Monte Carlo event generator validation and tuning for the LHC

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ABSTRACT: We summarise the motivation for, and the status of, the tools developed by CEDAR/MCnet for validating and tuning Monte Carlo event generators for the LHC against data from previous colliders. We then present selected preliminary results from studies of event shapes and hadronisation observables from e⁺e⁻ colliders, and of minimum bias and underlying event observables from the Tevatron, and comment on the approach needed with early LHC data to best exploit the potential for new physics discoveries at the LHC in the next few years.

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1. Introduction

The LHC is designed to discover what lies beyond the TeV scale, so tantalisingly probed by the Tevatron experiments. The most obvious theoretically motivated candidates for discovery are the Higgs boson and supersymmetry, but unitarity and renormalisation arguments mean that it is extremely likely that we will find *something* new.

One thing that we are absolutely certain to observe is the Standard Model! In particular, the LHC will be a probe of QCD machine as it has never been seen before: the proton will be probed in regions of high momentum transfer and low Björken x (requiring a new understanding of parton densities and hence new PDF fits), jets above 1 TeV will be seen, and the behaviour of the pp total cross-section and multiple parton interactions will be measured at values of \sqrt{s} where current data offer little constraint. It is certain that the SM will need to be measured and understood in this new regime before any new physics discovery can be claimed with confidence.

Key to the process of developing new physics analyses is the simulation of both background and signal events. Particularly with the rise of multivariate methods, such as have been discussed in other ACAT parallel sessions, the discrimination between signal and background is often tuned to predictions from Monte Carlo event generators. While published analyses must be virtually independent of such modelling assumptions, the accuracy of the physics description provided by the simulation codes is crucial for efficient exploitation of LHC data. In the first part of this talk, we summarise the current state of efforts to systematically check the validity of MC generator simulations, and to improve their performance

by systematic parameter tuning. In the second part, we will focus on the underlying event as an area of physics whose MC description can be improved before LHC running by use of Tevatron data, and which must be re-tuned to early LHC data when available, in order to make the most of LHC BSM studies in the early years of the collider.

2. Event generator tuning and validation tools

Despite the importance of MC simulation to the development of LHC physics studies, there has until recently been a dearth of coordinated MC validation studies. That is, while plenty of individual LHC physics analyses have considered private plots of a generator's predictions, there has not been a study broad enough to provide side-by-side displays of how different simulation codes perform, both with respect to each other and to data from previous experiments. This may be due to the awkwardness of the task: all generators are run in a different way, with different steering parameters; and ensuring that the event records produced by them can be manipulated to provide data which may be compared to that from existing experiments is a task ill-suited to experimental physicists under pressure to produce plots for one specific process. However, the task is important, as changes in generator parameter choices can profoundly affect their predictions, and a tuning which appears good for one observable may be unphysically awful for another.

The CEDAR project, on which we reported at the previous ACAT [1], was established to provide manpower to address this issue by developing tools for MC validation. Since then, CEDAR has been integrated into the MCnet EU research network, which is ideal for sharing developments between generator developers and LHC experimentalists. The main tools developed by (and now being used by) CEDAR are the validation tool *Rivet*, and the *Professor* tuning system. We have reported on these systems before, so our description will accordingly be brief and focussed on recent developments: anyone to whom these tools are entirely new is advised to check refs. [1–3].

2.1 Rivet

Rivet is an analysis framework for MC generator validation, intended originally to be a modern, C++1 successor to the venerable HZTOOL system. Key design features of Rivet are that the HepMC event record is the only data source (hence providing isolation from the temptation to query generator internals), that CPU-intensive computations of observables from stable particles are automatically cached for each event and shared between analyses, and that the reference data may be automatically exported to flat data files from the HEPDATA archive² [4]. The reference data is also used to define the binnings of MC histograms, automatically ensuring that there is no problem with synchronising arrays of bin edge positions.

Internally, Rivet analyses can be programmed using a very clean C++ API which isolate physicist users from the details of object memory and life-cycle management. Ex-

¹Not entirely an oxymoron... but certainly not a tautology!

