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Determination of Low Energy Constants and testing Chiral Perturbation Theory at order p^6 (NNLO)

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We present the results of a search for relations between observables that are independent of the Chiral Perturbation Theory (ChPT) Next-to-Next-to-Leading Order (NNLO) Low-Energy Constants (LECs). We have found some relations between observables in $\pi\pi$, πK scattering and K_{l4} decay which have been evaluated numerically using fit 10 in [1] for the NLO LECs. We also show some preliminary results for a new global fit of the NLO LECs

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1. Introduction

Testing the validity of Chiral Perturbation Theory (ChPT) is a challenging task because of the many unknown parameters, called the Low Energy Constants (LECs), entering into the theory. In particular, at NNLO 90 unknown constants, the C_i, appear in the p^6 Lagrangian.

One way to overcome this problem is to study different combinations of observables that depend on the C_i in the same way. These lead to C_i-independent relations which can be used to perform the test. Furthermore those combinations might be useful to gain information on the LECs too, since they let us isolate the same combinations of C_i using different observables.

In [2] we study 76 observables at NNLO and find 36 such relations. We compare ChPT NNLO predictions with data/dispersive results for 13 of these. The observables involved are the ones in $\pi\pi$ and πK-scattering and in $K_{\ell 4}$ decay. Here we first discuss how we perform the numerical analysis, the results of which appear in Tab.1, 2, 3 and 4, then we present for each process the relations studied. Finally we show some preliminary results for a new global fit of the L_i at NNLO.

2. Numerical Analysis

The numerical analysis of the C_i-independent relations has been done in the following way. First we evaluate the combinations of observables appearing in each side of the relations using experiment/dispersive (exp) results of [3] for $\pi\pi$ scattering, [4] for πK scattering and [5, 6] for $K_{\ell 4}$ decay. Then we use ChPT results up to order p^6 [7, 8, 9] setting the L_i to the values of fit 10 in [1]. Finally we subtract from the first (exp) evaluation the ChPT one. These differences will contain the C_i part and higher order corrections. They have been quoted in Tab.1, 2, 3 and 4 in the columns labeled remainder. This has been done for each side of the relations under study. To check whether a relation is well satisfied we compare the remainders of its left-hand-side (LHS) and right-hand-side (RHS). Since they contain the same C_i combinations, they should be equal within the uncertainties.

The errors quoted in the second columns of Tab.1, 2, 3 and 4 are obtained adding in quadrature the uncertainties in [3, 4, 5, 6]. This might result in an underestimate of the total error because of correlations. The theoretical errors due to the NLO LECs are shown in brackets in the columns of Tab.1, 2, 3 and 4 labeled NNLO 1-loop. They are obtained by varying all the L_i around the central values of fit 10 according to the full covariance matrix as obtained by the authors of [1] and exploring the region with $\chi^2/\mathrm{dof} \approx 1$. The error is then estimated as the maximum deviation observed. The error for the L_i contribution at NLO is never shown since it drops out of all the relations. No uncertainties due to higher order contributions have been added. The uncertainties due to theoretical errors are mostly on the last quoted digit.

3. $\pi\pi$ scattering

The $\pi\pi$ scattering amplitude can be written as a function $A(s,t,u)$ which is symmetric in t, u:

$$A(\pi^a\pi^b \to \pi^c\pi^d) = \delta^{a,b}\delta^{c,d}A(s,t,u) + \delta^{c,d}\delta^{b,d}A(t,u,s) + \delta^{a,d}\delta^{b,c}A(u,t,s)\,, \tag{3.1}$$

where s,t,u are the usual Mandelstam variables. The isospin amplitudes $T^I(s,t)$ $(I=0,1,2)$ are $T^0(s,t) = 3A(s,t,u)+A(t,u,s)+A(u,s,t)$, $T^1(s,t) = A(s,t,u)-A(u,s,t)$, $T^2(s,t) = A(t,u,s)+A(u,s,t)$, and are expanded in partial waves

$$T^I(s,t) \;=\; 32\pi \sum_{\ell=0}^{+\infty}(2\ell+1)P_\ell(\cos\theta)t_\ell^I(s), \tag{3.2}$$

