy to visualize. This curve can be constructed by placing the energy-momentum vectors of the partons one after the other in colour order as shown (schematically) in Figure 2.

The fragmentation process developed in [3] identifies the area in the area law with the area between the directrix and the "hadronic curve"[b],

[b] The string constant κ, or the energy per unit length in the string, will be set to unity

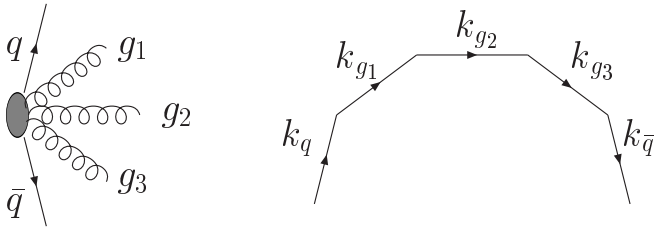

Figure 2. Schematic representation of the directrix for a configuration with a few resolved partons.

i.e., the curve obtained by placing the hadron energy momenta one after the other in rank order. The area used in the area law can be partitioned into contributions from the formation of each hadron in many different ways. Figure 3 shows one possible partitioning where a triangular region is associated with one particle (shaded region in the upper left part of the figure). This figure also illustrates the connection between the area in Figure 1 and the area between the directrix and the hadronic curve. The upper left part of the figure shows the same set of breakup vertices and hadrons as Figure 1. The vectors q_j in the lower half of the figure are obtained from the vertex vectors x_j by inverting one light-cone component of x_j, and are "dual" to the vectors x_j in this sense. They represent the energy momentum transfer between the two parts of the string formed because of the breakup at x_j. The triangular regions in the upper part of the figure can be geometrically mapped to the triangular regions in the lower part. But the sum of the triangular areas in the lower part is the area between the directrix and the hadronic curve whereas in the upper part it is the area as used in Eq. (1)(ignoring a dynamically uninteresting constant contribution of $\frac{1}{2}m^2$ for each hadron of mass m).

The hadronisation process in ALFS associates a quadrangular "plaquette", bounded by the hadron energy momentum vector, two 'vertex' vectors, and a section of the directrix, with the hadron. These plaquettes are not simple geometrical projections of the triangular areas shown in Figure 3, but their areas are related in such a way that the sum of the areas of the plaquettes is the same as the sum of the areas of the triangles. The 'vertex' vectors in ALFS indeed do correspond to the space time locations where quark antiquark pairs form along the string during fragmentation, for a flat string. But in a more general context, it is better to think of them as

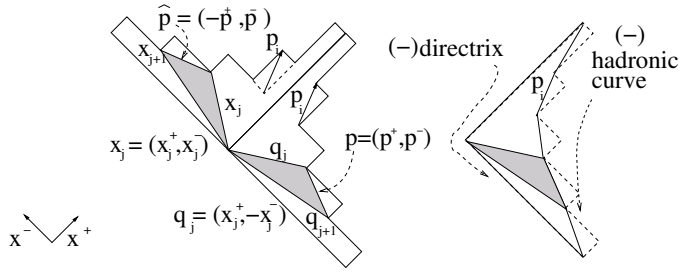

Figure 3. One possible way to partition the area of a fragmenting string into congributions for each hadron. It shows the connection between the area in the area law and the area between the directrix and the hadronic curve.

somewhat more complex dynamical variables.

String fragmentation (especially as implemented in ALFS) could be thought of in terms of energy momentum transfer or "ladder" diagrams like in Figure 4. A quark momentum k_q branches into a hadron momentum p_1 and an energy momentum transfer q_1, which then branches into a hadron vector p_2 and a new energy momentum transfer q_2, and so on. At each stage the hadron momentum forms from the energy momentum transfer vector comming into that stage and another independent vector which serves to define a longitudinal plane in space–time. This other vector is just the anti–quark vector for a flat string. More generally it is a section of the directrix. This completes our brief overview of string fragmentation

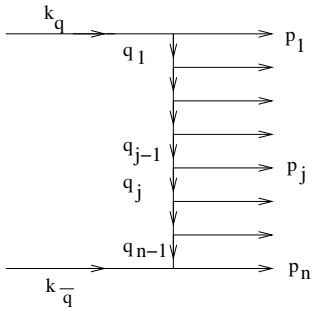

Figure 4. Lund Model can be thought of in terms of a ladder diagram involving energy momentum exchanges.

in ALFS. For a detailed treatment and the exact expressions the reader is referred to [3].