²An earlier CEDAR project, described in the previous ACAT proceedings, involved the upgrade of HEPDATA's database system and addition of a Java data model to make such exporting possible.

ternal analyses may be built as shared libraries and loaded at runtime by a simple "plugin" system. A Python interface to the Rivet and HepMC libraries, implemented using the SWIG wrapper generator, is used to provide a very user-friendly command-line interface to Rivet analyses, including analysis metadata querying. At present, there are roughly 40 key analyses from LEP, SLD and Tevatron experiments in Rivet: most development effort is focussed on adding QCD analyses from "missing" colliders such as ISR, SPPS, CLEO, and the b-factories. HERA analyses will primarily remain in HZTOOL, updated with a HepMC input layer and histogram output compatible with Rivet.

The current stable version of Rivet is 1.1.2 (with a 1.1.3 patch release expected soon). The 1.2.0 version will provide much-improved histogramming, after which the analysis infrastructure will be essentially complete and all effort will be on adding more analyses and exploiting Rivet for more advanced generator validation studies.

2.2 Professor

Professor is an extension of the Delphi generator tuning system, developed by a collaboration between MCnet, TU-Dresden, and Berlin Humboldt University. Unlike either the intrinsically sub-optimal "by-eye" tunings commonly delegated to unfortunate graduate students, or brute-force tunings — which rapidly fail to scale to large parameter spaces, even in these days of grid computing — it is based on parameterising the response of MC observable bins to correlated shifts in generator parameters via a polynomial, usually second order. Accordingly, there is an assumption that the generator responds in a sufficiently smooth way to parameter variations, but in practice this proves to be true — at least when the bin variations are combined together to compute some goodness of fit function (GoF), e.g. a heuristic χ^2 , against reference data. The parameterisation is determined by randomly sampling parameter vectors from the parameter hypercube and running the generator at each sampled point via a batch cluster or the LCG grid: a singular value decomposition is used to deterministically implement the "pseudoinverse" which determines each bin's best polynomial coefficients according to a least squares definition. It is usually possible to factorise the 30 or so main interesting parameters of a generator like Pythia into semiindependent groups of 5–10, and the scaling of the minimum number of runs — generally dependent on the polynomial order — gives $N_{\min} \sim \mathcal{O}(100)$. To obtain some estimate of the systematic error introduced by the procedure, we actually sample $N \sim 3N_{\rm min}$ runs and then construct $\mathcal{O}(100)$ different sets of parameterisations by randomly choosing N_{fit} of those runs, where $N_{\rm min} < N_{\rm fit} < N$. Finally, the Minuit numerical minimiser is used to minimise the parameterised GoFs and produce a set of (hopefully consistent) predicted optimal parameter sets. This procedure has the desirable feature of combined performance and tractability: as $\mathcal{O}(10\text{M})$ events may be needed for each parameter point (typically \approx 2–3 CPU days), minimising an analytic function created from hundreds of such runs in parallel is vastly preferable to running thousands of serial Minuit runs.

Several things about this procedure are worth noting: first, the choice of parameter ranges is the responsibility of the user and must be based on an understanding of the generator. Too wide choices will be insufficiently sensitive to the interesting region; too

narrow and the predicted minimum may be outside the sampled ranges³. Second, the choice of GoF function is flexible, most significantly in that typically one will consider certain distributions to be more significant than others and give them an extra *weight* accordingly. The choice of numeric weights to maintain a balanced minimisation is something of an art form, illustrating the ever-useful rule that when one demands optimal solutions, he should be careful about exactly what he asks for.

An extra technical point, which usually leads to questions from MC authors accustomed to integrating awkward functions, is the random sampling of the parameter hypercube. So far we have always sampled uniformly on the parameter space, without evident bias. For particularly non-linear parameters, non-linear sampling or parameter transformations could be used: in essence we want to sample according to the "prior reasonableness" of the parameters, and in the absence of other information a flat prior is the natural choice (and not a dangerous one since the ranges are bounded). However, this is another area where knowledge of the generator is useful: tunings are most definitely best done in collaboration with the generator authors.

Finally, note that there is nothing MC-specific about this method: it is a general method for minimising very expensive functions where there is no *a priori* estimate for the functional form. Accordingly, within HEP the Professor approach has been adapted to fitting the top quark mass and to choosing the parameters of unintegrated PDFs for the CCFM shower formalism [5].