where t and u have been written as $t=-\frac{1}{2}(s-4m_\pi^2)(1-\cos\theta)$, $u=-\frac{1}{2}(s-4m_\pi^2)(1+\cos\theta)$. Near threshold the t_ℓ^I are further expanded in terms of the threshold parameters

$$t_\ell^I(s) = q^{2\ell}(a_\ell^I + b_\ell^I q^2 + \mathscr{O}(q^4)), \qquad q^2 = \frac{1}{4}(s-4m_\pi^2), \tag{3.3}$$

where $a_\ell^I, b_\ell^I \dots$ are the scattering lengths, slopes,.... We studied the 11 parameters where a dependence on the C_i shows up. Using $s+t+u=4m_\pi^2$ we can write the amplitude to order p^6 as

$$A(s,t,u) = b_1 + b_2 s + b_3 s^2 + b_4(t-u)^2 + b_5 s^3 + b_6 s(t-u)^2 + \text{non polynomial part} \tag{3.4}$$

The tree level Feynman diagrams give polynomial contributions to $A(s,t,u)$ which must be expressible in terms of $b_1,\dots,b_6$. Therefore we expect and find 5 relations:

$$\left[5b_0^2 - 2b_0^0 - 27a_1^1 - 15a_0^2 + 6a_0^0\right]_{C_i} \;=\; -18\left[b_1^1\right]_{C_i}, \tag{3.5}$$

$$\left[3a_1^1 + b_0^2\right]_{C_i} \;=\; 20\left[b_2^2 - b_2^0 - a_2^2 + a_2^0\right]_{C_i}, \tag{3.6}$$

$$\left[b_0^0 + 5b_0^2 + 9a_1^1\right]_{C_i} \;=\; 90\left[a_2^0 - b_2^0\right]_{C_i}, \tag{3.7}$$

$$\left[3b_1^1 + 25a_2^2\right]_{C_i} \;=\; 10\left[a_2^0\right]_{C_i}, \tag{3.8}$$

$$\left[-5b_2^2 + 2b_2^0\right]_{C_i} \;=\; 21\left[a_3^1\right]_{C_i}, \tag{3.9}$$

where $[A]_{C_i} \equiv C_i^r$-dependent part of A. All quantities are expressed in units of $m_{\pi^+}^2$. In fact, since these relations hold for every contribution to the polynomial part, they are valid for the NLO tree level contribution as well and for two- and three-flavour ChPT. Thus they get L_i-contributions only at NNLO via the non polynomial part of Eq. (3.4).

In Tab. 1 we show our numerical results. We quote the left-hand-side (LHS) and right-hand-side (RHS) of each of the relations. In the second column we use the values of the threshold parameters of [3]. The next columns use the ChPT results of [7] and give the contributions from pure one-loop at NLO, the tree level NLO contribution, the pure two-loop contribution, and the L_i dependent part at NNLO (called NNLO 1-loop).

Comparing the remainders of the LHS with the RHS ones, we see that the first three relations are very well satisfied, while the last two work at a level around two sigma.

We can also check how the two-flavour predictions hold up. Since here the corrections are in powers of m_π^2 rather than in powers of m_K^2, the expansion should converge better. For the ChPT evaluation we use the threshold parameters as quoted in [3] for their best fit of the NLO LECs. The result is shown in Tab. 2. We see the same pattern as for the three-flavour case: the first three relations are very well satisfied while the last two are somewhat worse but below two sigma.