3. Physics of Bose Einstein Correlations in the Lund Model

There is a formal similarity between the Area Law in Eq. 1 and quantum mechanical transition probabilities. And even though hadronisation is a quantum mechanical process, the semiclassical approach in the Lund Model has been very successful in describing experimental data. It is not impossible therefore, that the underlying quantum mechanical process might have an amplitude which when squared resembles the area law.

In [2] Andersson and Ringnér argued that one can associate an amplitude of the form

$$\mu = e^{i(\kappa + ib/2)A} \tag{2}$$

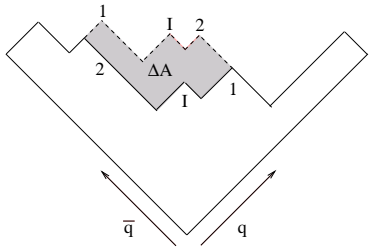

Figure 5. Interchanging rank order of two identical particles in the final state would require a different set of breakup vertices and a different area under the string.

where κ is the string constant, with the decay of a string into a set of hadrons. This amplitude trivially reproduces the Area Law in Eq. 1. But it also introduces interference effects for final states involving two or more identical particles, since for such final states the string fragmentation model allows many different ways to produce the same final state from a given initial state as illustrated in Fig 5. The figure shows two sets of breakup vertices which could lead to the same set of final state particles in the same flavour order. The particles labeled "1" and "2", assumed identical, have interchanged rank orders between the two schemes. The two schemes clearly involve different areas, and hence will have different amplitudes according to Eq. (2). This means the total squared amplitude for forming such a final state (assuming there are no other identical particles in the event) should be $|\mu|^2 = |\mu_1 + \mu_2|^2$, where μ_1 and μ_2 are the amplitudes of the two schemes shown in Figure 5. But a probabilistic Monte Carlo simulation would assign a probability $\mu_1^2 + \mu_2^2$ with such a state, which

does not account for the interference term. Thus, to associate the right probability with the events we may weight this event with an event weight

$$w = (1 + 2Re(\mu_1^* \mu_2)/(|\mu_1|^2 + |\mu_2|^2)) \tag{3}$$

The result can be generalised to the case of many identical particles, and to include the effect of transverse momentum generation during hadronisation, as described in [2].

Treatment of string states with gluonic excitations presents new problems. Since the multiplicity of the events rises with the number of gluonic excitations, the number of identical particles expected is larger. This presents a computational problem. More importantly though, in this case it is not always possible to find a string fragmentation scheme with only the rank order of two identical particles interchanged. When an exchanged scheme exists the calculation of true area differences and transverse momentum contributions to the amplitude is rather involved, if the exchanged particles were originally produced in different planar regions.

But in string fragmentation, the particle energy momenta are constructed from local momentum flow along the string world surface in the neighbourhood of the breakup vertices. Therefore, most of the energy momentum of a hadron is along the local longitudinal directions relative to the string. Figure 6 once again shows two identical particles formed in different regions in the string. But this time they do not belong to the same planar region on the string. It is clear that the "exchanged" scheme (shown to the right) would be highly unlikely to emerge from this string as the energy momenta are no longer nearly aligned with the local longitudinal directions.

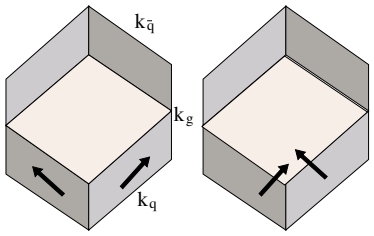

Figure 6. We show here the surface traced by a string in a system consisting of a quark, a single gluonic excitation, and an antiquark. Interchanging rank order of two identical particles in different string planes seems unnatural. The interchanged schemes would have very low probabilities to be produced during string fragmentation. It may help to think of the two surfaces represented here like two chairs facing the reader, for visualisation.

It was mentioned earlier that the fragmentation scheme in ALFS does not depend on explicit representations of the string surface such as the one in Figure 6. In that approach, it is sometimes possible to find another partonic configuration which may result in the exchanged scheme as one possible event. But if the partonic state is held fixed, such an exchange would be improbable for the reasons just mentioned.

As a first approximation therefore, it is reasonable to calculate BEC on multigluon strings by considering particle permutations in the planar regions of the string surface and ignoring the effects of exchange of particles across gluon corners. But the number of gluons and the size of planar regions on the string depend on the cut-off scale in the parton cascade used to generate the partons in an event generator. It would therefore seem that by making the cut-off sufficiently small we can make the planar regions so small that there would not be any instances of identical particles in one planar region anywhere in the event. To address this, we introduce the concept of coherence chains.