3. Validation and tuning of MC simulations to LEP and Tevatron data

Having summarised the Professor method for Monte Carlo tunings, we will now discuss the tuning of Pythia 6 parameters to e^+e^- and Tevatron data, constraining the parameters of the initial and final state parton showers, hadronisation, and multiple parton interactions (MPI) model. We will only consider the tune to Pythia's traditional virtuality-ordered parton shower and "old" MPI model — the full tune details of the newer p_\perp -ordered shower and more complex interleaved ISR/MPI model will be discussed in a forthcoming publication.

The number of major steering parameters in Pythia 6 is approximately 30, with the majority being associated with aspects of hadronisation. While practically achievable, albeit not easily, we would rather not have to tune in 30 dimensions since the likelihood of undersampling or of the minimiser failing to find the global minimum is relatively high. Fortunately, the parameters can be roughly factorised into sets of less than 10 which can be tuned almost independently: for example, the flavour composition of the final state particles is irrelevant when calculating event shape observables and hence the kinematic/flavour parameters can be treated independently. Similarly the treatment of tensor mesons and relative production of different diquark spin states.

3.1 Tuning of FSR and hadronisation to e⁺e⁻ data

Observing this approximate independence of certain parameter groups, we begin our tuning

³At least in this case, the course is clear: extend the range and run the generator for another weekend.

	default	new tune					
Kinematic parameters							
MSTJ(11)	4	5	Frag fn.				
PARJ(21)	0.36	0.325	σ_q				
PARJ(41)	0.3	0.5	Lund a				
PARJ(42)	0.58	0.6	Lund b				
PARJ(47)	1.0	0.67	r_b				
PARJ(81)	0.29	0.29	$\Lambda_{ m QCD}$				
PARJ(82)	1.0	1.65	PS cut-off				
Flavour parameters							
PARJ(1)	0.1	0.073	di-quark suppression				
PARJ(2)	0.3	0.2	strange suppression				
PARJ(3)	0.4	0.94	strange di-quark suppression				
PARJ(4)	0.05	0.032	spin-1 di-quark suppression				
PARJ(11)	0.5	0.31	spin-1 light meson				
PARJ(12)	0.6	0.4	spin-1 strange meson				
PARJ(13)	0.75	0.54	spin-1 heavy meson				
PARJ(25)	1.0	0.63	η suppression				
PARJ(26)	0.4	0.12	η' suppression				

Table 1: Parameters used in the tune of Pythia 6 to LEP/SLD data, and their resulting values.

of Pythia 6 with tuning of final state radiation (FSR) and hadronisation effects at the LEP and SLD e⁺e⁻ colliders. The relevant parameters are shown in Table 1, factorised into "kinematic" and "flavour" sets. The key sets of distributions to fit are the total charged multiplicity at LEP by the OPAL collaboration [6], identified particle multiplicities from the Particle Data Group [7], event shape variables from ALEPH [8] and DELPHI [9], and b-fragmentation from DELPHI [10]. All are influenced by shower and hadronisation kinematics, but only the second is sensitive to the flavour parameters. However, if we only consider ratios of identified particle rates, they are roughly independent of kinematic parameters. Hence our e⁺e⁻ tuning is implemented in 2 stages, with the second stage fixing the parameters determined in the first:

- 1. Tune flavour parameters to the identified particle rates, relative to the π^{\pm} rate;
- 2. Tune kinematic parameters to event shapes and total charged multiplicity.

Each stage of the tune was performed using 300 random runs of 200k events in e^+e^- configuration at the LEP 1 CoM energy of 91.2 GeV. The resulting tune, which has been checked for robustness against many choices of runs and observable weights, are listed in Table 1. The mean charged multiplicity, being a single number of disproportionate importance, was given an effective weight of 220 (spread between several measurements), while most other distributions were given weights less than 10 — most were set to 1.

