LU TP 09-14
June 2009

$K \to \pi\pi$ Decays in $SU(2)$ Chiral Perturbation Theory

Johan Bijnens and Alejandro Celis

Department of Theoretical Physics, Lund University,
Sölvegatan 14A, SE 223-62 Lund, Sweden

Abstract

We study the decays $K \to \pi\pi$ in one-loop two-flavour Chiral Perturbation Theory. We provide arguments why the calculation of the coefficient of the pionic chiral logarithm $\ell_M = M^2 \log M^2$ is unique and then perform the calculation. As a check we perform the reduction of the known three-flavour result. The underlying arguments are expected to be valid for heavier particles and other processes as well.

Keywords: Kaon Decays, Chiral Perturbation Theory.
PACS:12.39.Fe Chiral Lagrangians, 13.20.Eb Decays of K mesons, 11.30.Rd Chiral symmetries

1 Introduction

Calculating nonleptonic decays precisely from first principles is a longstanding problem. Progress has been made both on the short distance front and on the long-distance front. Lattice QCD provides a way to take care of the latter but is at present limited in the light quark masses that can be reached. A final extrapolation in the light quark masses is still needed. For this extrapolation Chiral Perturbation Theory (ChPT) [1, 2, 3] is used but in the nonleptonic sector it has been found that the one-loop corrections for nonleptonic decays are rather sizable [4, 5, 6]. The same has also been observed for the quenched and partially quenched extensions, see e.g. [7] and references therein.

For static kaon properties like its mass, decay constant and the B_K parameter an alternative is to use two-flavour ChPT with kaons included. This was first used for the mass and πK scattering in [8], see also [9, 10], and later extended to the decay constant and B_K and used for lattice chiral extrapolations [11]. This same method was used for $K_{\ell 3}$ at q^2_{max} where the standard power counting works [12] as well as for general q^2 [12]. In the latter case the standard ChPT power counting schemes do not work because of the presence of a large momentum pion. However the authors of [12] argued that also in this case the coefficient of the chiral logarithm $m_\pi^2 \log(m_\pi^2)$ is calculable.

In this letter we extend the arguments of [12] to the case of $K \to \pi\pi$ decays and calculate the pionic chiral logarithm for these decays. We expect that this type of arguments can be applied to more general processes as well as discussed in Sect. 3. These results are also discussed in the thesis [13].

In Sect. 2 we discuss two-flavour ChPT and include the kaon as a heavy particle [8, 11] and add the nonleptonic weak decay sector to it. Sect. 3 describes the general argument why we expect that also hard pions can be treated using ChPT and give in particular the argument for the case of $K \to \pi\pi$. Sect. 4 presents the results of the one-loop calculations in two-flavour ChPT while in Sect. 5 we check that the three-flavour result contains the same logarithms. In Sect. 6 we summarize our results.

2 Two-flavour ChPT

2.1 Strong and semileptonic Lagrangian

Two-flavour ChPT in the meson sector is given in [2]. We use here the exponential notation for the pion field instead. The notation is the same as in [14]. The lowest order Lagrangian is

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{F^2}{4} \left(\langle u_\mu u^\mu \rangle + \langle \chi_+ \rangle \right) , \tag{1}$$

with

$$\begin{aligned} u_\mu &= i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\} , \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u , \end{aligned}$$

$$
\begin{aligned}
u &= \exp\left(\frac{i}{\sqrt{2}F}\phi\right) , \\
\chi &= 2B(s+ip), \\
\phi &= \left(\begin{array}{cc} \frac{1}{\sqrt{2}}\pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 \end{array}\right) .
\end{aligned} \tag{2}
$$

The field u transforms under a chiral transformation $g_L \times g_R \in SU(2)_L \times SU(2)_R$ as

$$
u \longrightarrow g_R u h^\dagger = h u g_L^\dagger . \tag{3}
$$

h depends on u and g_L, g_R and is the socalled compensator field. Under this transformation $u_\mu \longrightarrow h u_\mu h^\dagger$. The notation $\langle X \rangle$ stands for trace over up and down quark indices and all matrices are 2×2 matrices.

