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Chiral perturbation theory in the meson sector

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The present status of Chiral Perturbation Theory in the meson sector is discussed concentrating on recent developments. This write-up contains short discussions on a listing a few historical papers, the principles behind ChPT, two-flavour ChPT including some comments about the pion polarizability, three-flavour ChPT with a discussion of the recently found relations as tests of ChPT and preliminary results of a new fit of the NLO low-energy-constants (LECs). It discusses somewhat deeper $\eta \to 3\pi$ and the arguments for the existence of a "hard pion ChPT" and its application to $K \to 2\pi$.

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1. Introduction

Chiral Perturbation Theory (ChPT) and effective field theory (EFT) play a major role at this now 15 year old conference series. In this talk I review some aspects of mesonic ChPT. This talk has a large overlap with earlier talks given at Lattice 2007 [1] and EFT09 [2] as well as with my review on ChPT at two-loop order [3]. I concentrate on some of the newer results in this written version, consult the earlier references for the topics discussed here in less detail.

In the talk I discussed several topics not included in this write-up. The discussion and references for the partially quenched calculations and recent progress in renormalization group and effective field theory can be found in [2] and for the results for ChPT for the weak interaction a recent review is [4].

This write-up contains short discussions on a listing a few historical papers, the principles behind ChPT, two-flavour ChPT including some comments about the pion polarizability, three-flavour ChPT with a discussion of the recently found relations and preliminary results of a new fit of the NLO low-energy-constants (LECs). I devote quite some space to $\eta \to 3\pi$ and the arguments for the existence of a "hard pion ChPT" and its application to $K \to 2\pi$.

2. Some History: 50, 40, 35, 30, 25, 20 and 15 years ago

In this "capital city" of Chiral Perturbation Theory it is appropriate to look back at some of its history. ChPT has a 50 year history by now as was reviewed by S. Weinberg in his talk [5]. It should also be remarked that this conference series is now 15 years old.

I picked some papers which fell at or close to jubileum years. About 50 years ago our subject started with the Goldberger-Treiman relation [6] and the advent of PCAC, the partially conserved axial-current [7], and how this reproduced the Goldberger-Treiman relation. About 40 years ago a lot of work had been done within the framework of PCAC but 1968 and 1969 saw some very important papers: the Gell-Mann–Oakes–Renner relation [8] and the proper way how to implement chiral symmetry in all generality in phenomenological Lagrangians [9] after Weinberg's derivation for the two flavour case [10]. Shortly afterwards loop calculations started with e.g. loop results for $\pi\pi$ scattering [11] and $\eta \to 3\pi$ [12]. 30 years ago the start with the modern way of including higher order Lagrangians and performing a consistent renormalization came with [13]. At the same time there was also the beautiful paper by Gasser and Zepeda about the types of non-analytical corrections that can appear [14]. The seminal papers by Gasser and Leutwyler of 25 years ago then put the entire subject on a modern firm footing [15, 16]. The same period also had my own entry into the subject [17]. Many one-loop calculations were done and the understanding that the coefficients in the higher-order Lagrangians could be understood from the contributions of resonances was put on a firm footing 20 years ago [15, 18]. Let me close this historical part with two 15 year old papers, a very clear discussion of the basics of ChPT [19] and the first full two-loop calculation [20].

3. Chiral Perturbation Theory: ChPT, CHPT or χPT

ChPT is best described as " *Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques*" and a clear discussion about its

derivation and underlying assumptions is in [19]. Some reviews are [21, 3]. More reviews and references to introductory lectures can be found on the webpage [22].

For effective field theories, there are three principles that are needed and for ChPT they are

- **Degrees of freedom:** Goldstone Bosons from the spontaneous chiral symmetry breakdown.
- **Power counting:** This is what allows a systematic ordering of terms and is here essentially dimensional counting in momenta and masses.
- **Expected breakdown scale:** The scale of the not explicitly included physics, here resonances, so the scale is of order M_ρ, but this is channel dependent.

I will not go into more details here, links to lectures can be found on the website [22], short introductions can be found in [2] and [3].

4. Two-flavour ChPT at NNLO

References to order p^2 and p^4 work can be found in [3]. The first work at NNLO used dispersive methods to obtain the nonanalytic dependence on kinematical quantities, q^2, s, t, u at NNLO. This was done for the vector (electromagnetic) and scalar form-factor of the pion in [23] (numerically) and [24] (analytically) and for $\pi\pi$-scattering analytically in [25].

Basically all processes of interest are calculated to NNLO in ChPT: $\gamma\gamma \to \pi^0\pi^0$ [20, 26], $\gamma\gamma \to \pi^+\pi^-$ [27, 28], F_π and m_π [27, 29, 30], $\pi\pi$-scattering [29], the pion scalar and vector form-factors [30] and pion radiative decay $\pi \to \ell\nu\gamma$ [31]. The pion mass is known at order p^6 in finite volume [32]. Recently $\pi^0 \to \gamma\gamma$ has been done to this order as discussed in the talk by Moussallam[33].