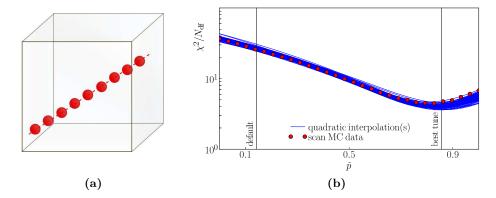


Figure 1: Pythia 6 (Q^2 shower) $\chi^2/N_{\rm df}$ variation along a line in the parameter hyperspace, as illustrated in (a). The line shown in (b) runs between the default and Professor tunes for the flavour parameter tuning. The red dots are the true generator χ^2 values, and the blue lines an ensemble of parameterisations from the Professor procedure. The Professor result is clearly superior, although it does not match the true optimum exactly.

This boosted weight helps to compensate for the down-weighting that is implicit in any distributions with a fewer than typical number of bins, and which is hence most significant for single-bin observables. The robustness of the approach is illustrated in Figure 1, which illustrates the GoF measure as predicted by Professor and as realised by the generator along a line scan in flavour parameter space between the default and Professor tunes: the Professor result is clearly near-optimal, while the default appears relatively arbitrary.

3.2 Tuning of ISR and MPI to $p\overline{p}$ data

The final state parameters derived in the tunes to e^+e^- data can now be used as a base around which to tune the parameters controlling initial state effects in hadron collisions. For this we use Tevatron data, primarily from the CDF experiment: the CDF measurement of the Z p_{\perp} spectrum [11], the DØ measurement of dijet azimuthal angle decorrelation [12], and CDF measurements of the "underlying event" (MPI) from both Run I and Run II [13–17]. In general, such tunes are sensitive to the choice of parton density set used: in this tune we use the Pythia 6 default, the leading order CTEQ5L fit [18]. We are currently repeating this tuning with PDF sets useful for LHC experiment production simulations in 2009, and for the new Monte Carlo specific "modified LO" sets such as the LO* and LO** variations on the MSTW LO PDF sets [19].

This time, no factorisation of the parameters listed in Table 2 is required. The Z p_{\perp} spectrum is sensitive particularly to the ISR and intrinsic p_{\perp} parameters, since these are the only way that transverse momentum can be given to the Z boson. If this is ignored, the fit to MPI observables tends to destroy the Z p_{\perp} description: this was the motivation for the evolution of the Tevatron "Tune A" to "Tune AW". Similarly, the ISR/intrinsic p_{\perp} contributions must be constrained to the DØ dijet angular decorrelation — a measure of how the initial state effects disturb the back-to-back picture of dijet events — which was the motivation for the Tevatron "Tune DW". We incorporate these into our Tevatron tune with weights of 40 and 2 respectively: the latter weight is not set particularly high because

-	default	tune DW	new tune	
PARP(62)	1.0	1.25	2.97	ISR cut-off
PARP(64)	1.0	0.2	0.12	ISR scale factor for α_s
PARP(67)	4.0	2.5	2.74	max. virtuality
PARP(82)	2.0	1.9	2.1	p_{\perp}^0
PARP(83)	0.5	0.5	0.84	matter distribution
PARP(84)	0.4	0.4	0.5	matter distribution
PARP(85)	0.9	1.0	0.82	colour connection
PARP(86)	0.95	1.0	0.91	colour connection
PARP(90)	0.16	0.25	0.17	p_{\perp}^{0} energy evolution
PARP(91)	2.0	2.1	2.0	intrinsic k_{\perp}
PARP(93)	5.0	15.0	5.0	intrinsic k_{\perp} cut-off

Table 2: Parameters used in the tune of Pythia 6 initial state physics to Tevatron data, and their resulting values.

no setting of Pythia 6 seems to describe this data *very* well, hence it acts as a veto on bad tunes rather than a driver of very good ones.

The run conditions for this data are more complex than for the e⁺e⁻ case, since several energies and process types (QCD and Drell-Yan) are involved, and because to efficiently fill profile histograms binned in leading jet or Z p_{\perp} requires several runs with kinematic ME cuts to be combined. The required statistics are also larger, varying between 1–2M events per run, as compared to $\mathcal{O}(100\text{k})$ events for the e⁺e⁻ tune. The resulting parameter set, again checked for robustness, is listed in Table 2.

It should be noted that the constraint of the $p\overline{p}$ cross-section energy evolution, which is a crucial number for LHC physics, is weakly constrained by this tune, since the only contributing energies are 1800 and 1960 GeV; this is remedied by the tunes in our forth-coming publication, which include data from 630 GeV, and we hope also to include data from 200 GeV pp runs at RHIC and earlier hadron colliders.

