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Eta and Eta' physics[1]

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Abstract

This talk describes the reasons why η and η' decays are an interesting topic of study for both theory and experiment. The main part discusses the results of the recent calculation of $\eta \to 3\pi$ at two-loop order in ChPT. Some puzzling aspects of the results compared to earlier dispersive calculations are highlighted. I also like to remind the reader of the use of η and η' decays for studying the anomaly.

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ETA AND ETA' PHYSICS

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Abstract

This talk describes the reasons why η and η' decays are an interesting topic of study for both theory and experiment. The main part discusses the results of the recent calculation of $\eta \to 3\pi$ at two-loop order in ChPT. Some puzzling aspects of the results compared to earlier dispersive calculations are highlighted. I also like to remind the reader of the use of η and η' decays for studying the anomaly.

1 Introduction

This conference has a lot of talks related to η and η', both on decays, production and in a hadronic medium. The production is treated in a plenary talk by Krusche and decays experimentally in the talk by Wolke. There were also a lot of talks for both production and decay in the parallel sessions. In this talk I will concentrate on decays and in particular mainly on $\eta \to 3\pi$. There are lots of references treating η and η' physics. Many of them can be found in the proceedings of two recent conferences devoted to them [1, 2]. There have also been more recent workshops in Jülich (ETA06) and Peniscola (ETA07).

This talk first discusses why η and η' are interesting, then reminds the reader of some of the aspects of Chiral Perturbation Theory (ChPT) after which the main part, devoted to $\eta \to 3\pi$ comes. I close by pointing out some properties of $\eta' \to \eta\pi\pi, \pi\pi\pi$ decays and the anomaly. Earlier reviews covering similar material are Refs. [3, 4].

2 Why are η and η' Interesting?

The η and η' are particles that decay strongly but all their decays are suppressed. That means that they are good laboratories to study non-dominant strong interaction effects. Weak decays can happen but do occur at branching ratios of order 10^{-11} or lower. So, if charge conjugation violation would be discovered it would be very important. On the other hand, most standard extensions of the standard model do not predict such effects at an observable level in η or η' decays.

But let us first see why pseudo-scalars are special. The QCD Lagrangian is

$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\overline{q}_L \not{D} q_L + i\overline{q}_R \not{D} q_R - m_q \left(\overline{q}_R q_L + \overline{q}_L q_R \right) \right] \tag{1}$$

So if $m_q = 0$ then the left and right handed quarks are decoupled and they can be interchanged freely among themselves leading to a global symmetry $G = U(3)_L \times U(3)_R$. This symmetry is clearly broken in the hadron spectrum, the proton and the S_{11}, as well as the ρ and the a_1 have very different masses[1]. The chiral symmetry group G must thus be spontaneously broken, only the vector part of the group is clearly visible in the spectrum.

As a consequence there must be a set of light particles, the pseudo-Goldstone boson, whose interactions vanish at zero momentum as follows from Goldstone's theorem. There are eight fairly light particles around with the right quantum number, π^0, $\pi^\pm$, $K^\pm$, K^0, $\overline{K^0}$ and η. But the next candidate with the correct quantum numbers, the η', is heavy. We write the group G in terms of simple groups,

$$G = U(3)_L \times U(3)_R = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A\,, \tag{2}$$

and notice that the breaking pattern of $G = SU(3)_L \times SU(3)_R \longrightarrow H = SU(3)_V$ gives eight light particles as observed. The reason is that the $U(1)_A$ part of G is a good symmetry of the classical action but not of the full quantum theory. The divergence of its current has a part coming from the anomaly which couples to gluons via

$$\partial_\mu A^{0\mu} = 2\sqrt{N_f}\omega \quad \text{with} \quad \omega = \frac{1}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\,G_{\mu\nu}G_{\alpha\beta}\,. \tag{3}$$

ω is strongly interacting thus the divergence of the singlet axial-current cannot be treated as zero. So the η' can be heavy as is seen experimentally. Quantum effects break thus the $U(1)_A$, however the r.h.s. of (3) is a total divergence, so how can it have an effect? The answer was found by 't Hooft [7]. Gauge field configurations with non-zero winding number, instantons, can produce an effect. This in turn led to the so-called strong CP problem *but* solved the η' mass problem. A conclusion is thus that the η' has potentially

[1] There is a discussion at present whether chiral symmetry is restored for higher hadron masses. This is not relevant for this talk. Recent references can be traced from [5, 6].

large and very interesting non-perturbative effects and interactions with gluonic degrees of freedom that differ from other hadrons. Since $\hat{m} \neq m_s$ this also affects η physics via mixing.