The LECs have been fitted in several processes. $\bar{l}_4$ from fitting to the pion scalar radius [31, 34], $\bar{l}_3$ from an estimate of the pion mass dependence on the quark masses [15, 34] and $\bar{l}_1$, $\bar{l}_2$ from the agreement with $\pi\pi$-scattering [34], $\bar{l}_6$ from the pion charge radius [30] and $\bar{l}_6 - \bar{l}_5$ from the axial form-factor in $\pi \to \ell\nu\gamma$. There is also a recent determination of $\bar{l}_5$ from hadronic tau decays [35]. The final best values are [30, 31, 34, 35]

$$\begin{array}{llll} \bar{l}_1 = -0.4 \pm 0.6\,, & \bar{l}_2 = 4.3 \pm 0.1\,, & \bar{l}_3 = 2.9 \pm 2.4\,, & \bar{l}_4 = 4.4 \pm 0.2\,, \\ \bar{l}_6 - \bar{l}_5 = 3.0 \pm 0.3\,, & \bar{l}_6 = 16.0 \pm 0.5 \pm 0.7\,, & \bar{l}_5 = 12.24 \pm 0.21\,. & \end{array} \tag{4.1}$$

Values of $\bar{l}_3$ and $\bar{l}_4$ have also been obtained by the lattice as discussed in several talks at this conference. There is information on some combinations of p^6 LECs. These are basically via the curvature in the vector and scalar form-factor of the pion [30] and two combinations from $\pi\pi$-scattering [34] from the knowledge of b_5 and b_6 in that reference. The order p^6 LECs c_i^r are estimated to have a small effect for m_π, f_π and $\pi\pi$-scattering.

A possible problem for ChPT are the pion polarizabilities. These are cleanly predicted in ChPT, the latest numbers from ChPT [28] and experiment [36] are:

$$\text{ChPT}: \ (\alpha_1 - \beta_1)_{\pi^\pm} = (5.7 \pm 1.0) \cdot 10^{-4} \text{ fm}^3 \tag{4.2}$$

$$\text{Exp}: \ (\alpha_1 - \beta_1)_{\pi^\pm} = (11.6 \pm 1.5_{stat} \pm 3.0_{syst} \pm 0.5_{mod}) \cdot 10^{-4} \text{ fm}^3 \tag{4.3}$$

A possible problem in this experiment is the background from direct $\gamma N \to \gamma N\pi$ production. Large values also follow from the older Primakoff experiments and a dispersive analysis [37] from $\gamma\gamma \to$

$\pi\pi$:

$$\text{Primakoff}: \ (\alpha_1-\beta_1)_{\pi^\pm} = (13.6 \pm 2.8_{stat} \pm 2.4_{syst}) \cdot 10^{-4} \text{ fm}^3 \tag{4.4}$$

$$\text{dispersive}: \ (\alpha_1-\beta_1) = (13.0 + 3.6 - 1.9) \cdot 10^{-4} \text{fm}^3 . \tag{4.5}$$

The latter value has been criticized in [38] who argue that [37] has a large uncontrolled model dependence and conclude " "Our calculations... are in reasonable agreement with ChPT for charged pions". See their presentations in these proceedings for more details. ChPT in our present understanding cannot produce a value as large as in (4.3).

5. Three-flavour ChPT

5.1 Calculations

In this section I discuss several results at NNLO in mesonic three-flavour ChPT. The formulas here are much larger than in two-flavour ChPT and while the expressions have been reduced to a series of well-defined two-loop integrals, the latter are evaluated numerically. Both are the consequence of the different masses present. The vector two-point functions [39, 40] and the isospin breaking in the $\rho\omega$ channel [41] were among the first calculated. The flavour disconnected scalar two-point function relevant for bounds on L_4^r and L_6^r was worked out in [42]. The remaining scalar two-point functions are known, available from the speaker but unpublished. Masses and decay constants as well as axial-vector two-point functions were the first calculations requiring full two-loop integrals, done in the π and η [39, 43] and the K channel [39]. Including isospin breaking contributions to masses and decay constants was done in [44]. After $K_{\ell 4}$ had also been evaluated to NNLO [45] a fit to the LECs was done as described below. The vacuum expectation values in the isospin limit were done in [45], with isospin breaking in [44] and at finite volume in [46].

Vector (electromagnetic) form-factors for pions and kaons were calculated in [47, 48] and in [48] a NNLO fit for L_9^r was performed. L_{10}^r can be had from hadronic tau decays [35] or the axial form-factor in $\pi, K \to \ell\nu\gamma$. The NNLO calculation is done, but no data fitting was performed [49]. A rather important calculation is the $K_{\ell 3}$ form-factor. This calculation was done in [50, 51] and a rather interesting relation between the value at zero, the slope and the curvature for the scalar form-factor obtained [50]. Isospin-breaking has been included as well [52].

Scalar form-factors including sigma terms and scalar radii [53] and $\pi\pi$ [54] and πK-scattering [55] are known and used to place limits on L_4^r and L_6^r. Finally, the relations between the l_i^r, c_i^r and L_i^r, C_i^r have been extended to the accuracy needed to compare order p^6 results in two and three-flavour calculations [56] and there has been some progress towards fully analytical results for m_π^2 [57] and πK-scattering lengths [58]. The most recent results are $\eta \to 3\pi$ [59], isospin breaking in $K_{\ell 3}$ [52].