4. Comparisons of generators/tunes

The important thing about a generator tune, of course, is whether or not it describes the data. This is not necessarily guaranteed, even with a procedure like Professor: there is a subjectivity in the choice of observables and the weights they are afforded, and — more fundamentally — there is no guarantee that the generator/model is capable of describing the data well at several energies or even at one energy. Fortunately, Pythia 6 proves itself up to the challenge in this case. In this section, we shall show a small selection of the distributions to which Pythia 6 has been tuned, comparing to other tunes of the same generator, and finally compare the quality of the data description offered by the Professor tune of Pythia 6 to that from other shower/hadronisation codes in various states of tunelessness.

4.1 Comparisons of Pythia 6 tunes

In Figures 2 and 3, the improvements of the Professor tune with respect to the default tune are shown for selected e^+e^- and $p\bar{p}$ observables. The improvements in the e^+e^- case are small — unsurprising, since the default Pythia 6 tune is based on the original Delphi version of the Professor procedure — but significant, especially in the case of the b-quark fragmentation function. The Tevatron observables see improvements over the "AW" tune in several areas, particularly the minimum bias and Drell-Yan $\langle p_{\perp} \rangle$ vs. $N_{\rm charged}$ distributions, and are a major improvement over the default tune (not shown).

The traces shown in the figures also include two tunes of the newer Pythia p_{\perp} -ordered shower and new MPI model, which are of interest because a) they demonstrate that the newer MPI model is capable of describing the bump at ~ 20 GeV in the CDF 2008 leading jets analysis and b) the Atlas tune is badly wrong in many areas, especially the description of all Drell-Yan UE data. The problems of the Atlas tune, and the capabilities of the new MPI model will be addressed in the forthcoming publication of the Professor tunes of both Pythia 6 shower/MPI models.

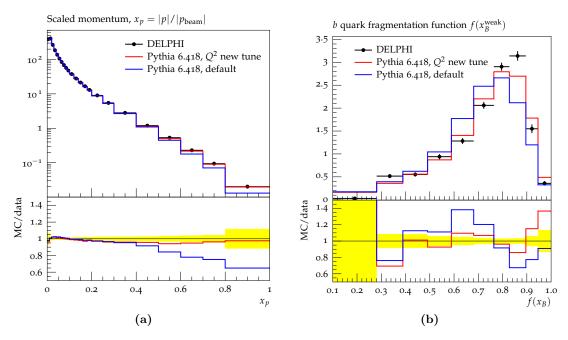


Figure 2: Pythia 6 (Q^2 shower) comparative performance on LEP scaled momentum and b-quark fragmentation function data.

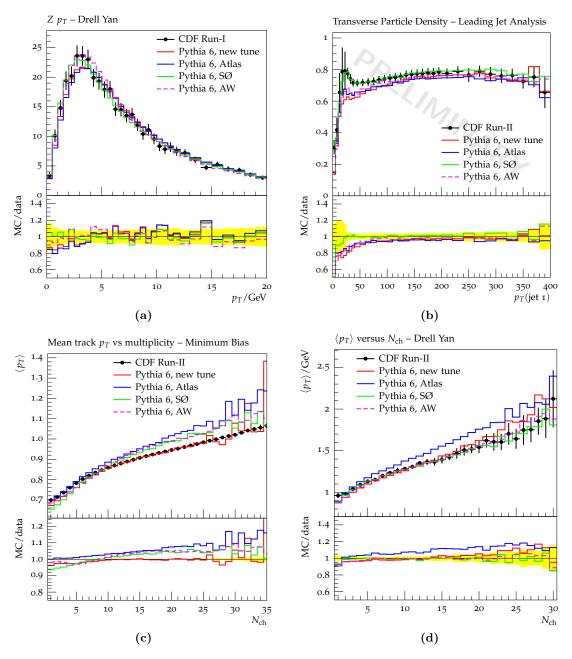


Figure 3: Pythia 6 (Q^2 shower, "old" MPI) comparative performance on Tevatron minimum bias and underlying event data at 1800 and 1960 GeV.