4. Coherence Chains

When the cut-off scale in the ordering variable (gluon transverse momentum, for example) is made small, softer and softer gluons are resolved. For a relatively soft gluon, the two planes in Figure 6 will be only slightly inclined with respect to each other, and the exchanged scheme would not appear so unnatural. If such exchanges are permitted in ALFS, the new partonic states created will not be outrageously different from the one we started with.

However, parton showers are probabilistic in the Monte Carlos. Information about phases involved with different partonic configurations are "lost". To analyse permutations of identical hadrons across gluon cornors, we need to consider interference effects between results of hadronisation from two slightly different partonic configurations. This appears to be problematic as we need both the phase information from the string fragmentation and the phase information from the partonic stage while calculating the interference terms and event weights.

Infrared stability of string fragmentation, on the other hand, suggests that the detailed properties of the hadronic states should not be extremely sensitive to gluon emission around hadronic mass scales. In a sense the string state is resolved at the hadron mass scales by the fragmentation process. One interesting consequence of this was observed for the set of

hadrons emerging out of the fragmentation of multigluon strings in ALFS.

The energy momenta of the hadrons could be collected into sets, such that inside each set, the energy momenta are aligned in a plane in space time upto a small scale in transverse momentum fluctuations. This suggests that at least some aspects of the hadronic phenomena might be insensitive to the softest gluons generated by the parton showers.

With an analysis of BEC in mind we call these groups of particles in the final state as "Coherence Chains". They describe the regions on the string over which coherent interference effects between hadrons should be considered. As we have seen, it seems quite unnatural to consider symmetrisation across hard gluon cornors, cf. Fig 6, whereas symmetrisation across soft gluons is necessary. The transverse momentum resolution scale used to define the coherence chains should be chosen such that it distinguishes between these situations.

The approximation being made in the analysis of BEC through the coherence chains could be stated as follows: we ignore the possible effects on BEC, of the slightly different amplitudes of different partonic states which may give rise to one coherence chain after hadronisation. To calculate BE weights, we treat the hadronic state as if it came from a simpler string state which has only those planes in it which are present in the coherence chains. Symmetrisation is then carried out separately for each plane and the squared amplitudes multiplied and a suitable event weight calculated.

The hadron energy momenta are not directly altered as in PYBOEI (the BE subroutine in PYTHIA), but different events receive different weights.

There is a tendency for events with higher multiplicity to yield higher weights. Since multiplicity is a function of gluonic activity, it is not possible to retune parameters pertaining to hadronisation to compensate for the multiplicity dependence of weights, unless we associate a total of one hadronic state for each partonic configuration. This leaves only the possibility of a rejection weighting on the hadronized states based on their BE weights in a Monte Carlo. This procedure is much slower than PYBOEI. But the purpose of this exercise is to provide a physical picture for the phenomenon inside the Lund Model.

5. Priliminary Results and Concluding Remarks

The interference based analysis of BEC in the Lund Model has been extended so as to be applicable to multigluon string fragmentation as implemented in ALFS. Modules for BEC calculations have also been introduced

into ALFS. A priliminary analysis shows the expected enhancement of the two particle correlation function at small momentum differences. For events with a few prominent jets, BEC tends to decrease with the number of jets if the total λ-measure for the strings is kept fixed, cf Figure 7. No significant correlation is seen between oppositely charged pions, cf Figure 8. A detailed study of the properties of coherence chains, how they affect the analysis of BEC and further studies of this model for BEC itself will be presented elsewhere.

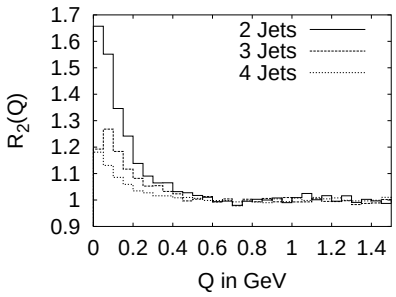

Figure 7. Two particle correlation function from ALFS for systems with few jets. "String length" or λ-measure was kept fixed.

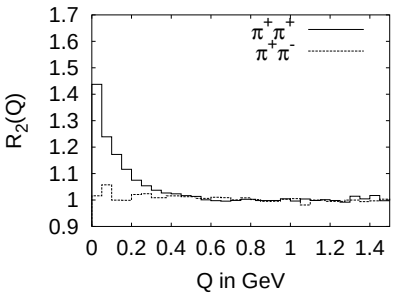

Figure 8. This plot shows that no significant correlation effects are expected between oppositely charged pions in this model.

Acknowledgements

This project began as a collaboration between the Late Prof. Bo Andersson, my collegue Fredrik Söderberg and myself. Even though it is still an unfinished project and new developments are being made, we are indebted to Prof. Andersson for the numerous insights he provided, while he was with us.

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