We now introduce a kaon field K that is a doublet under isospin

$$
K = \left(\begin{array}{c} K^+ \\ K^0 \end{array}\right) , \tag{4}
$$

which transforms under a chiral transformation as

$$
K \longrightarrow hK . \tag{5}
$$

We can define a covariant derivative for objects that transform as (5) and for those transforming as $A \longrightarrow hAh^\dagger$ via

$$
\begin{aligned}
\nabla_\mu A &= \partial_\mu A + [\Gamma_\mu, A] , \\
\nabla_\mu K &= \partial_\mu K + \Gamma_\mu K , \\
\Gamma_\mu &= \frac{1}{2}\{u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i l_\mu)u^\dagger\} .
\end{aligned} \tag{6}
$$

The fields s, p $r_\mu = v_\mu + a_\mu$, $l_\mu = v_\mu - a_\mu$ are the standard external scalar, pseudoscalar, left- and right-handed vector fields introduced by Gasser and Leutwyler. The mass term for the light quarks is introduced by setting

$$
s = \left(\begin{array}{cc} m_u & \\ & m_d \end{array}\right) . \tag{7}
$$

In this paper we always work in the isospin limit $m_u = m_d = \hat{m}$.

The effective ChPT Lagrangian contributing to pion-kaon scattering up to second chiral order is given by [8]

$$
\begin{aligned}
\mathcal{L}^{(1)}_{\pi K} &= \nabla_\mu K^\dagger \nabla^\mu K - \overline{M}_K^2 K^\dagger K , \\
\mathcal{L}^{(2)}_{\pi K} &= A_1 \langle u_\mu u^\mu \rangle K^\dagger K + A_2 \langle u^\mu u^\nu \rangle \nabla_\mu K^\dagger \nabla_\nu K + A_3 K^\dagger \chi_+ K + A_4 \langle \chi_+ \rangle K^\dagger K .
\end{aligned} \tag{8}
$$

The chiral order associated with each class of terms corresponds to the chiral order of the leading tree-contributions and is indicated as an upper index (i). In (8) we introduced the notation $\overline{M}_K^2$ for the kaon mass in the limit where $\hat{m} = 0$. Similarly we use the $M^2 = 2B\hat{m}$ for the lowest order pion mass.

The kaon mass up to order $\hat{m}$ has no chiral logarithms [8] and those for the pion mass are well known [2]

$$\begin{aligned} M_\pi^2 &= M^2\left(1 - \frac{1}{2F^2}\overline{A}(M^2) + 2\frac{M^2}{F^2}l_3^r + \cdots\right), \\ M_K^2 &= \overline{M}_K^2 - 2M^2\left(A_3 + 2A_4\right) + \cdots . \end{aligned} \tag{9}$$

Here we introduced the one-loop function

$$\overline{A}(M^2) = -\frac{M}{16\pi^2}\log\left(\frac{M^2}{\mu^2}\right). \tag{10}$$

The decay constant for the pion is treated in the usual way with [2]

$$F_\pi = F\left(1 + \frac{1}{F^2}\overline{A}(M^2) + \frac{M^2}{F^2}l_4^r\right). \tag{11}$$

The kaon decay constant needs the introduction of the weak current

$$\bar{s}_L\gamma_\mu u_L . \tag{12}$$

This can be done by introducing a spurion field $t_{L\mu}$ transforming such that $t_L \longrightarrow g_L t_L^\dagger$ under $SU(2)_L$. The combination $(t_{L\mu}^\dagger)^i\,\bar{s}_L\gamma^\mu q_{Li}$ with $q_1 = u$ and $q_2 = d$ is then chirally invariant. The Lagrangian coupling the kaons is thus given by [11, 12]

$$\mathcal{L}_{Kus} = w_1 t_{L\mu}^\dagger u^\dagger \nabla^\mu K + w_2 t_{L\mu}^\dagger u^\dagger u^\mu K + h.c.. \tag{13}$$

From this one can derive the correction to F_K [11]

$$F_K = \overline{F}_K\left(1 + \frac{3}{8F^2}\overline{A}(M^2) + \cdots\right). \tag{14}$$

$\overline{F}_K$ is the kaon decay constant in the limit $\hat{m} = 0$ and the dots stand for terms of order $\hat{m}$ but no logarithms. The terms in (13) are zeroth and first order in the chiral counting for F_K and $K_{\ell 3}$ at q^2_{max}.

