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Partially Quenched Chiral Perturbation Theory to NNLO

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Abstract. This paper summarizes the recent calculations of the masses and decay constants of the pseudoscalar mesons at the two-loop level, or NNLO, in Partially Quenched Chiral Perturbation theory (PQχPT). Possible applications include chiral extrapolations of Lattice QCD, as well as the determination of the low-energy constants (LEC:s) of QCD.

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INTRODUCTION

Chiral Perturbation Theory (χPT) is an effective field theory approximation to QCD which allows for a description of low-energy hadronic properties in terms of a systematic expansion in momenta, energies and quark masses [1]. χPT is by now a well developed field, such that many quantites, e.g. meson masses, decay constants and vacuum expectation values, are known up to NNLO, or $\mathcal{O}(p^6)$. As χPT is a non-renormalizable theory, an increasing, but certainly not overwhelming, number of low-energy constants (LEC:s) appears at each order in the calculations. These LEC:s are not determined by chiral symmetry, but are in principle calculable from full QCD. In addition, some of the LEC:s can also be fixed by fitting χPT expressions to experimental data.

At the present time, however, the only known way of deriving quantitative information on the low-energy properties of QCD from first principles is by means of Lattice QCD simulations, where QCD is 'solved' by numerical Monte Carlo techniques on a discretized space-time lattice. Ideally, one would then use χPT to extract information on the LEC:s of QCD, as well as for the extrapolation of Lattice QCD results to the continuum limit. Unfortunately, χPT is insufficient for this purpose as Lattice QCD simulations have, for reasons of computational efficiency, to be performed in the so-called partially quenched approximation (PQQCD), where the masses of the sea quarks are different from and typically much larger than the valence quark masses. However, much progress is being made and at present, simulations are practicable for light sea quark masses of $\sim m_s/4$, where m_s is the strange quark mass. Nevertheless, this is still quite far from the physical light quark masses of $\sim m_s/25$.

Partially Quenched χPT (PQχPT) is the extension of χPT to the case of unequal valence and sea quark masses [2, 3], and provides a way of making contact with present Lattice QCD simulations if the sea quark masses are small enough so that the chiral expansion is applicable. The possiblity of producing Lattice QCD data for different values of the sea and valence quark masses also allows for the investigation of a more comprehensive set of LEC:s. In PQχPT, disconnected valence quark loops are eliminated by introduction of bosonic 'ghost' quarks. PQχPT at NLO as well as the treatment of the super-singlet Φ_0 has been discussed in great detail in [4].

PQχPT AT NNLO

Since QCD may be obtained as a continuous limit of PQQCD, the LEC:s of PQQCD with n_f sea quarks should, in general, be expected to be the same as for (unquenched) n_f flavor QCD. A slight complication is introduced by the Cayley-Hamilton relations which are used to decrease the number of independent LEC:s in the unquenched case, since these no longer hold for partially quenched theories. Nevertheless, the LEC:s of unquenched QCD may be expressed as linear combinations of the PQQCD ones, so that a set of values for the partially quenched LEC:s may be directly translated into the physical ones.

The expressions obtained at NNLO in PQχPT are extremely long compared to the ones in unquenched χPT, even though the basic Feynman diagrams are similar. The NNLO Feynman diagrams for the calculation of the decay constants of the pseudoscalar mesons are shown in Fig. 1. One source of added complexity is the appearance of many more possible combinations of quark masses in the calculations, but most importantly, the meson propagators in the neutral sector of PQχPT contain both single and double poles, the residues of which consist of complicated rational functions of the sea and valence quark masses. Fortunately, these residues satisfy a multitude of nontrivial relations which can, although at great effort, be used to compress the end results by significantly more than an order of magnitude, thereby bringing them to a publishable form.