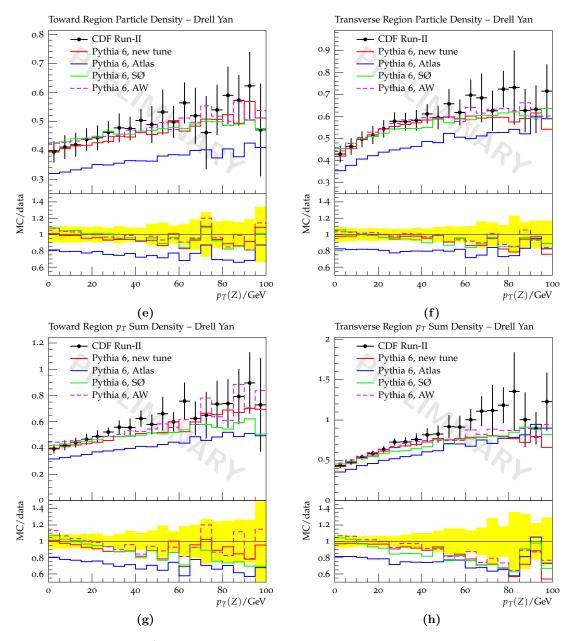


Figure 3: Pythia 6 (Q^2 shower, "old" MPI) comparative performance on Tevatron minimum bias and underlying event data at 1800 and 1960 GeV.

4.2 Comparisons of various generator descriptions of tune observables

For interest, we also show comparisons of the default tunes of several generators on the same set of analysis distributions. This is of interest in that it highlights the degree of variation possible between very similar models and tunes: it is clear that for LHC purposes the tuning of many generators other than Pythia 6 has a lot of room for improvement. All generators shown in the plots of Figure 4, with the exception of the Pythia 6 "AW" tune, are incapable of describing minimum bias data, and hence are cut off below a leading jet p_{\perp} of 30 GeV for the QCD distributions.

Fortran Herwig [20] (plus the Jimmy [21] UE model) has been shown both with the Jimmy default and the Atlas tune, but obviously suffers from not having been tuned as extensively by LEP and Tevatron experiments as Pythia 6. This should be of concern to Atlas, who are using this generator for SUSY simulation due to its incorporation of spin correlations in SUSY decays.

Herwig++ [22] has been brute-force tuned to the CDF 2001 jets UE analysis, and hence its fit is rather good for most distributions. Interestingly, the CDF 2008 Drell-Yan UE study, which was not available for the brute-force tuning, shows a deficiency in the UE model tune: this data appears to break a degeneracy between several ways of obtaining good fits to initial state data, and will be addressed by a Professor tune of Herwig++. Similarly Sherpa [23], which has only been tuned "by eye" so far, will undergo a Professor tune in the near future.

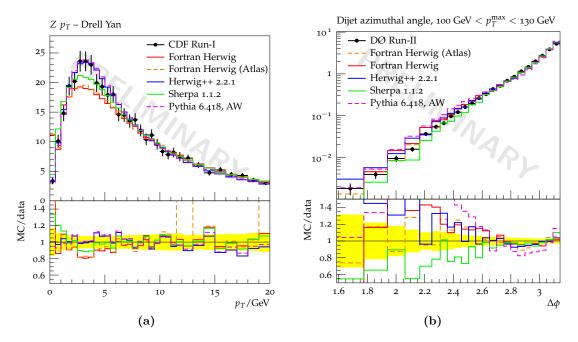


Figure 4: Comparisons of several LO MC event generators on Tevatron minimum bias and underlying event analyses at 1800 and 1960 GeV.