2.2 The nonleptonic Lagrangian

At the quark level the two dominant $\Delta S = -1$ operators are given by

$$(\bar{s}_L\gamma_\mu u_L)(\bar{u}\gamma^\mu d_L) \qquad \text{and} \qquad (\bar{s}_L\gamma_\mu d_L)(\bar{u}\gamma^\mu u_L). \tag{15}$$

We can again makes these terms fully chirally invariant by adding a spurion t_k^{ij} transforming as $t_k^{ij} \longrightarrow t_{k'}^{i'j'} = t_k^{ij}(g_L)_{k'}{}^k(g_L^\dagger)_i{}^{i'}(g_L^\dagger)_j{}^{j'}$. The term

$$t_k^{ij}(\bar{s}\gamma_\mu q_{Li})(\bar{q}_L^k\gamma^\mu q_{Lj}) \tag{16}$$

is then fully chirally invariant. We can actually simplify a little since the operators in (15) transform as a doublet or triplet, $\Delta I = 1/2$ or $3/2$, under $SU(2)_L$. The double combination of the operators can be made invariant by a single spurion $t_{1/2}$ transforming as $t^i_{1/2} \longrightarrow t^{i'}_{1/2} = t^i_{1/2}(g_L^\dagger)_i{}^{i'}$.

The actual operators then correspond to the values $t^1_{1/2} = 0$, $t^2_{1/2} = 1$ for the $\Delta I = 1/2$ and $t^{12}_1 = t^{21}_1 = -t^{22}_2 = 1$, others zero, for the $\Delta I = 3/2$ operator.

In constructing possible terms, we can use the identities $2u_\mu u^\mu = \langle u_\mu u^\mu \rangle$ and $\langle u_\mu \rangle = 0$, as well as the equations of motion. When calculating for our case here, i.e. $\chi = \mathrm{diag}(\hat{m}, \hat{m})$, we have in addition $\langle \chi_- \rangle = 0$ and $\langle \chi_+ \rangle = 2\chi_+$.

We have ordered the terms here by the counting in derivatives and powers of χ, but how they do contribute is discussed in Sect. 3.

The $\Delta I = 1/2$ terms are using the quantity $\tau_{1/2} = t_{1/2} u^\dagger$

$$\begin{aligned} \mathcal{L}_{1/2} \;=\;& iE_1\,\tau_{1/2}K + E_2\,\tau_{1/2}u^\mu \nabla_\mu K + iE_3\langle u_\mu u^\mu\rangle \tau_{1/2}K + iE_4\tau_{1/2}\chi_+ K + iE_5\langle \chi_+\rangle \tau_{1/2}K \\ & + E_6\tau_{1/2}\chi_- K + E_7\langle \chi_-\rangle \tau_{1/2}K + iE_8\langle u_\mu u_\nu\rangle \tau_{1/2}\nabla^\mu\nabla^\nu K + \cdots + h.c.\,. \end{aligned} \tag{17}$$

By using the equations of motion the first term can be traded for $\tau_{1/2}\nabla_\mu\nabla^\mu K$. The terms with zero or two derivatives or one power of χ are a complete set. We have kept one term with four derivatives to show that the arguments presented in Sect. 3 work for that example. The factors of i are chosen such that a real coefficient corresponds to a CP conserving term.

For the $\Delta I = 3/2$ case, we introduce the quantity $\tau^{ij}_k \equiv t^{i'j'}_{k'}(u^\dagger)_{i'}{}^i (u^\dagger)_{j'}{}^j u_{k'}{}^k$ and get the Lagrangian to second order in derivatives or first order in χ

$$\begin{aligned} \mathcal{L}_{3/2} \;=\;& iD_1\tau^{ij}_k (u_\mu)_i{}^k (u^\mu K)_j + D_2\tau^{ij}_k (u_\mu)_i{}^k (\nabla^\mu K)_j + iD_3\tau^{ij}_k (\chi_+)_i{}^k K_j \\ & + D_4\tau^{ij}_k (\chi_-)_i{}^k K_j + \cdots + h.c.\,. \end{aligned} \tag{18}$$

A term like $i\tau^{ij}_k (u_\mu u^\mu)_i{}^k K_j$ never contributes since t^{ij}_k is such that the trace part of the first factor does not contribute. This also means that in the isospin limit the D_3 and D_4 terms never contribute. Here we have not included any terms with more derivatives.