5.2 Testing ChPT and estimates of the order p^6 LECs

Most numerical analysis at order p^6 use a (single) resonance approximation to the order p^6 LECs. The main underlying motivation is the large N_c limit and phenomenological success at order p^4 [18]. There is a large volume of work on this, some references are [60]. The numerical work I will report has used a rather simple resonance Lagrangian [18, 29, 44, 45]. The estimates of

	Roy-Steiner	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	remainder
LHS (I)	5.4 ± 0.3	0.16	0.97	0.77	-0.11	0.6 ± 0.3
RHS (I)	6.9 ± 0.6	0.42	0.97	0.77	-0.03	1.8 ± 0.6
10 LHS (II)	0.32 ± 0.01	0.03	0.12	0.11	0.00	0.07 ± 0.01
10 RHS (II)	0.37 ± 0.01	0.02	0.12	0.10	-0.01	0.14 ± 0.01
100 LHS (III)	-0.49 ± 0.02	0.08	-0.25	-0.17	0.05	-0.21 ± 0.02
100 RHS (III)	-0.85 ± 0.60	0.03	-0.25	0.11	-0.03	-0.71 ± 0.60
100 LHS (IV)	0.13 ± 0.01	0.04	0.00	0.01	0.03	0.05 ± 0.01
100 RHS (IV)	0.01 ± 0.01	0.01	0.00	0.00	0.00	-0.01 ± 0.01
10^3 LHS (V)	0.29 ± 0.05	0.09	0.00	0.06	0.01	0.13 ± 0.03
10^3 RHS (V)	0.31 ± 0.07	0.03	0.00	0.06	0.05	0.17 ± 0.07

Table 1: πK-scattering: The numerical results for relations I-V, both left- (LHS) and right-hand side for the dispersive result from [62] and the NLO, NNLO 2-loop and NNLO L_i-dependent part (1-loop). The tree level for LHS and RHS of (I) is 3.01 and vanishes for the others. The equality of the remainder of LHS and RHS gives a test of ChPT.

the C_i^r is the weakest point in the numerical fitting at present, however, many results are not very sensitive to this. The main problem is that the C_i^r which contribute to the masses, are estimated to be zero, except for η' effects, and how these might affect the determination of the others. The estimate is μ-independent while the C_i^r are not.

The fits done in [44, 45, 53] tried to check this by varying the total resonance contribution by a factor of two, varying the scale μ from 550 to 1000 MeV and compare estimated C_i^r to experimentally determined ones. The latter works well, but the experimentally well determined ones are those with dependence on kinematic variables only, not ones relevant for quark-mass dependence.

A new fit is in progress but in order to check whether ChPT with three flavours works, one would like a test that is as much as possible independent of the estimated values of the C_i. In [61] we studied 76 observables leading to 35 combinations that are independent of the C_i, or 35 relations. For these we found 13 with good "data," $\pi\pi$ [34] and πK [62]threshold parameters from dispersion theory and $K_{\ell 4}$ from experiment [63, 64].

The results for the 5 relations found in πK scattering are shown in Tab. 1. The equality of the remainder of LHS and RHS gives a test of ChPT. The results are encouraging but not 100% conclusive. More details and more results can be found in [61, 65].

5.3 The fitting and results

The inputs used for the standard fit, as discussed more extensively in [44, 45], are

- $K_{\ell 4}$: $F(0)$, $G(0)$, λ from E865 at BNL[63].
- $m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$, electromagnetic corrections include the violation of Dashen's theorem.
- F_{π^+} and F_{K^+}/F_{π^+}.
- $m_s/\hat{m} = 24$. Variations with $m_s/\hat{m}$ were studied in [44, 45].
- L_4^r, L_6^r the main fit, 10, has them equal to zero, but see below and the arguments in [42].

Some results of this fit are given in Tab. 2. The errors are very correlated, see Fig. 6 in [45] for an example. Varying the values of L_4^r, L_6^r as input can be done with a reasonable fitting chi-squared when varying $10^3 L_4^r$ from -0.4 to 0.6 and L_6^r from -0.3 to 0.6 [53]. The variation of many quantities with L_4^r, L_6^r (including the changes via the changed values of the other L_i^r) are shown in [53, 54, 55]. Fit B was one of the fits with a good fit to the pion scalar radius and fairly small corrections to the sigma terms [53] while fit D [66] is the one that gave agreement with $\pi\pi$ and πK-scattering threshold quantities.

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.53 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54
$10^3 L_9^r$	5.93 ± 0.43	7.0	–	–
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
m_π^2: p^4, p^6	0.006,0.258	$0.009, \equiv 0$	-0.138,0.009	-0.091,0.133
m_K^2: p^4, p^6	0.007,0.306	$0.075, \equiv 0$	-0.149,0.094	-0.096,0.201
m_η^2: p^4, p^6	-0.052,0.318	$0.013, \equiv 0$	-0.197,0.073	-0.151,0.197
m_u/m_d	0.45±0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}$: p^4, p^6	0.169,0.051	$0.22, \equiv 0$	0.153,0.067	0.159,0.061

Table 2: The fits of the L_i^r and some results, see text for a detailed description. They are all quoted at $\mu = 0.77$ GeV. Table with values from [44, 48, 53, 55, 66].

Note that $m_u/m_d = 0$ is never even close to the best fit and this remains true for the entire variation with L_4^r, L_6^r. The value of F_0, the pion decay constant in the three-flavour chiral limit, can vary significantly, even though I believe that fit B is an extreme case.