	[3]	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	remainder
LHS (3.5)	0.009 ± 0.039	0.054	−0.044	−0.041	−0.002(3)	0.041 ± 0.039
RHS (3.5)	-0.102 ± 0.002	−0.009	−0.044	−0.060	−0.008(6)	0.018 ± 0.002
10 LHS (3.6)	0.334 ± 0.019	0.209	0.097	0.103	0.029(11)	-0.105 ± 0.019
10 RHS (3.6)	0.322 ± 0.008	0.177	0.097	0.120	0.034(13)	-0.107 ± 0.008
LHS (3.7)	0.216 ± 0.010	0.166	0.029	0.053	0.016(6)	-0.047 ± 0.010
RHS (3.7)	0.189 ± 0.003	0.145	0.029	0.049	0.020(7)	-0.054 ± 0.003
10 LHS (3.8)	0.213 ± 0.005	0.137	0.032	0.053	0.035(12)	-0.043 ± 0.005
10 RHS (3.8)	0.175 ± 0.003	0.121	0.032	0.050	0.029(10)	-0.057 ± 0.003
10^3 LHS (3.9)	0.92 ± 0.07	0.36	0.00	0.56	−0.01(13)	0.00 ± 0.07
10^3 RHS (3.9)	1.18 ± 0.04	0.42	0.00	0.57	0.03(13)	0.15 ± 0.04

Table 1: The relations found in the $\pi\pi$-scattering. The lowest order contribution is always zero by construction. The NLO LEC part satisfies the relation, as it should. Notice the extra factors of ten for some of them. All quantities are in the units of powers of m_{π^+}.

	[3]	two-flavour [3]	remainder
LHS (3.5)	0.009 ± 0.039	−0.003	0.007 ± 0.039
RHS (3.5)	-0.102 ± 0.002	−0.097	-0.005 ± 0.002
10 LHS (3.6)	0.334 ± 0.019	0.332	0.002 ± 0.019
10 RHS (3.6)	0.322 ± 0.008	0.318	0.004 ± 0.075
LHS (3.7)	0.216 ± 0.010	0.206	0.010 ± 0.010
RHS (3.7)	0.189 ± 0.003	0.189	0.000 ± 0.003
10 LHS (3.8)	0.213 ± 0.005	0.204	0.009 ± 0.005
10 RHS (3.8)	0.175 ± 0.003	0.176	-0.001 ± 0.003
10^3 LHS (3.9)	0.92 ± 0.07	1.00	-0.08 ± 0.07
10^3 RHS (3.9)	1.18 ± 0.04	1.15	0.04 ± 0.04

Table 2: The relations found in the $\pi\pi$-scattering evaluated in two-flavour ChPT. In the second column we have used the NNLO results quoted in [3]. Notice the extra factors of ten for some of them. All quantities are in units of powers of m_{π^+}.

4. πK scattering

The πK scattering has amplitudes $T^I(s,t,u)$ in the isospin channels $I = 1/2, 3/2$. As for $\pi\pi$ scattering we introduce the partial wave expansion of the isospin amplitudes

$$T^I(s,t,u) \;=\; 16\pi \sum_{\ell=0}^{+\infty}(2\ell+1)P_\ell(\cos\theta)t^I_\ell(s), \tag{4.1}$$

and we define scattering lengths a^I_ℓ, b^I_ℓ by expanding the $t^I_\ell(s)$ near threshold:

$$t^I_\ell(s) \;=\; \frac{1}{2}\sqrt{s}q^{2\ell}_{\pi K}\left(a^I_\ell + b^I_\ell q^2_{\pi K} + \mathcal{O}(q^4_{\pi K})\right), \qquad q^2_{\pi K} = \frac{s}{4}\left(1-\frac{(m_K+m_\pi)^2}{s}\right)\left(1-\frac{(m_K-m_\pi)^2}{s}\right),$$

and $t = -2q_{\pi K}^2(1-\cos\theta)$, $\quad u = -s-t+2m_K^2+2m_\pi^2$. Again we studied only those observables where a dependence on the C_i shows up.