3 Chiral Perturbation Theory

The chiral symmetry of QCD and its spontaneous breaking has many consequences. The best method to exploit these is Chiral Perturbation Theory (ChPT) which is best defined via

ChPT $\equiv$ "Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques."

A derivation which clearly brings out all the assumptions involved is [8]. Lectures and review articles can be found in my Lattice07 talk [9] or on the webpage [10]. The original modern references are [11, 12].

ChPT uses as power-counting essentially dimensional counting in terms of a generic momentum p. Momenta and meson masses are counted as order p. Because of the Gell-Mann-Oakes-Renner relation, $m_M^2 \propto m_q$, quark masses and external scalar and pseudo-scalar fields are counted as order p^2 and the covariant derivative requires external vector and axial-vector field to be counted as order p. With this counting there is no term of order p^0 in the chiral Lagrangian. The lowest order Lagrangian is given by

$$\mathcal{L}_2 = \frac{F_0^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger\rangle\}\,, \tag{4}$$

with U parameterizing the Goldstone Boson manifold G/H with

$$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)\,, \quad \Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2\,\eta_8}{\sqrt{6}} \end{pmatrix}. \tag{5}$$

The external fields are in the covariant derivative, $D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu$ for the left and right external currents: $r(l)_\mu = v_\mu + (-)a_\mu$ and the external scalar and pseudo-scalar external densities are in $\chi = 2B_0(s + ip)$. Quark masses come via the scalar density $s = \mathcal{M} + \cdots$ and traces are over (quark) flavours $\langle A\rangle = Tr_F(A)$. The number of parameters increases fast at higher orders, there are 10+2 at order p^4 [13] and 90+4 at order p^6 [14] for three-flavour mesonic ChPT.

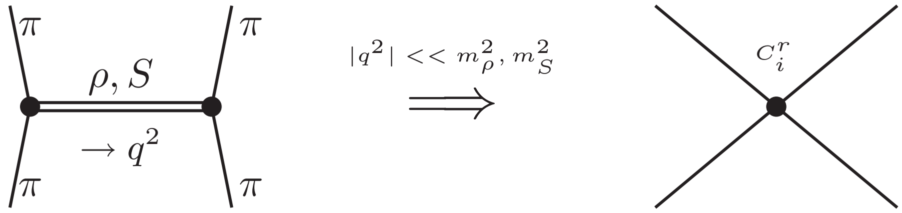


Figure 1: Resonance saturation of the order p^6 low-energy-constants C_i^r via resonate exchange for $\pi\pi$-scattering.

The main uses of ChPT are that it contains all the $SU(3)_V$ relations automatically and in addition relates processes with different numbers of pseudo-scalars and it includes the nonanalytic dependences on masses and kinematical quantities, often referred to as chiral logarithms. As an example, the pion mass in two-flavour ChPT is given by [12]

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2}\log\frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu)\right] + \cdots, \qquad (6)$$

with $M^2 = 2B\hat{m}$ and $B \neq B_0$, $F \neq F_0$ because of two versus three-flavour ChPT. In (6) we see the logarithm and the occurrence of the higher order parameter $\ell_3^r(\mu)$. Eq. (6) also shows some of the choices that need to be made when performing higher order ChPT calculations: Which subtraction scale μ and which quantities should be used to express the results. Lowest order masses or physical meson masses and dito for the decay constants and other kinematical quantities as s, t, u in $\pi\pi$-scattering. There is clearly no unique choice and the choice can influence the *apparent* convergence of the ChPT series quite strongly. Another problem is that typically, not all the higher order parameters that show up in the calculations are known experimentally. Thus one needs to make estimates of these, usually via a version of resonance saturation originally introduced in ChPT in [15]. This is schematically depicted in Fig. 1. More recent references on resonance saturation and possible pitfalls are [16,17]. Discussions on this problem can also be found in the papers on order p^6 ChPT and the review [18].