We, JB and I. Jemos, are working on a new general fit. The preferred value of F_K/F_π is changed and NA48 has measured the $K_{\ell 4}$ formfactors more accurately. In addition, we want to include more constraints directly. Some preliminary results are also discussed in I. Jemos talk [65]. For this work, all the calculated processes are being programmed in $C++$ to allow for a more uniform treatment and an easier handling of the LECs. This program is only partly completed but first fitting results are given in Tab. 3. The column labeled fit 10 iso uses the input as used for fit 10 in [44] but without isospin breaking. The results are essentially identical to those of the fit including isospin breaking. This column is included as reference. The estimates for the C_i^r used are the same as used in [44] for all the fits shown in the table except the last. The other columns always have some more constraints included. The small boxes indicate the LECs which have changed most. First we add the better information on the $K_{\ell 4}$ form-factors from NA48 [64]. This produces sizable changes in L_1^r and L_3^r. The newer value of the PDG for $F_K/F_\pi = 1.193$ then changes the fitted value of L_5^r which influences L_8^r via the fitted masses. L_5^r gets lowered somewhat more when

	fit 10 iso	NA48	F_K/F_π	Scatt	All	All ($C_i^r=0$)
$10^3 L_1^r$	0.40 ± 0.12	0.98	0.97	0.97	0.98 ± 0.11	0.75
$10^3 L_2^r$	0.76 ± 0.12	0.78	0.79	0.79	0.59 ± 0.21	0.09
$10^3 L_3^r$	-2.40 ± 0.37	-3.14	-3.12	-3.14	-3.08 ± 0.46	-1.49
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$	0.71 ± 0.67	0.78
$10^3 L_5^r$	0.97 ± 0.11	0.93	0.72	0.56	0.56 ± 0.11	0.67
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$	0.15 ± 0.71	0.18
$10^3 L_7^r$	-0.30 ± 0.15	-0.30	-0.26	-0.23	-0.22 ± 0.15	-0.24
$10^3 L_8^r$	0.61 ± 0.20	0.59	0.48	0.44	0.38 ± 0.18	0.39
χ^2 (dof)	0.25 (1)	0.17 (1)	0.19 (1)	5.38 (5)	1.44 (4)	1.51 (4)

Table 3: The changes in the L_i compared to the isospin symmetric fit to the same input as the old fit 10 of [44]. The other columns include one new effect at a time, see text.

we include the scattering lengths a_0^0, a_0^2, $a_0^{1/2}$ and $a_0^{3/2}$. Adding the pion scalar radius requires a nonzero for one of L_4^r and L_6^r. This is what is shown in the column labeled "All." In the last column we have set the estimated value of the $C_i^r = 0$. This is work in progress but some puzzles appear at this stage, the large N_c relation $2L_1^r = L_2^r$ is now badly broken and the central value of L_4^r is not small compared to L_5^r. We are working on including more scattering lengths and trying to include also some lattice results on the meson masses and decay constants. This is especially important since the simple estimate of the C_i^r used has none contributing to masses and decay constants.

6. $\eta \to \pi\pi\pi$

In the limit of conserved isospin, no electromagnetism and $m_u = m_d$, the η is stable. Direct electromagnetic effects are small [67]. The decay thus proceeds mainly through the quark-mass difference $m_u - m_d$. The lowest order was done in [68], order p^4 in [69] and recently the full order p^6 has been evaluated [59]. The momenta for the decay $\eta \to \pi^+\pi^-\pi^0$ we label as p_η, p_+, p_- and p_0 respectively and we introduce the kinematical Mandelstam variables $s = (p_+ + p_-)^2, t = (p_+ + p_0)^2, u = (p_- + p_0)^2$. These are linearly dependent, $s+t+u = m_{\pi^o}^2 + m_{\pi^-}^2 + m_{\pi^+}^2 + m_\eta^2 \equiv 3s_0$ .The amplitudes for the charged, $A(s,t,u)$, and neutral, $\overline{A}(s,t,u)$ are related

$$\overline{A}(s_1,s_2,s_3) = A(s_1,s_2,s_3) + A(s_2,s_3,s_1) + A(s_3,s_1,s_2)\,. \tag{6.1}$$

The relation in (6.1) is only valid to first order in $m_u - m_d$. The overall factor of $m_u - m_d$ can be put in different quantities, two common choices are

$$A(s,t,u) = \frac{\sqrt{3}}{4R} M(s,t,u) \quad \text{or} \quad A(s,t,u) = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\,\frac{\mathscr{M}(s,t,u)}{3\sqrt{3}F_\pi^2}\,, \tag{6.2}$$

with $R = (m_s - \hat{m})/(m_d - m_u)$ or $Q^2 = R(m_s + m_d)/(2\hat{m})$ pulled out. The lowest order result is

$$M(s,t,u)_{LO} = \left((4/3)m_\pi^2 - s\right)/F_\pi^2\,. \tag{6.3}$$

The tree level determination of R in terms of meson masses gives with (6.3) a decay rate of 66 eV which should be compared with the experimental results of 295±17 eV[70]. In principle, since the

Exp.	a	b	d
KLOE	$-1.090\pm0.005^{+0.008}_{-0.019}$	$0.124\pm0.006\pm0.010$	$0.057\pm0.006^{+0.007}_{-0.016}$
Crystal Barrel	-1.22 ± 0.07	0.22 ± 0.11	0.06 ± 0.04 (input)
Layter *et al.*	-1.08 ± 0.014	0.034 ± 0.027	0.046 ± 0.031
Gormley *et al*	-1.17 ± 0.02	0.21 ± 0.03	0.06 ± 0.04

Table 4: Measurements of the Dalitz plot distributions in $\eta\to\pi^+\pi^-\pi^0$. Quoted in the order cited in [74]. The KLOE result f is $f=0.14\pm0.01\pm0.02$.