It is also customary to introduce the crossing symmetric and antisymmetric amplitudes $T^{\pm}(s,t,u)$

$$3T^+(s,t,u) \;=\; T^{1/2}(s,t,u)+T^{3/2}(s,t,u), \qquad T^-(s,t,u) = T^{1/2}(s,t,u)-T^{3/2}(s,t,u), \tag{4.2}$$

which can be expanded around $t=0$, $s=u$ using $\nu = (s-u)/(4m_K)$ (subthreshold expansion):

$$T^+(s,t,u) \;=\; \sum_{i,j=0}^{\infty} c_{ij}^+ t^i \nu^{2j}, \qquad T^-(s,t,u) = \sum_{i,j=0}^{\infty} c_{ij}^- t^i \nu^{2j+1}. \tag{4.3}$$

There are 10 subthreshold parameters that have tree level contributions from the NNLO LECs. In c_{01}^- and c_{20}^- the same combination $-C_1+2C_3+2C_4$ appears [8]:

$$16\rho^2 \left[c_{20}^-\right]_{C_i} = 3\left[c_{01}^-\right]_{C_i}. \tag{4.4}$$

Therefore in the isospin odd channel only three subthreshold parameters get independent contributions from the C_i. So for the 7 differences $a_\ell^- = a_\ell^{1/2}-a_\ell^{3/2}$ and $b_\ell^- = b_\ell^{1/2}-b_\ell^{3/2}$ getting contributions at NNLO and three subthreshold parameters we expect four relations:

$$\begin{aligned}
&\left(\rho^4+3\rho^3+3\rho+1\right)\left[a_1^-\right]_{C_i} = 2\rho^2(\rho+1)^2\left[b_1^-\right]_{C_i} - \frac{2}{3}\rho\left(\rho^2+1\right)\left[b_0^-\right]_{C_i} \\
&+\frac{1}{2\rho}\left(\rho^2+\frac{4}{3}\rho+1\right)\left(\rho^2+1\right)\left[a_0^-\right]_{C_i}, &(4.5)\\
&5(\rho+1)^2\left[b_2^-\right]_{C_i} = \frac{(\rho-1)^2}{\rho^2}\left[a_1^-\right]_{C_i} - \frac{\rho^4+\frac{2}{3}\rho^2+1}{4\rho^4}\left[a_0^-\right]_{C_i} + \frac{\rho^2-\frac{2}{3}\rho+1}{2\rho^2}\left[b_0^-\right]_{C_i}, &(4.6)\\
&5\left(\rho^2+1\right)\left[a_2^-\right]_{C_i} = \left[a_1^-\right]_{C_i} + 2\rho\left[b_1^-\right]_{C_i}, &(4.7)\\
&7\left(\rho^2+1\right)\left[a_3^-\right]_{C_i} = \left[a_2^-\right]_{C_i} + 2\rho\left[b_2^-\right]_{C_i}, &(4.8)
\end{aligned}$$

the threshold parameters are expressed in units of m_{π^+} and we use the symbol $\rho = m_K/m_\pi$.

T^+ brings in 7 more combinations of threshold parameters, $a_\ell^+ = a_\ell^{1/2}+2a_\ell^{3/2}$ and $b_\ell^+ = b_\ell^{1/2}+2b_\ell^{3/2}$, but there are 6 independent subthreshold parameters so we find only one more relation:

$$7\left[a_3^+\right]_{C_i} \;=\; \frac{1}{2\rho}\left[a_2^+\right]_{C_i} - \left[b_2^+\right]_{C_i} + \frac{1}{5\rho}\left[b_1^+\right]_{C_i} - \frac{1}{60\rho^3}\left[a_0^+\right]_{C_i} - \frac{1}{30\rho^2}\left[b_0^+\right]_{C_i}. \tag{4.9}$$

Again these relations hold for all tree-level contributions up to NNLO. The numerical check is shown in Tab. 3. The columns in Tab. 3 have the same meaning as in Tab. 1.

The first relation is reasonably satisfied, somewhat below two sigma. The second relation has a large discrepancy but if we assume a theory error of about half the NNLO contribution it seems reasonable. The third relation is well satisfied but the RHS has a rather large experimental error. The fourth relation does not work well, mainly due to the fact that we seem to underestimate the value for a_3^-. The last relation works well.