4 $\eta \to 3\pi$

In the limit of conserved isospin, i.e. we turn off electromagnetism and set $m_u = m_d$, the η is stable. Direct electromagnetic effects have been known to be small since long ago [19, 20]. It should thus proceed mainly through the quark-mass difference $m_u - m_d$. The lowest order was done in [21, 22], order p^4 in [23] and recently the full order p^6 has been evaluated [24]. In this section I will mainly present the new results of [24].

The momenta for the decay $\eta \to \pi^+\pi^-\pi^0$ are labeled as p_η, p_+, p_- and p_0 respectively and we introduce the kinematical Mandelstam variables

$$s = (p_+ + p_-)^2\,, t = (p_+ + p_0)^2\,, u = (p_- + p_0)^2\,.$$

These are linearly dependent, $s+t+u = m_{\pi^o}^2 + m_{\pi^-}^2 + m_{\pi^+}^2 + m_\eta^2 \equiv 3s_0$ .The amplitude is for the charged and neutral decay

$$\begin{aligned}
\langle \pi^0\pi^+\pi^-\mathrm{out}|\eta\rangle &= i\,(2\pi)^4\,\delta^4\left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right)\,A(s,t,u)\,,\\
\langle \pi^0\pi^0\pi^0\mathrm{out}|\eta\rangle &= i\,(2\pi)^4\,\delta^4\left(p_\eta - p_1 - p_2 - p_3\right)\,\overline{A}(s_1,s_2,s_3)\,,\\
\overline{A}(s_1,s_2,s_3) &= A(s_1,s_2,s_3) + A(s_2,s_3,s_1) + A(s_3,s_1,s_2)\,.
\end{aligned} \tag{7}$$

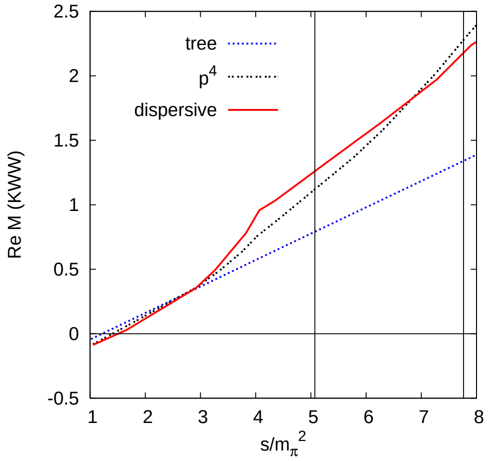
The relation in the last line of (7) is only valid to first order in $m_u - m_d$. The factor of $m_u - m_d$ can be pulled out in various ways. Two common ones are

$$A(s,t,u) = \frac{\sqrt{3}}{4R} M(s,t,u) \quad \text{or} \quad A(s,t,u) = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\,\frac{\mathcal{M}(s,t,u)}{3\sqrt{3}F_\pi^2}\,, \tag{8}$$

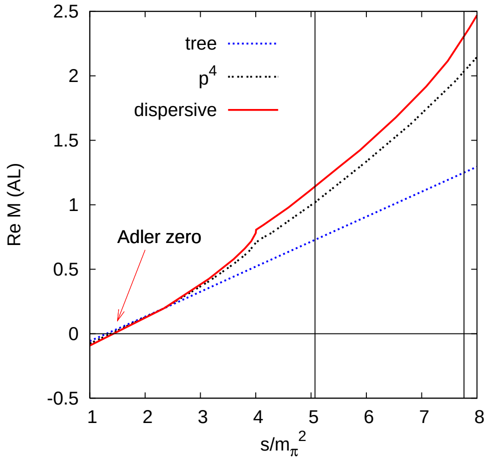
with quark-mass ratios $R = (m_s - \hat{m})/(m_d - m_u)$ and $Q^2 = R(m_s + m_d)/(2\hat{m})$. The lowest order result corresponds to

$$M(s,t,u)_{LO} = \left((4/3)\,m_\pi^2 - s\right)/F_\pi^2\,. \tag{9}$$

The tree level determination of R in terms of meson masses gives with (9) a decay rate of 66 eV which should be compared with the experimental results of 295±17 eV [25]. In principle, since the decay rate is proportional to $1/R^2$ or $1/Q^4$, this should allow for a precise determination of R and Q. However, the change required seems somewhat large. The order p^4 calculation [23] increased the predicted decay rate to 150 eV albeit with a large error. About half of the enhancement in the amplitude came from $\pi\pi$ rescattering and the other half from other effects like the chiral logarithms [23]. The rescattering effects have been studied at higher orders using dispersive methods in [26] and [27]. Both calculations found a similar enhancement in the decay rate bringing it to about 220 eV but differ in the way the Dalitz plot distributions look. This can be seen in Fig. 2 where I show the real part of the amplitude as a function of s along the line $s = u$. The calculations use a very different formalism but make similar approximations, they mainly differ in the way the subtraction constants are determined. That discrepancy and the facts that in