	A_0^2	a	b	d	f
LO	120	-1.039	0.270	0.000	0.000
NLO	314	-1.371	0.452	0.053	0.027
NLO ($L_i^r=0$)	235	-1.263	0.407	0.050	0.015
NNLO	538	-1.271	0.394	0.055	0.025
NNLO ($\mu=0.6$ GeV)	543	-1.300	0.415	0.055	0.024
NNLO ($\mu=0.9$ GeV)	548	-1.241	0.374	0.054	0.025
NNLO ($C_i^r=0$)	465	-1.297	0.404	0.058	0.032
NNLO ($L_i^r=C_i^r=0$)	251	-1.241	0.424	0.050	0.007

Table 5: Theoretical estimate of the Dalitz plot distributions in $\eta\to\pi^+\pi^-\pi^0$.

decay rate is proportional to $1/R^2$ or $1/Q^4$, this should allow for a precise determination of R and Q. However, the change required seems large. The order p^4 calculation [69] increased the predicted decay rate to 150 eV albeit with a large error. About half of the enhancement in the amplitude came from $\pi\pi$ rescattering and the other half from other effects like the chiral logarithms [69]. The rescattering effects have been studied at higher orders using dispersive methods in [71] and [72]. Both calculations found an enhancement in the decay rate to about 220 eV but differ in the way the Dalitz plot distributions look. That difference and the facts that in $K_{\ell4}$ the dispersive estimate [73] was about half the full ChPT calculation [45] and at order p^4 the dispersive effect was about half of the correction for $\eta\to3\pi$ makes it clear that a full order p^6 calculation was desirable. The calculation [59] generalized the methods of [44] to deal with π^0-η mixing. The correction found in [59] at order p^6 is 20-30% in amplitude, larger in magnitude than the dispersive estimates [71, 72] but with a shape similar to [72].

The Dalitz plot in $\eta\to3\pi$ is parameterized in terms of x and y defined in terms of the kinetic energies of the pions T_i and $Q_\eta=m_\eta-2m_{\pi^+}-m_{\pi^0}$ for the charged decay and z defined in terms of the pion energies E_i. The amplitudes are expanded in $x=\sqrt{3}\left(T_+-T_-\right)/Q_\eta$, $y=3T_0/Q_\eta-1$, $z=(2/3)\sum_{i=1,3}\left(3E_i-m_\eta\right)^2/\left(m_\eta-3m_{\pi^0}\right)^2$, via

$$|M(s,t,u)|^2 = A_0^2\left(1+ay+by^2+dx^2+fy^3+\cdots\right), \quad |\overline{M}(s,t,u)|^2=\overline{A}_0^2\left(1+2\alpha z+\cdots\right). \tag{6.4}$$

Recent experimental results for these parameters are shown in Tabs. 4 and 6. There are discrepancies among the experiments but the latest precision measurements of α agree. The predictions from ChPT to order p^6 with the input parameters as described earlier are given in Tabs. 5 and 7. The different lines corresponds to variations on the input and the order of ChPT. The lines labeled NNLO are the central results. The agreement with experiment is not too good and clearly needs

Exp.	α
Crystal Ball (MAMI C)	-0.032 ± 0.003
Crystal Ball (MAMI B)	$-0.032 \pm 0.002 \pm 0.002$
WASA/COSY	$-0.027 \pm 0.008 \pm 0.005$
KLOE	$-0.027 \pm 0.004^{+0.004}_{-0.006}$
Crystal Ball (BNL)	-0.031 ± 0.004
WASA/CELSIUS	$-0.026 \pm 0.010 \pm 0.010$
Crystal Barrel	$-0.052 \pm 0.017 \pm 0.010$
GAMS2000	-0.022 ± 0.023
SND	$-0.010 \pm 0.021 \pm 0.010$

Table 6: Measurements of the Dalitz plot distribution in $\eta \to \pi^0\pi^0\pi^0$. Quoted in the order cited in [76].

	$\overline{A}_0^2$	α
LO	1090	0.000
NLO	2810	0.013
NLO ($L_i^r = 0$)	2100	0.016
NNLO	4790	0.013
NNLO ($C_i^r = 0$)	4140	0.011
NNLO ($L_i^r, C_i^r = 0$)	2220	0.016
dispersive [71]	—	$-(0.007$—$0.014)$
tree dispersive	—	-0.0065
absolute dispersive	—	-0.007
Borasoy [75]	—	-0.031
error	160	0.032

Table 7: Theoretical estimates of the Dalitz plot distribution in $\eta \to \pi^0\pi^0\pi^0$. [77, 59]

further study. Especially puzzling is that α is consistently positive while the dispersive calculations as well as [75] give a negative value. The inequality $\alpha \leq \left(d + b - a^2/4\right)/4$ derived in [59] shows that α has rather large cancellations inherent in its prediction and that the overestimate of b is a likely cause of the wrong sign for α. The fairly large correction gives in the end larger values of Q compared to those derived from the masses [59].

7. Hard pion ChPT?

In this section I discuss some recent work that argues that chiral effects should also be calculable for processes with hard pions. This type of arguments was given by Flynn and Sachrajda for $K_{\ell 3}$ decays away from $q^2_{\rm max}$ [78]. It was argued that those arguments apply much more generally in [79] and there they were also applied to $K \to \pi\pi$.

The underlying argument is that the main predictions of ChPT, namely chiral logarithms come from soft pion lines. In pure ChPT as discussed above the powercounting works since all lines are considered soft. In the baryon sector, power counting has also been developed. There the meson lines are soft and the baryons are always close to their mass-shell. The heavy momentum always follows a baryon line. Here two momentum regions are important, those close to the baryon momentum $p = M_B v + k$ and the soft ones $p = k$, k soft for both. As argued in [80], the M_B-dependence of loop-diagrams is analytic and can be absorbed in the LECs. This is a broad area of research as can be judged from many of the talks in working group 2. Similarly, ChPT with mesons with a heavy-quark relies on having two momentum regimes with one line always carrying the large momentum, [81, 82]. Thereafter it has been argued that ChPT could also be constructed for unstable particles near their mass-shell [83], see especially the discussion in [84]. In all these cases, the underlying argument is always the same. The heavy-mass dependence is analytic and can be absorbed in the LECs.