	[4]	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	remainder
LHS (4.5)	5.4 ± 0.3	0.16	0.97	0.77	$-0.11(11)$	0.6 ± 0.3
RHS (4.5)	6.9 ± 0.6	0.42	0.97	0.77	$-0.03(7)$	1.8 ± 0.6
10 LHS (4.7)	0.32 ± 0.01	0.03	0.12	0.11	$0.00(2)$	0.07 ± 0.01
10 RHS (4.7)	0.37 ± 0.01	0.02	0.12	0.10	$-0.01(2)$	0.14 ± 0.01
100 LHS (4.6)	-0.49 ± 0.02	0.08	-0.25	-0.17	$0.05(3)$	-0.21 ± 0.02
100 RHS (4.6)	-0.85 ± 0.60	0.03	-0.25	0.11	$-0.03(13)$	-0.71 ± 0.60
100 LHS (4.8)	0.13 ± 0.01	0.04	0.00	0.01	$0.03(1)$	0.05 ± 0.01
100 RHS (4.8)	0.01 ± 0.01	0.01	0.00	0.00	$0.00(1)$	-0.01 ± 0.01
10^3 LHS (4.9)	0.29 ± 0.03	0.09	0.00	0.06	$0.01(2)$	0.13 ± 0.03
10^3 RHS (4.9)	0.31 ± 0.07	0.03	0.00	0.06	$0.05(3)$	0.17 ± 0.07

Table 3: The relations found in the πK-scattering. The tree level contribution to the LHS and RHS of relation 1 is 3.01 and vanishes for the others. The NLO LECs part satisfies the relation. Notice the extra factors of ten for some of them. All quantities are in the units of powers of m_{π^+}

5. $\pi\pi$ and πK scattering

If we consider the $\pi\pi$ and πK system together we get two more relations due to the identities

$$[b_5]_{C_i} = \left[c^+_{30}\right]_{C_i} + \frac{3}{\rho}\left[c^-_{20}\right]_{C_i}, \qquad [b_6]_{C_i} = \frac{1}{4\rho}\left[c^-_{20}\right]_{C_i} + \frac{1}{16\rho^2}\left[c^+_{11}\right]_{C_i}, \tag{5.1}$$

where c^-_{ij} (c^+_{ij}) are expressed in units of $m_\pi^{2i+2j+1}$(m_π^{2i+2j}). We can express these relations in terms of the threshold parameters (all quantities expressed in powers of m_{π^+}):

$$6\left[a^1_3\right]_{C_i} = (1+\rho)\left[a^+_3 + 3a^-_3\right]_{C_i}, \tag{5.2}$$

$$3\left[(1+\rho)^2\left[b^2_2\right]_{C_i} + 7(1-\rho)^2\left[a^1_3\right]_{C_i}\right] = (1+\rho)\left[7\left(1-4\rho+\rho^2\right)\left[a^-_3\right]_{C_i} + \left[a^+_2 + 2\rho b^+_2\right]_{C_i}\right]. \tag{5.3}$$

The numerical results are quoted in Tab. 4. The first relation does not work but the second is well satisfied. If we look in the numerical results we see that a^-_3 plays a minor role in the RHS of the second relation but is important in the first, so this could be the same problem of relation (4.8). A related analysis can be found in [10].

6. $K_{\ell 4}$

The decay $K^+(p) \to \pi^+(p_1)\pi^-(p_2)e^+(p_\ell)\nu(p_\nu)$ is given by the amplitude [11]

$$T = \frac{G_F}{\sqrt{2}} V^\star_{us}\bar{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_\ell)(V^\mu - A^\mu) \tag{6.1}$$

where V^μ and A^μ are parametrized in terms of four formfactors: F, G, H and R (but the R-formfactor is negligible in decays with an electron in the final state). Using partial wave expansion and neglecting d wave terms one obtains [12]:

$$\begin{aligned} F &= f_s + f'_s q^2 + f''_s q^4 + f'_e s_e/4m_\pi^2 + f_t\sigma_\pi X\cos\theta + \dots, \\ G_p &= g_p + g'_p q^2 + g''_g q^4 + g'_e s_e/4m_\pi^2 + g_t\sigma_\pi X\cos\theta + \dots \end{aligned} \tag{6.2}$$

