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Renormalization and additional degrees of freedom within the chiral effective theory for spin-1 resonances

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Abstract

We study in detail various aspects of the renormalization of the spin-1 resonance propagator in the effective field theory framework. First, we briefly review the formalisms for the description of spin-1 resonances in the path integral formulation with the stress on the issue of propagating degrees of freedom. Then we calculate the one-loop 1^{--} meson self-energy within the Resonance chiral theory in the chiral limit using different methods for the description of spin-one particles, namely the Proca field, antisymmetric tensor field and the first order formalisms. We discuss in detail technical aspects of the renormalization procedure which are inherent to the power-counting non-renormalizable theory and give a formal prescription for the organization of both the counterterms and one-particle irreducible graphs. We also construct the corresponding propagators and investigate their properties. We show that the additional poles corresponding to the additional one-particle states are generated by loop corrections, some of which are negative norm ghosts or tachyons. We count the number of such additional poles and briefly discuss their physical meaning.

Contents

1 Introduction **2**

2 Propagators and poles **5**
2.1 Proca formalism 5
2.1.1 General properties of the propagator 5
2.1.2 Additional degrees of freedom in the path integral formalism 8
2.2 Antisymmetric tensor formalism 10
2.2.1 Path integral formulation 12
2.3 First order formalism 15

3 Organization of the counterterms **18**

4 The self-energies at one loop **24**
4.1 The Proca field case 24
4.2 The antisymmetric tensor case 27
4.3 The first order formalism 30
4.4 Note on the counterterms 35

5 From self-energies to propagators **36**
5.1 The number of poles using Argument principle 36
5.2 The Källén-Lehman representation and nature of the poles 39
5.3 Examples of the poles 43

6 Summary and discussion **47**

A Additional degrees of freedom in the path integral - the Proca field **49**

B The additional degrees of freedom in the path integral-the antisymmetric tensor case **52**

C Path integral formulation of the first order formalism **55**

D The parameters α_i and β_i in terms of LECs **58**
D.1 The Proca field case 58
D.2 The antisymmetric tensor case 58
D.3 The first order formalism 59

E Proof of the positivity of the spectral functions **61**

1 Introduction

As is well known, in the low energy region the dynamical degrees of freedom of QCD are not quarks and gluons but the low lying hadronic states and, as a consequence, a non-perturbative description of the their dynamics is inevitable. An approach using effective Lagrangians appears to be very efficient for this purpose and it has made a considerable progress recently. In the very low energy region ($E \ll \Lambda_H \sim 1\text{GeV}$), the octet of the lightest pseudoscalar mesons (π, K, η) represents the only relevant part of the QCD spectrum. The Chiral Perturbation Theory (χPT) [1, 2, 3] based on the spontaneously broken chiral symmetry $SU(3)_L \times SU(3)_R$ grew into a very successful model-independent tool for the description of the Green functions (GF) of the quark currents and related low-energy phenomenology. The pseudoscalar octet is treated as the octet of pseudo-Goldstone bosons (PGB) and χPT is organized according to the Weinberg power-counting formula [1] as a rigorously defined simultaneous perturbative expansion in small momenta and the light quark masses. Recently, the calculations are performed at the next-to-next-to-leading order $O(p^6)$ (for a comprehensive review and further references see [4]).

In the intermediate energy region ($\Lambda_H \leq E < 2\text{GeV}$), where the set of relevant degrees of freedom includes also the low lying resonances, the situation is less satisfactory. This region is not separated by a mass gap from the rest of the spectrum and, as a consequence, there is no appropriate scale playing the role analogous to that of Λ_H in χPT. Therefore, the effective theory in this region cannot be constructed as a straightforward extension of the χPT low energy expansion by means of introducing resonances *e.g.* as homogenously (but nonlinearly) transformed matter fields in the sense of [5], [6] and pushing the scale Λ_H to 2GeV.