3 An argument why $K \to \pi\pi$ can be treated

A general reason why we expect that there might be some predictions possible also for processes with large momentum pions is that chiral logarithms are caused by small momentum pion propagators. Soft pion couplings are related directly using the soft pion theorem,

$$\lim_{q\to 0}\langle \pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle \alpha|\left[Q_5^k, O\right]|\beta\rangle\,, \tag{19}$$

to matrix elements without the soft pion. The states α and β can also contain large momentum pions. The underlying problem is to find a chirally invariant description of the right side in (19). What we propose here is to use an effective Lagrangian description

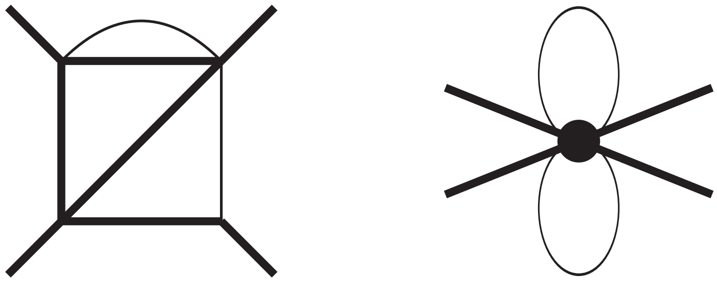

Figure 1: An example of the argument used. The thick lines contain a large momentum, the thin lines a soft momentum. Left, a general Feynman diagram with hard and soft lines. Right: the contracted version where we assume the "vertex" is correctly described by an effective Lagrangian.

which describes $\langle\alpha|O|\beta\rangle$ and *nearby* processes in a chiral invariant way. This Lagrangian could have also imaginary coefficients if that is needed to describe the nearby underlying processes.

For a general loop calculation, the hard part, should be describable by an effective Lagrangian as long as none of the external momenta changes very much. This is illustrated in Fig. 1 and is essentially the analysis of possible infrared divergences as discussed in Sect. 8.3.1 in [15]. Related thoughts can be found in [16] and in the work on asymptotic expansions of loop integrals [17] and in the first study of baryon ChPT [18]. This effective Lagrangian should then provide a sufficiently complete description of the process in the neighbourhood of $\langle\alpha|O|\beta\rangle$. Finding a complete description in the relevant neighbourhood is thus the crux. For the case of $K_{\ell 3}$ decays at a general q^2 this was accomplished in [12] by showing that matrix-elements of higher order operators were related to the matrix-elements of the lowest order operator to the order needed.

Let us generalize their argument now to the case at hand, $K \to 2\pi$ decays. We look at matrix-elements of the type $\langle\pi(p_1)\pi(p_2)|O|K(p_K)\rangle$ where O is any of the operators in $\mathcal{L}_{1/2}$ or with a higher number of derivatives. We show here that these matrix-elements are all proportional to the lowest order one up to terms of order $\hat{m}$ times order one coefficients. We will formulate it in terms of the expansion in powers of M^2. The lowest order is 1, then M^2 (plus logarithms), M^4,.... The combinations of hard momenta are $p_1^2 = p_2^2 = M_\pi^2$, $p_K^2 = M_K^2$ and $p_1.p_K = p_2.p_K = M_K^2/2$. Neither of the masses has a chiral logarithm of the type $\ell_M = M^2 \log(M^2)$. Terms which contain powers of χ will not contribute to the order 1 or ℓ_M but only start at M^2. We thus need to look only at terms with derivatives ∇_μ or u_μ. Lorentz indices always come in pairs. Let us first look at the case where all the derivatives are from ∇_μ. If the derivative hits a soft pion, the underlying soft part of the loop integrals is $\int d^d p\, p_\mu/(p^2 - M^2)$ which contributes no terms of order ℓ_M. So the only parts that can contribute are when the extra derivatives both hit either the kaon or the two hard pions, we will in the below thus always only consider the hard particles. All of these can always be related to the lowest order term. First, if both derivatives hit the same hard particle, it produces their mass which contains no extra ℓ_M. If they hit both pions, we can perform a partial integration where only one derivative hits a pion and the other the kaon plus mass term contributions. But, $K \to \pi\pi$ is symmetric under the interchange of the pions, there must thus be an identical term with the other pion momentum, the pion momenta in this form are thus always $p_1 + p_2$ but that means that that derivative can always be to the kaon as well and turned into a kaon mass. This takes care of all terms with extra powers of $\nabla_\mu \ldots \nabla_\mu$. What happens now with terms with u_μ, where the derivatives must be on the hard pions. The remaining terms are those of the type E_2, E_3 or E_8 in (17). These can all be related to the E_1 term up to order M^2. We use the identity