Ref. [79] argued that the same type of reasoning works for processes with hard pions. The reasoning is depicted in Fig. 1: Take a process with a given external momentum configuration, identify the soft lines and cut them. The resulting part is analytic in the soft stuff and should thus be describable by an effective Lagrangian with coupling constants depending on the given

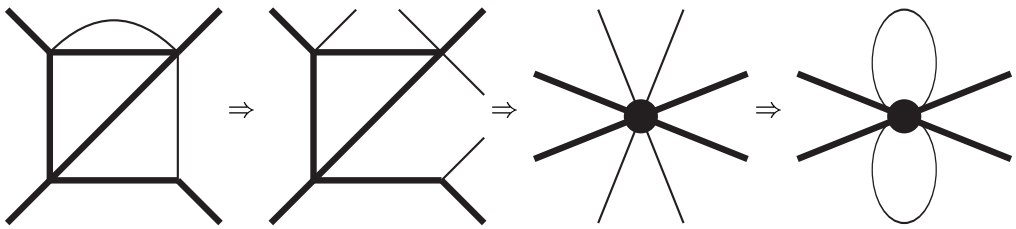

Figure 1: An example of the argument for a "hard pion ChPT." The thick lines contain a large momentum, the thin lines a soft momentum. Left: a general Feynman diagram with hard and soft lines. Middle-left: we cut the soft lines to remove the soft singularity. Middle-right: The contracted version where the hard part is assumed to be correctly described by a "vertex" of an effective Lagrangian. Right: the contracted version as a loop diagram. This is expected to reproduce the chiral logarithm of the left diagram.

external momenta. The Lagrangian should be complete in the *neighbourhood*, in both momenta and processes, and should respect the symmetries present in the problem. Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses.

In [78] and [79] it was proven that respectively for $K_{\ell 3}$-formfactors and $K \to \pi\pi$-decays the lowest order Lagrangian is sufficiently complete to be able to calculate uniquely the pionic chiral logarithm. [79] explicitly kept some higher order terms to illustrate the argument and found that

$$\begin{aligned} A_0^{NLO} &= A_0^{LO}\left(1+\frac{3}{8F^2}\overline{A}(M^2)\right)+\lambda_0 M^2+\mathscr{O}(M^4)\,,\\ A_2^{NLO} &= A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M^2)\right)+\lambda_2 M^2+\mathscr{O}(M^4)\,, \end{aligned} \tag{7.1}$$

with $\overline{A}(M^2) = -1/(16\pi^2)M^2\log(M^2/\mu^2)$ and M^2 the lowest order pion mass. λ_0 and λ_2 depend on higher order terms in the Lagrangian and are not calculable. Another check was that the three-flavour ChPT did have the same pionic chiral logarithms. Notice that the logarithms in (7.1) are not due to the final state interaction, that effect goes into the couplings in this approach, and actually go against the $\Delta I = 1/2$-rule.

8. Conclusions

ChPT in the meson sector is progressing and finds new application areas. I have shortly reviewed two- and three-flavour ChPT for mesons and discussed or provided a references to several areas where there has been recent progress.

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References

[1] J. Bijnens, PoS **LAT2007** (2007) 004 [arXiv:0708.1377 [hep-lat]].

[2] J. Bijnens, PoS **EFT09** (2009) 022 [arXiv:0904.3713 [hep-ph]].

[3] J. Bijnens, Prog. Part. Nucl. Phys. **58** (2007) 521 [hep-ph/0604043].

[4] G. D'Ambrosio, PoS **EFT09** (2009) 061

[5] S. Weinberg, *Effective field theories – past and future,* these proceedings.

[6] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110** (1958) 1178.

[7] Y. Nambu, Phys. Rev. Lett. **4** (1960) 380; M. Gell-Mann and M. Levy, Nuovo Cim. **16** (1960) 705.

[8] M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. **175** (1968) 2195.

[9] S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. G. Callan, S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2247 (1969).

[10] S. Weinberg, Phys. Rev. **166** (1968) 1568.

[11] G. Ecker and J. Honerkamp, Nucl. Phys. B **52** (1973) 211.

[12] P. Langacker and H. Pagels, Phys. Rev. D **10** (1974) 2904.

[13] S. Weinberg, Physica A **96** (1979) 327.

[14] J. Gasser and A. Zepeda, Nucl. Phys. B **174** (1980) 445.

[15] J. Gasser and H. Leutwyler, Annals Phys. **158** (1984) 142.

[16] J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 465.

[17] J. Bijnens, H. Sonoda and M. B. Wise, Phys. Rev. Lett. **53** (1984) 2367.

[18] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321** (1989) 311; G. Ecker *et al.*, Phys. Lett. B **223** (1989) 425; J. F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. D **39** (1989) 1947.

[19] H. Leutwyler, Annals Phys. **235** (1994) 165. [hep-ph/9311274].

[20] S. Bellucci, J. Gasser and M. E. Sainio, Nucl. Phys. B **423** (1994) 80 [Erratum-ibid. B **431** (1994) 413] [hep-ph/9401206].