(a)



(b)

Figure 2: (a) Decay amplitude obtained by use of extended Khuri-Treiman equations [26] along the line $s = u$. (b) Alternative dispersive analysis for the decay amplitude [27]. Figs. from [4], adapted from [26, 27].

$K_{\ell 4}$ the dispersive estimate [28] was about half the full ChPT calculation [29] and at order p^4 the dispersive effect was about half of the correction for $\eta \to 3\pi$ makes it clear that also for this process a full order p^6 calculation is desirable. This has been done recently in [24].

Ref. [24] generalizes the methods of [30] to deal with π^0-η mixing to processes with mixing on more than one external leg. The input parameters are from the main order p^6 fit, called fit 10, of [30] and the needed order p^6 constants are determined by resonance exchange as discussed earlier. Details can be found in [24]. In Fig. 3 I show the numerical result for the amplitude along two lines in the Dalitz plot, $t = u$ and $s = u$. The latter can be compared directly with the dispersive result of Fig. 2. The correction found in [24] at order p^6 is 20-30% in amplitude, larger in magnitude than the dispersive estimates [26, 27] but with a shape similar to [27].

The Dalitz plot in $\eta \to 3\pi$ is parameterized in terms of x and y defined in terms of the kinetic energies of the pions T_i and $Q_\eta = m_\eta - 2m_{\pi^+} - m_{\pi^0}$ for the charged decay and z defined in terms of the pion energies E_i. The amplitudes are expanded in x, y, z.

$$
\begin{aligned}
x &= \sqrt{3}\frac{T_+ - T_-}{Q_\eta}\,, \quad y = \frac{3T_0}{Q_\eta} - 1\,, \quad z = \frac{2}{3}\sum_{i=1,3}\left(\frac{3E_i - m\eta}{m_\eta - 3m_{\pi^0}}\right)^2\,,\\
|M(s,t,u)|^2 &= A_0^2\left(1 + ay + by^2 + dx^2 + fy^3 + \cdots\right)\,,\\
|\overline{M}(s,t,u)|^2 &= \overline{A}_0^2\,(1 + 2\alpha_2 + \cdots)\,. \qquad (10)
\end{aligned}
$$

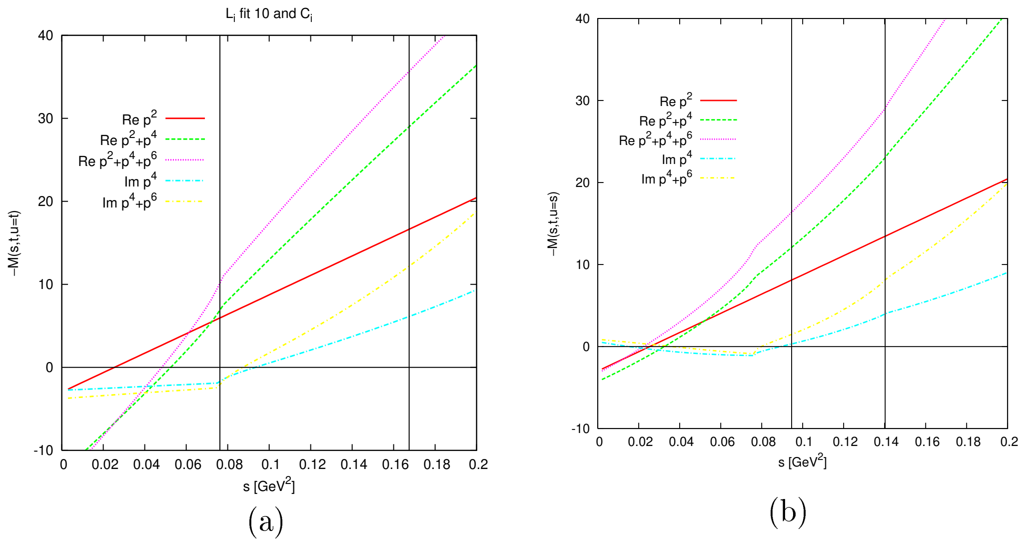


Figure 3: (a) The amplitude $M(s,t,u)$ along the line $t=u$. The vertical lines indicate the physical region. Shown are the real and imaginary parts with all parts summed up to the given order. (b) Similar plot but along the line $s=u$. Figs. from [24].