$$\partial_\mu \left(\tau_{1/2}\tilde{K}\right) = \frac{1}{2}\tau_{1/2}u_\mu\tilde{K} + \tau_{1/2}\nabla_\mu\tilde{K}\,, \tag{20}$$

valid for any $\tilde{K}$ transforming as $\tilde{K} \longrightarrow h\tilde{K}$. The matrix element of a total derivative vanishes since $p_1 + p_2 = p_K$. Using $\tilde{K} = u^\mu K$ and $\tilde{K} = \nabla^\mu K$ we get

$$\begin{aligned} 0 &= \frac{1}{2}\tau_{1/2}u_\mu u^\mu K + \tau_{1/2}\nabla_\mu u^\mu K + \tau_{1/2}u_\mu\nabla^\mu K\,, \\ 0 &= \frac{1}{2}\tau_{1/2}u_\mu\nabla^\mu K + \tau_{1/2}\nabla_\mu\nabla^\mu K\,. \end{aligned} \tag{21}$$

This shows that the E_2 and E_3 terms can be reduced to the E_1 term. The E_8 term can also be removed, perform a partial integration on one of the ∇_μ hitting the Kaon. This produces either a $\nabla_\mu u^\mu$ which is of order M^2 or a $\nabla_\mu u_\nu$. But in the latter case we can use that $\nabla_\mu u_\nu = \nabla_\nu u_\mu + f_{-\mu\nu}$ [14] where the extra term vanishes for zero external fields as is the case for $K \to \pi\pi$. The remainder is then of a form already discussed. We have thus shown that for $K \to \pi\pi$ matrix elements all operators have matrix elements that up to terms of order M^2 are proportional to the lowest order operator.

The same type of arguments goes through for all $\Delta I = 3/2$ operators. We can also show that the terms with D_1 and D_2 in (18) are equivalent in the same way by considering $\partial_\mu\left(\tau^{ij}_k (u^\mu)_i{}^k K_j\right)$.

The above argument does not work for relating $K \to 2\pi$ to $K \to 3\pi$, however the principle can again be applied if one of the pions is soft. We have not checked whether additional operators can already occur at lowest order for this case.

The type of arguments presented above are clearly applicable to many more processes with hard momenta, in particular we expect that they can be applied to matrix-elements needed for B and D decays as well, but again, we have not performed such an analysis.

4 The one-loop calculation for $K \to \pi\pi$

There are three measured decays $K \to \pi\pi$: $K_S \to \pi^0\pi^0$, $K_S \to \pi^+\pi^-$ and $K^+ \to \pi^0\pi^+$ and their charge conjugates. $K_S = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0)$ is the even CP eigenstate and $K_L = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$ is an odd eigenstate. The amplitudes for the three decays can be written in

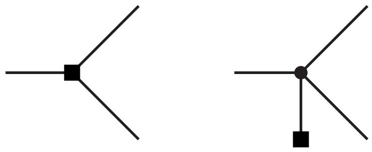

Figure 2: Diagrams contributing to $K \to \pi\pi$ at tree level. A black box indicates a vertex from the weak Lagrangian, (17) or (18), and a black circle represent a vertex from the strong Lagrangian, (1) or (8).

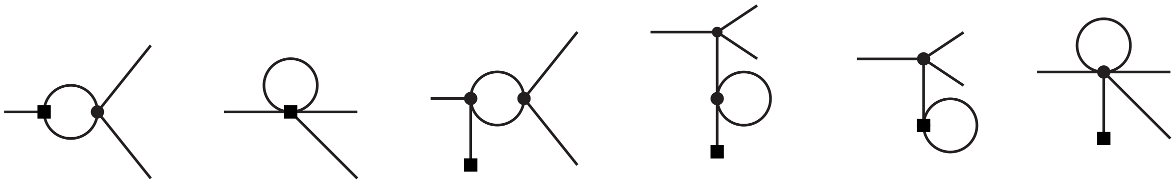

Figure 3: Diagrams contributing to $K \to \pi\pi$ at one loop. Vertices as in Fig. 2.