[21] G. Ecker, Prog. Part. Nucl. Phys. **35** (1995) 1 [hep-ph/9501357]; A. Pich, Rept. Prog. Phys. **58** (1995) 563 [hep-ph/9502366]; V. Bernard *et al.*, Int. J. Mod. Phys. E **4** (1995) 193 [hep-ph/9501384]; V. Bernard and U. G. Meissner, Ann. Rev. Nucl. Part. Sci. **57** (2007) 33 [hep-ph/0611231].

[22] `http://www.thep.lu.se/~bijnens/chpt.html`

[23] J. Gasser and U. G. Meissner, *Nucl. Phys.* B **357** (1991) 90.

[24] G. Colangelo, M. Finkemeier and R. Urech, *Phys. Rev.* D **54** (1996) 4403 [hep-ph/9604279].

[25] M. Knecht *et al.*, *Nucl. Phys.* B **457** (1995) 513 [hep-ph/9507319].

[26] J. Gasser, M. A. Ivanov and M. E. Sainio, *Nucl. Phys.* B **728** (2005) 31 [hep-ph/0506265].

[27] U. Burgi, *Phys. Lett.* B **377** (1996) 147 [hep-ph/9602421]; *Nucl. Phys.* B **479** (1996) 392 [hep-ph/9602429].

[28] J. Gasser, M. A. Ivanov and M. E. Sainio, Nucl. Phys. B **745** (2006) 84 [hep-ph/0602234].

[29] J. Bijnens *et al.*, *Phys. Lett.* B **374** (1996) 210 [hep-ph/9511397]; *Nucl. Phys.* B **508** (1997) 263 [Erratum-ibid. B **517** (1998) 639] [hep-ph/9707291].

[30] J. Bijnens, G. Colangelo and P. Talavera, *J. High Energy Phys.* **9805** (1998) 014 [hep-ph/9805389].

[31] J. Bijnens and P. Talavera, *Nucl. Phys.* B **489** (1997) 387 [hep-ph/9610269].

[32] G. Colangelo and C. Haefeli, *Nucl. Phys.* B **744** (2006) 14 [hep-lat/0602017].

[33] K. Kampf and B. Moussallam, 0901.4688 [hep-ph], B. Moussallam, these proceedings.

[34] G. Colangelo, J. Gasser and H. Leutwyler, *Nucl. Phys.* B **603** (2001) 125 [hep-ph/0103088].

[35] M. Gonzalez-Alonso *et al.*, Phys. Rev. D **78** (2008) 116012 [arXiv:0810.0760 [hep-ph]].

[36] J. Ahrens *et al.*, Eur. Phys. J. A **23**, 113 (2005) [arXiv:nucl-ex/0407011].

[37] L. V. Fil'kov and V. L. Kashevarov, Phys. Rev. C **73** (2006) 035210 [arXiv:nucl-th/0512047]; L.V. Fil'kov, these proceedings

[38] B. Pasquini, D. Drechsel and S. Scherer, Phys. Rev. C **77** (2008) 065211 [arXiv:0805.0213 [hep-ph]]; D. Drechsel, these proceedings.

[39] G. Amorós *et al.*, *Nucl. Phys.* B **568** (2000) 319 [hep-ph/9907264].

[40] E. Golowich and J. Kambor, *Nucl. Phys.* B **447** (1995) 373 [hep-ph/9501318]; S. Dürr and J. Kambor, *Phys. Rev.* D **61** (2000) 114025 [hep-ph/9907539].

[41] K. Maltman, *Phys. Rev.* D **53** (1996) 2573 [hep-ph/9504404].

[42] B. Moussallam, *J. High Energy Phys.* **0008** (2000) 005 [hep-ph/0005245].

[43] E. Golowich and J. Kambor, *Phys. Rev.* D **58** (1998) 036004 [hep-ph/9710214].

[44] G. Amorós *et al.*, *Nucl. Phys.* B **602** (2001) 87 [hep-ph/0101127].

[45] G. Amorós *et al.*, *Phys. Lett.* B **480** (2000) 71 [hep-ph/9912398]; *Nucl. Phys.* B **585** (2000) 293 [Erratum-ibid. B **598** (2001) 665] [hep-ph/0003258].

[46] J. Bijnens and K. Ghorbani, *Phys. Lett.* B **636** (2006) 51 [hep-lat/0602019].

[47] P. Post and K. Schilcher, *Phys. Rev. Lett.* **79** (1997) 4088 [hep-ph/9701422]; *Nucl. Phys.* B **599** (2001) 30 [hep-ph/0007095].

[48] J. Bijnens and P. Talavera, *J. High Energy Phys.* **0203** (2002) 046 [hep-ph/0203049].

[49] C. Q. Geng, I. L. Ho and T. H. Wu, *Nucl. Phys.* B **684** (2004) 281 [hep-ph/0306165].

[50] J. Bijnens and P. Talavera, *Nucl. Phys.* B **669** (2003) 341 [hep-ph/0303103].

[51] P. Post and K. Schilcher, *Eur. Phys. J.* C **25** (2002) 427 [hep-ph/0112352].

[52] J. Bijnens and K. Ghorbani, arXiv:0711.0148 [hep-ph].

[53] J. Bijnens and P. Dhonte, *J. High Energy Phys.* **0310** (2003) 061 [hep-ph/0307044].

[54] J. Bijnens, P. Dhonte and P. Talavera, *J. High Energy Phys.* **0401** (2004) 050 [hep-ph/0401039].

[55] J. Bijnens, P. Dhonte and P. Talavera, *J. High Energy Phys.* **0405** (2004) 036 [hep-ph/0404150].

[56] J. Gasser *et al.*, Phys. Lett. B **652** (2007) 21 [arXiv:0706.0955 [hep-ph]], Phys. Lett. B **675** (2009) 49 [arXiv:0903.0801 [hep-ph]]; M. A. Ivanov, these proceedings.