Table 1: Measurements of the Dalitz plot distributions in $\eta\to\pi^+\pi^-\pi^0$. The KLOE result [31] for f is $f=0.14\pm0.01\pm0.02$.

Exp.	a	b	d
KLOE [31]	$-1.090{\pm}0.005^{+0.008}_{-0.019}$	$0.124{\pm}0.006\pm0.010$	$0.057{\pm}0.006^{+0.007}_{-0.016}$
CB [32]	-1.22 ± 0.07	0.22 ± 0.11	$0.06{\pm}0.04$ (input)
[33]	-1.08 ± 0.014	0.034 ± 0.027	0.046 ± 0.031
[34]	-1.17 ± 0.02	0.21 ± 0.03	0.06 ± 0.04

Recent experimental results for these parameters are shown in Tabs. 1 and 2. There are discrepancies among the experiments but the two latest precision experimental measurements of α agree. The predictions from ChPT to order p^6 with the input parameters fixed as described earlier are give in Tabs. 3 and 4. The predictions from the dispersive analysis as well as [38] have not been included. The different lines corresponds to variations on the input and the order of ChPT. The lines labeled NNLO are the central results. The agreement with experiment is not too good and clearly needs further study. Especially puzzling is the α is consistently positive while the dispersive calculations as well as [38] give a negative value. The inequality $\alpha\le\left(d+b-a^2/4\right)/4$ derived in [24] shows that α has rather large cancellations inherent in its prediction and that the overestimate of b is a likely

Table 2: Measurements of the Dalitz plot distribution in $\eta \to \pi^0\pi^0\pi^0$.

Exp.	α
KLOE [35]	$-0.027 \pm 0.004^{+0.004}_{-0.006}$
Crystal Ball [36]	-0.031 ± 0.004
WASA/CELSIUS [37]	$-0.026 \pm 0.010 \pm 0.010$

Table 3: Theoretical estimate of the Dalitz plot distributions in $\eta \to \pi^+\pi^-\pi^0$.

	A_0^2	a	b	d	f
LO	120	-1.039	0.270	0.000	0.000
NLO	314	-1.371	0.452	0.053	0.027
NLO ($L_i^r = 0$)	235	-1.263	0.407	0.050	0.015
NNLO	538	-1.271	0.394	0.055	0.025
NNLO ($\mu = 0.6$ GeV)	543	-1.300	0.415	0.055	0.024
NNLO ($\mu = 0.9$ GeV)	548	-1.241	0.374	0.054	0.025
NNLO ($C_i^r = 0$)	465	-1.297	0.404	0.058	0.032
NNLO ($L_i^r = C_i^r = 0$)	251	-1.241	0.424	0.050	0.007

cause of the wrong sign obtained for α. In addition, the fairly large correction obtained gives in the end somewhat larger values of Q compared to those derived from the masses [24].

5 Other Remarks

I would simply like to repeat here some remarks made earlier, see e.g. [4]. The hadronic decays of the η' are interesting, they are predicted to be small at lowest order. $\eta' \to 3\pi$ agrees reasonably well with expectations but $\eta' \to \eta\pi\pi$ has very large higher order corrections since the lowest order is suppressed by a factor of m_π^2. I would also like to emphasize once more that the decay

Table 4: Theoretical estimates of the Dalitz plot distribution in $\eta \to \pi^0\pi^0\pi^0$.

	$\overline{A}_0^2$	α
LO	1090	0.000
NLO	2810	0.013
NLO ($L_i^r = 0$)	2100	0.016
NNLO	4790	0.013
NNLO ($C_i^r = 0$)	4140	0.011
NNLO ($L_i^r = C_i^r = 0$)	2220	0.016

of η and η' allow many tests of the triangle, quadrangle, ... anomaly.

Acknowledgments

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