terms of the $\Delta I = 1/2$ and $3/2$ amplitudes A_0 and A_2.

$$
\begin{aligned}
A[K_S \to \pi^0\pi^0] &= \sqrt{\frac{2}{3}}A_0 - \frac{2}{\sqrt{3}}A_2\,,\\
A[K_S \to \pi^+\pi^-] &= \sqrt{\frac{2}{3}}A_0 + \frac{2}{\sqrt{3}}A_2\,,\\
A[K^+ \to \pi^0\pi^+] &= \frac{\sqrt{3}}{2}A_2\,.
\end{aligned}
\tag{22}
$$

The tree level diagrams are shown in Fig. 2 and lead to

$$
\begin{aligned}
A_0^{LO} &= \frac{\sqrt{3}i}{2F^2}\left[-\frac{1}{2}E_1 + (E_2 - 4E_3)\,\overline{M}^2_K + 2E_8\overline{M}^4_K + A_1E_1\right] + \mathcal{O}(\ell_M)\,,\\
A_2^{LO} &= \sqrt{\frac{3}{2}}\frac{i}{F^2}\left[(-2D_1 + D_2)\,\overline{M}^2_K\right] + \mathcal{O}(\ell_M)\,.
\end{aligned}
\tag{23}
$$

We have kept here redundant terms to check explicitly the arguments of Sect. 3 and have dropped all terms of order M^2. These come with new free coefficients as can be seen from the extra terms in (17) and (18). The term with A_1E_1 is the only part coming from the tadpole diagram of Fig. 2(b).

The one-loop diagrams are shown in Fig. 3 and there are in addition contributions from wave-function renormalization. These diagrams are not shown in Fig. 3. Kaon wave-function renormalization has no terms of order ℓ_M but pion wave-function renormalization contributes to this order.

Diagram	A_0	A_2
Z	$-\frac{2F^2}{3}A_0^{LO}$	$-\frac{2F^2}{3}A_2^{LO}$
(a)	$\sqrt{3}i\left(-\frac{1}{3}E_1+\frac{2}{3}E_2\overline{M}_K^2\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{2}{3}D_2\overline{M}_K^2\right)$
(b)	$\sqrt{3}i\left(-\frac{5}{96}E_1-\left(\frac{7}{48}E_2+\frac{25}{12}E_3\right)\overline{M}_K^2+\frac{25}{24}E_8\overline{M}_K^4\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{61}{12}D_1+\frac{77}{24}D_2\right)\overline{M}_K^2$
(e)	$\sqrt{3}i\frac{3}{16}A_1E_1$	
(f)	$\sqrt{3}i\left(\frac{1}{8}E_1+\frac{1}{3}A_1E_1\right)$	

Table 1: The coefficients of $\overline{A}(M^2)/F^4$ in the contributions to A_0 and A_2 from the different diagrams in Fig. 3. Z denotes the part from wave-function renormalization.

The tadpole diagrams (c-f) do not contribute to A_2, only to A_0, as expected. Diagrams (a) and (c) have $\pi\pi$ and $K\pi$ intermediate states. All diagrams are nonzero but only a few have terms of order ℓ_M. Diagram (d) has no contribution but neither has (c). For diagram (a) only the $\pi\pi$ intermediate state provides a contribution of order ℓ_M. The $K\pi$ intermediate state did contribute for $K_{\ell 3}$. The contributions from the different diagrams are given in Tab. 1. Putting all the diagrams together, we do indeed find a universal coefficient for all the ℓ_M terms:

$$\begin{aligned} A_0^{NLO} &= A_0^{LO}\left(1+\frac{3}{8F^2}\overline{A}(M^2)\right)+\lambda_0 M^2+\mathcal{O}(M^4)\,, \\ A_2^{NLO} &= A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M^2)\right)+\lambda_2 M^2+\mathcal{O}(M^4)\,. \end{aligned} \tag{24}$$

Since we included redundant terms this also provides a check of the arguments given in Sect. 3.