	[3],[4] [5],[6]	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	remainder
10^3 LHS (5.2)	0.34 ± 0.01	0.12	0.00	0.16	0.00(4)	0.05 ± 0.01
10^3 RHS (5.2)	0.38 ± 0.03	0.12	0.00	0.05	0.04(2)	0.16 ± 0.03
10 LHS (5.3)	-0.13 ± 0.01	-0.12	0.00	-0.05	0.02(2)	0.01 ± 0.01
10 RHS (5.3)	-0.09 ± 0.02	-0.05	0.00	-0.02	$-0.01(1)$	-0.01 ± 0.02
LHS (6.4)	-0.73 ± 0.10	-0.23	0.00	-0.15	$-0.05(6)$	-0.29 ± 0.10
RHS (6.4)	0.50 ± 0.07	0.19	0.00	0.10	0.03(4)	0.18 ± 0.07

Table 4: The relations found between $\pi\pi$ and πK-scattering lengths and between the curvature in F in $K_{\ell 4}$ and πK scattering. All quantities are in the units of powers of m_{π^+}.

Here $s_\pi(s_e)$ is the invariant mass of dipion (dilepton) system, and $q^2 = s_\pi/(4m_\pi^2) - 1$. θ is the angle of the pion in their rest frame w.r.t. the kaon momentum and $t - u = -2\sigma_\pi X \cos\theta$. Using NNLO ChPT results [8, 9] we find one relation between the quantities defined in (6.2) and πK scattering:

$$\sqrt{2}\left[f_s''\right]_{C_i} = 64\rho F_\pi \left[c_{30}^+\right]_{C_i} . \tag{6.3}$$

This leads to a relation between πK threshold parameters and f_s'' which, with all quantities expressed in units of m_{π^+}, reads:

$$\sqrt{2}\left[f_s''\right]_{C_i} = 32\pi\frac{\rho}{1+\rho}F_\pi\left[\frac{35}{6}\left(2+\rho+2\rho^2\right)\left[a_3^+\right]_{C_i} - \frac{5}{4}\left[a_2^+ + 2\rho b_2^+\right]_{C_i}\right] . \tag{6.4}$$

Numerical results for (6.4) are shown in Tab. 4. The experimental results is taken from [5] for f_s''/f_s and from [6] for f_s. This should be an acceptable combination since the central value for f_s'/f_s and f_s''/f_s from [6] are in good agreement with those of [5]. This relation is not satisfied: the sign is even different on the two sides. Notice that, in both cases, we also see that the ChPT series has a large NNLO contribution.

It has been already noticed, see [1] and Fig. 1, that ChPT, at present, underestimates the curvature f_s''. On the other hand there are indications that dispersive analysis techniques might help solving this problem: Fig. 7 in [1] shows that the dispersive result of [13] has a larger curvature then the two-loop result. Therefore, we do not consider this discrepancy a major problem for ChPT.

7. New fits of the NLO constants (preliminary results)

As remarked in [14], many NNLO calculations are now available in three-flavour ChPT. Besides, new lattice and dispersive results and further experimental data are at our disposal too. A study of the predictive power of NNLO ChPT is needed, and therefore also an update of the L_i fit. For this reason we are working on a new program to perform this fit with many more observables implemented. So far we have included masses and decay constants, $K_{\ell 4}$ formfactors, $\pi\pi$ and πK scattering lengths and the scalar pion radius. For now we rely on the resonance estimates of the C_i used in [1], although our plan is to achieve more information on them.

Our first preliminary results are summarized in Tab. 5. In the second column we quote fit 10 of [1]. This was found using the available linear fit for $K_{\ell 4}$ of [6], $F_K/F_\pi = 1.22$, the kaon and