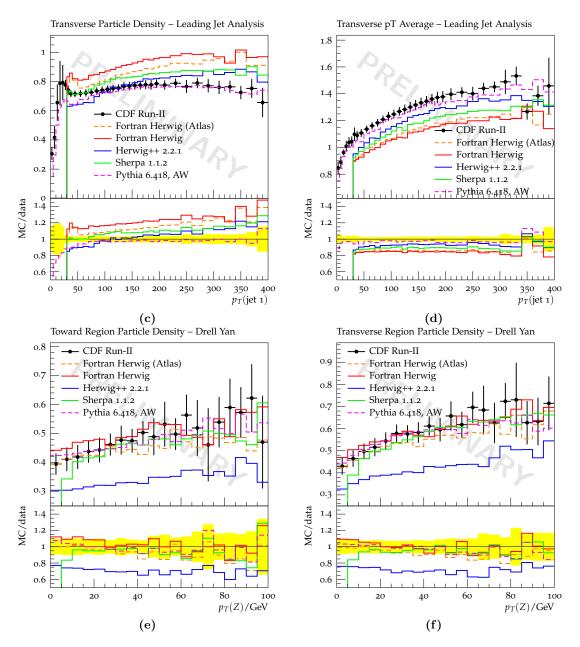


Figure 4: Comparisons of several LO MC event generators on Tevatron minimum bias and underlying event analyses at 1800 and 1960 GeV.

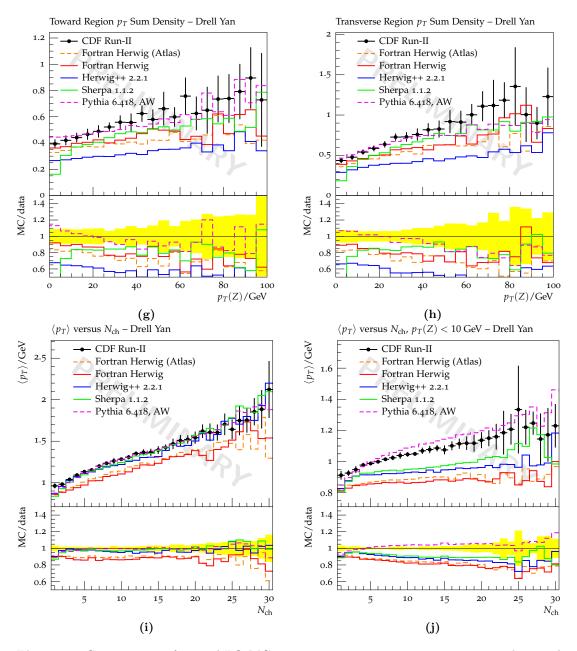


Figure 4: Comparisons of several LO MC event generators on Tevatron minimum bias and underlying event analyses at 1800 and 1960 GeV.

5. Conclusions

We have summarised the work on Monte Carlo event generator validation and tuning that has been done in the past year by the CEDAR collaboration, now integrated in the MCnet research network. The most visible aspects of this development work are the Rivet validation analysis system and the Professor tuning system. The latter is an extension of earlier systematic tuning work, and uses fitted bin-by-bin parameterisations of data response functions to predict the overall goodness of fit to experimental data, from which point a numerical optimisation is tractable.

Using the Professor system to tune data produced by Rivet from events generated by the Pythia 6 generator code (in the Q^2 -ordered parton shower mode), we have obtained a tune of Pythia 6 which combines a good description of data from LEP to the Tevatron. This tune has been extended to the Pythia 6 p_{\perp} -ordered parton shower/interleaved MPI model and will be documented in a forthcoming publication. Extensions of the Rivet library to data from the RHIC, SPPS and ISR colliders will allow for more extensive tunes, in particular constraining the evolution of the total pp/p \bar{p} QCD cross-section, an important feature of minimum bias and underlying event modelling for the LHC experiments.

The MC generator and SM groups on Atlas and CMS are currently beginning to use Rivet for MC validation and Professor tunes will be provided for the main generators—Pythia 6, Herwig 6/Jimmy, Sherpa, Herwig++ and Pythia 8— to be used as base configurations by both collaborations. This work is just beginning, and the details depend on the choice of PDFs for experiment LO generator production.

The phenomenological nature of low- p_{\perp} QCD modelling means that even the best fits to UE energy extrapolation can be disrupted by new data at LHC energies: the effect of Drell-Yan UE data on the otherwise good fits of the Herwig++ and Atlas Pythia 6 tunes stands as testament to the ability of new data to surprise. Accordingly, we intend for Professor to be available for private use within the LHC collaborations, to allow rapid response of generator tunes to first QCD data. This updating is necessary for good understanding of LHC QCD backgrounds to BSM physics, and is hence in every experimentalist's interest. The next year will see many physics surprises: our hope is that with systematic tools to evaluate and improve their behaviours, the MCnet Monte Carlo event generators will prove up to the task.

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