5 Comparison with the three-flavour result

Three flavour ChPT has been used a lot for $K\to\pi\pi$ decays. The isospin conserving calculations were done first in [4] and recalculated in [5] and [6]. The calculations of the logarithmic terms go back even further. By taking the published expressions from [5] and performing the limit $M^2\to 0$ carefully we can compare with our results of two-flavour ChPT. The lowest order result there reads

$$\begin{aligned} A_0^{(3)LO} &= -\frac{i\sqrt{6}CF_0^4}{\overline{F}_K F^2}\left(G_8+\frac{1}{9}G_{27}\right)\overline{M}_K^2\,, \\ A_2^{(3)LO} &= -\frac{i10\sqrt{3}CF_0^4}{9\overline{F}_K F^2}G_{27}\overline{M}_K^2\,, \end{aligned} \tag{25}$$

and can be used to determine the two-flavour LECs in terms of the three-flavour LECs by comparing (23) and (25).

We can now check whether the full three-flavour one-loop result also produces the same ℓ_M terms as were calculated here. To do this one must take into account that the lowest order result in [5] was expressed in terms of F_K and F_π. To compare with (24) we thus need to take into account the ℓ_M terms present in (11) and (14). Doing this we do obtain the same result as in (24) with A_i^{LO} replaced by $A_i^{(3)LO}$. Note that the corrections terms $\lambda_i M^2$ in three-flavour perturbation are also free at NLO there since they contain undetermined LECs.

6 Conclusions

We have argued that it is possible to have a "hard pion" ChPT and provided explicit arguments that in nonleptonic $K \to 2\pi$ the correction of order ℓ_M is calculable. The arguments given in Sect. 3 provide the main basis of this work. We then performed the calculation explicitly in Sect. 4 keeping some of the redundant terms and showed that the arguments also worked out in the explicit calculation. Equation (24) is the main analytical result of this paper. As a final check we performed the matching to the known three-flavour one-loop ChPT result.

Acknowledgements

This work is supported in part by the European Commission RTN network, Contract MRTN-CT-2006-035482 (FLAVIAnet), European Community-Research Infrastructure Integrating Activity "Study of Strongly Interacting Matter" (HadronPhysics2, Grant Agreement n. 227431) and the Swedish Research Council. FORM [19] was used for the calculations.

References

[1] S. Weinberg, Physica A **96** (1979) 327.

[2] J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 465.

[3] J. Gasser and H. Leutwyler, Annals Phys. **158** (1984) 142.

[4] J. Kambor, J. H. Missimer and D. Wyler, Phys. Lett. B **261** (1991) 496.

[5] J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B **521** (1998) 305 [arXiv:hep-ph/9801326].

[6] J. Bijnens, P. Dhonte and F. Persson, Nucl. Phys. B **648** (2003) 317 [arXiv:hep-ph/0205341].

[7] S. Li and N. H. Christ, arXiv:0812.1368 [hep-lat].

[8] A. Roessl, Nucl. Phys. B **555** (1999) 507 [arXiv:hep-ph/9904230].

[9] S. M. Ouellette, arXiv:hep-ph/0101055.

[10] M. Frink, B. Kubis and U. G. Meissner, Eur. Phys. J. C **25** (2002) 259 [arXiv:hep-ph/0203193].

[11] C. Allton *et al.* [RBC-UKQCD Collaboration], Phys. Rev. D **78** (2008) 114509 [arXiv:0804.0473 [hep-lat]].

[12] J. M. Flynn and C. T. Sachrajda [RBC Collaboration and UKQCD Collaboration], Nucl. Phys. B **812** (2009) 64 [arXiv:0809.1229 [hep-ph]].

[13] A. Celis, LU TP 09-12, Master thesis Lund University.

[14] J. Bijnens, G. Colangelo and G. Ecker, JHEP **9902** (1999) 020 [arXiv:hep-ph/9902437].

[15] C. Itzykson and J.-B. Zuber, Quantum Field Theory, McGraw-Hill, New York, 1980.

[16] J. Bijnens, P. Gosdzinsky and P. Talavera, JHEP **9801** (1998) 014 [arXiv:hep-ph/9708232].

[17] M. Beneke and V. A. Smirnov, Nucl. Phys. B **522** (1998) 321 [arXiv:hep-ph/9711391].

[18] J. Gasser, M. E. Sainio and A. Svarc, Nucl. Phys. B **307** (1988) 779.

[19] J. A. Vermaseren, math-ph/0010025.