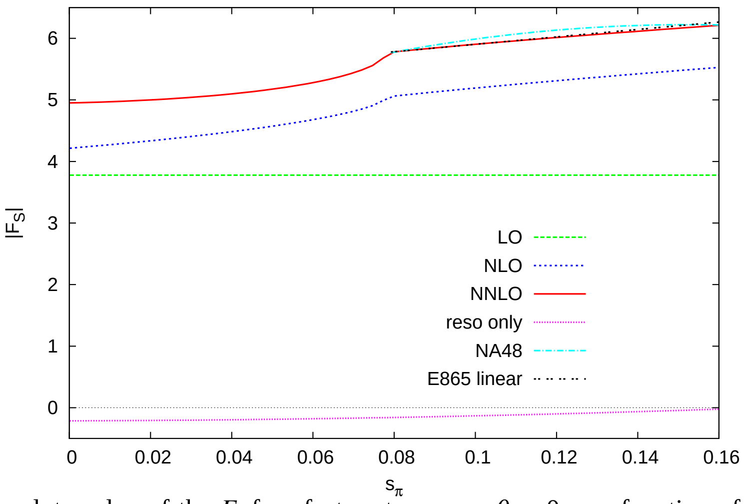

Figure 1: The absolute value of the F_s formfactor at $s_\ell = \cos\theta = 0$ as a function of s_π (in Gev^2 units) above and below threshold. The NNLO result nicely reproduces the linear fit quoted in [6], but not the large negative curvature in [5]. The line at the bottom is the contribution coming from the C_i, which has a positive curvature.

	fit 10 [1]	fit 10 iso	NA48/2	F_K/F_π	All
$10^3 L_1^r$	0.43	0.40 ± 0.12	$\boxed{0.98}$	0.97	0.99 ± 0.13
$10^3 L_2^r$	0.73	0.76 ± 0.12	0.78	0.79	0.60 ± 0.22
$10^3 L_3^r$	-2.35	-2.40 ± 0.37	$\boxed{-3.14}$	-3.12	-3.07 ± 0.59
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\boxed{0.65 \pm 0.64}$
$10^3 L_5^r$	0.97	0.97 ± 0.11	0.93	$\boxed{0.72}$	$\boxed{0.53 \pm 0.10}$
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\boxed{0.07 \pm 0.65}$
$10^3 L_7^r$	-0.31	-0.30 ± 0.15	-0.30	-0.26	-0.21 ± 0.15
$10^3 L_8^r$	0.59	0.61 ± 0.20	0.59	$\boxed{0.48}$	0.37 ± 0.17
χ^2 (dof)		0.25 (1)	0.17 (1)	0.19 (1)	0.78 (4)

Table 5: Preliminary results for the fits. $L_9 \equiv 0.59 \times 10^{-3}$ everywhere, as found from the vector pion radius in [15]. See text for a longer discussion

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eta masses with isospin breaking corrections included and setting $L_4 \equiv L_6 \equiv 0$. In the column labeled fit 10 iso we quote the fit we find using the same input as fit 10 but without including isospin breaking. As you see the two fits are in good agreement. The column NA48/2 relies on the new experimental data from [5]. We checked that the fit does not change including the curvature f_s''. With this fit ChPT predicts the value $f_s'' = -0.90$ to be compared with the experimental one $f_s'' = -1.58 \pm 0.064$. Note that the fit in [5] shows large correlations between the slope and the curvature of the F_s formfactor which have not been taken into account yet. The values of L_1 and L_3 change drastically. The third column shows the fit obtained changing the ratio F_K/F_π to 1.19. This affects mainly L_5 and L_8. The last column shows the fit obtained letting L_4 and L_6 free, and adding a_0^0, a_0^2, $a_0^{1/2}$, $a_0^{3/2}$ and the scalar pion radius. The value obtained for L_4 is larger then expected. Some more comment can be found in [16].

8. Conclusions

We have performed a systematic search for relations between observables that allow a test of ChPT at NNLO order in a C_i-independent way. We studied in detail the relations for the $\pi\pi$, πK scattering and $K_{\ell 4}$ since for these cases enough experimental and/or dispersion theory results exist.

The resulting picture is that ChPT at NNLO mostly works but there are troublesome cases. The $\pi\pi$ system alone works well. The πK system alone works satisfactorily but with some discrepancies. The same can be said for the combinations of both systems. A common part in these two cases is the presence of a_3^- . Comparing πK scattering and $K_{\ell 4}$ leads to a clear contradiction which needs further investigation.

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