[57] R. Kaiser, *J. High Energy Phys.* **0709** (2007) 065 [arXiv:0707.2277 [hep-ph]].

[58] R. Kaiser and J. Schweizer, *J. High Energy Phys.* **0606** (2006) 009 [hep-ph/0603153].

[59] J. Bijnens and K. Ghorbani, *J. High Energy Phys.* **0711** (2007) 030 [arXiv:0709.0230 [hep-ph]].

[60] M. Knecht and A. Nyffeler, *Eur. Phys. J.* C **21** (2001) 659 [hep-ph/0106034]; S. Peris *et al.*, *J. High Energy Phys.* **9805** (1998) 011 [hep-ph/9805442]; J. Bijnens *et al.*, *J. High Energy Phys.* **0304** (2003) 055 [hep-ph/0304222]. V. Cirigliano *et al.*, *Nucl. Phys.* B **753** (2006) 139 [hep-ph/0603205].

[61] J. Bijnens and I. Jemos, arXiv:0906.3118 [hep-ph], to be published in Eur. Phys. J. C;

[62] P. Buettiker, S. Descotes-Genon and B. Moussallam, *Eur. Phys. J.* C **33** (2004) 409 [hep-ph/0310283].

[63] S. Pislak *et al.* [BNL-E865], *Phys. Rev. Lett.* **87** (2001) 221801 [hep-ex/0106071]; *Phys. Rev.* D **67** (2003) 072004 [hep-ex/0301040].

[64] J. R. Batley *et al.* [NA48/2 Collaboration], Eur. Phys. J. C **54** (2008) 411; B. Bloch-Devaux, these proceedings.

[65] I. Jemos, these proceedings.

[66] J. Bijnens, N. Danielsson and T. A. Lahde, Acta Phys. Polon. B **38** (2007) 2777 [hep-ph/0701267].

[67] D. G. Sutherland *Phys. Lett.* **23**(1966)384; J. S. Bell and D. G. Sutherland, *Nucl. Phys.* **B4**(1968)315.

[68] J. A. Cronin, *Phys. Rev.* **161** (1967) 1483; H. Osborn and D. J. Wallace, *Nucl. Phys.* **B20** (1970) 23.

[69] J. Gasser and H. Leutwyler, *Nucl. Phys.* **B250** (1985) 539.

[70] W. M. Yao *et al.* [Particle Data Group], *J. Phys.* **G33** (2006) 1.

[71] J. Kambor *et al.*, *Nucl. Phys.* **B465** (1996) 215 [hep-ph/9509374].

[72] A. V. Anisovich and H. Leutwyler, *Phys. Lett.* **B375** (1996) 335 [hep-ph/9601237].

[73] J. Bijnens *et al.*, *Nucl. Phys.* **B427** (1994) 427 [hep-ph/9403390].

[74] F. Ambrosino *et al.* [KLOE], *J. High Energy Phys.* **0805** (2008) 006 [arXiv:0801.2642 [hep-ex]]; A. Abele *et al. Phys. Lett.* **B417** (1998) 197; J. G. Layter *et al.*, *Phys. Rev.* **D7** (1973) 2565; M. Gormley *et al.*, *Phys. Rev.* **D2** (1970) 501.

[75] B. Borasoy and R. Nissler, *Eur. Phys. J.* **A26** (2005) 383 [hep-ph/0510384].

[76] S. Prakhov *et al.*, Phys. Rev. C **79** (2009) 035204 [arXiv:0812.1999 [hep-ex]]; M. Unverzagt *et al.*, Eur. Phys. J. A **39** (2009) 169 [arXiv:0812.3324 [hep-ex]]; C. Adolph *et al.* [WASA-at-COSY Collaboration], arXiv:0811.2763 [nucl-ex]. F. Ambrosino *et al.*arXiv:0707.4137 [hep-ex]; W. B. Tippens *et al. Phys. Rev. Lett.* **87** (2001) 192001; M. Bashkanov *et al.*, Phys. Rev. C **76** (2007) 048201 [arXiv:0708.2014 [nucl-ex]]. A. Abele *et al.*, Phys. Lett. B **417** (1998) 193. . Alde *et al.*, Z. Phys. C **25** (1984) 225 [Yad. Fiz. **40** (1984) 1447]. M. N. Achasov *et al.*, JETP Lett. **73** (2001) 451.

[77] J. Bijnens and J. Gasser, *Phys. Scripta* **T99** (2002) 24 [hep-ph/0202242].

[78] J. M. Flynn and C. T. Sachrajda [RBC Collaboration and UKQCD Collaboration], Nucl. Phys. B **812** (2009) 64 [arXiv:0809.1229 [hep-ph]].

[79] J. Bijnens and A. Celis, arXiv:0906.0302 [hep-ph], to be published in Phys. Lett. B.

[80] J. Gasser, M. E. Sainio and A. Svarc, Nucl. Phys. B **307** (1988) 779.

[81] M. B. Wise, Phys. Rev. D **45** (1992) 2188.

[82] G. Burdman and J. F. Donoghue, Phys. Lett. B **280** (1992) 287.

[83] E. E. Jenkins, A. V. Manohar and M. B. Wise, Phys. Rev. Lett. **75**, 2272 (1995) [arXiv:hep-ph/9506356].

[84] J. Bijnens, P. Gosdzinsky and P. Talavera, JHEP **9801** (1998) 014 [arXiv:hep-ph/9708232].