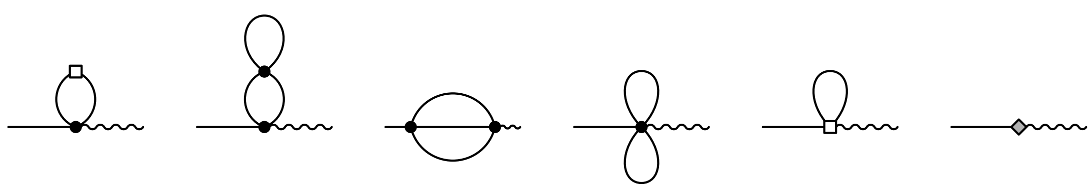

FIGURE 1. NNLO Feynman diagrams for the calculation of the pseudoscalar meson decay constant. Solid lines are pseudoscalars and wiggly lines are external axial currents. Solid dots, open squares and diamonds represent vertices of the $\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ PQχPT Lagrangians, respectively. Additional wavefunction renormalization contributions are not shown.

The masses and decay constants at NNLO in PQχPT depend on a number of as yet largely unknown LEC:s K_i^r from the $\mathcal{O}(p^6)$ counterterms. As expected from earlier work at two loops in χPT, the contributions from the chiral logarithms are very large at NNLO, but are significantly moderated when realistic values of the L_i^r are used. Numerical results up to NNLO for masses and decay constants for $n_f = 2$, with all K_i^r set to zero, are shown in Fig. 2. The quantities plotted are $\Delta_F \equiv F/F_0 - 1$ for the decay constant, and $\Delta_M \equiv M^2/M_0^2 - 1$ for the mass, where F_0 and M_0^2 denote the lowest order PQχPT results.

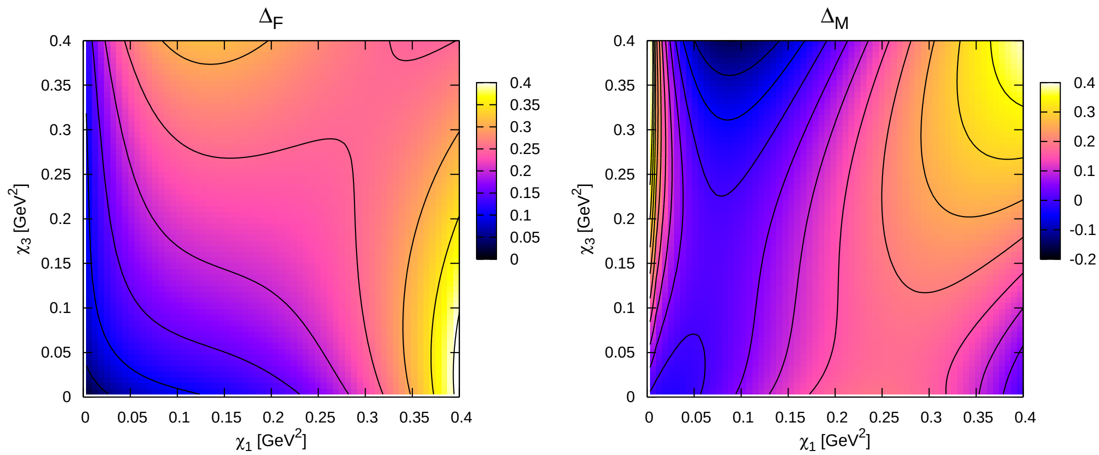

FIGURE 2. The relative deviation from the tree-level result in $n_f = 2$ PQχPT at NNLO, for the pseudoscalar meson decay constant (Δ_F) and mass (Δ_M), as a function of the valence (m_1) and sea quark (m_3) masses defined through $\chi_i \equiv 2B_0 m_i$, for the set of LEC:s called 'fit 10' in ref. [5].

The following NNLO calculations in PQχPT have already been published: The pseudoscalar meson mass for $n_f = 3$ sea quark flavors, for equal sea quark and equal valence quark masses [6], the decay constant for $n_f = 3$ up to two nondegenerate sea quarks, and a complete calculation of masses and decay constants for $n_f = 2$ [7]. The full results for $n_f = 3$ are being written up, along with a longer introduction to PQχPT at NNLO.

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