In order to introduce another type of effective Lagrangian description, the considerations based on the large N_C expansion together with the high-energy constraints derived from perturbative QCD and OPE appear to be particularly useful. In the limit $N_C \to \infty$, the chiral symmetry is enlarged to $U(3)_L \times U(3)_R$ and the spectrum relevant for the correlators of the quark bilinears consists of an infinite tower of free stable mesonic resonaces exchanged in each channel and classified according to the symmetry group $U(3)_V$. An appropriate description should therefore require an infinite number of resonance fields entering the $U(3)_L \times U(3)_R$ symmetric effective Lagrangian. Because the quasi-classical expansion is correlated with the large N_C expansion, the interaction vertices are suppressed by an appropriate power of $N_C^{-1/2}$ according to the number of the meson legs. At the leading order only the tree graphs have to be taken into account . An approximation to this general picture where we limit the number of the resonance fields to one in each channel and matching the resulting theory in the high energy region with OPE is known as the Resonance Chiral Theory (RχT) (it was introduced in seminal papers [7, 8]). Integrating out the resonance fields from the Lagrangian of RχT in the low energy region and the subsequent matching with χPT has become very successful tool for the estimates of the resonance contribution to the values of the $O(p^4)$ [7] and $O(p^6)$ [9, 10] low energy constants (LEC) entering the χPT Lagrangian. Therefore, studying RχT can help us to understand not only the dynamics of resonances but also the origin of LECs in χPT.

However, even when restricting to the case of the matter field formalism, it is known from the very beginning [8] that the form of the RχT Lagrangian is not determined uniquely. The reason is that the resonances with a given spin can be described in many ways using fields with different Lorentz structure. For example, for the spin-one resonances one can use *i.a.* the Proca vector field or the antisymmetric tensor field or both (within the first order formalism [11, 12]).

Though the theories based on different types of fields with Lagrangians which contain only finite number of operators are not strictly equivalent already on the tree level (in general, it is necessary to include nonlocal interaction or infinite number of operators and contact terms to ensure the complete equivalence, see [12]), we can always ensure a weak equivalence of all three formalisms up to a given fixed chiral order (this was established to $O(p^4)$ in [8] and enlarged to $O(p^6)$ in [12]).

As we have mentioned above, the lack of the mass gap (which could provide us with a scale playing the role analogous to Λ_H) prevents us from using a straightforward extension of the Weinberg power-counting formula [1] taking the resonance masses and momenta of the order $O(p)$ on the same footing as for PGB. Also the usual chiral power counting which takes the resonance masses as an additional heavy scale (which is counted as $O(1)$) fails within the RχT in a way analogous to the χPT with baryons [13]. Nevertheless, it seems to be fully legitimate to go beyond the tree level RχT and calculate the loops [14, 15, 16, 17, 18, 19, 20, 21, 22].

Being suppressed by one power of $1/N_C$, the loops allow to encompass such NLO effects in the $1/N_C$ expansion as resonance widths, resonance cuts and the final state interaction and (by means matching with χPT) to determine the NLO resonance contribution to LEC (and their running with renormalization scale).

However, we can expect both technical and conceptual complications connected with the renormalization of the effective theory for which no natural organization of the expansion (other than the $1/N_C$ counting) exists. Especially, because there is no natural analog of the Weinberg power counting in RχT, we can expect mixing of the naive chiral orders in the process of the renormalization (*e.g* the loops renormalize the $O(p^2)$ LEC and also counterterms of unusually high chiral orders are needed). Also a straightforward construction of the propagator from the self-energy using the Dyson re-summation can bring about the appearance of new poles in the GF. Because the spin-one particles are described using fields transforming under the reducible representation of the rotation group and due to the lack of an appropriate protective symmetry, some of these additional poles can correspond to new degrees of freedom, which are frozen at the tree level. The latter might be felt as a pathological artefact of the not carefully enough formulated theory, particularly because these extra poles might be negative norm ghosts or tachyons [23]. On the other hand, however, we could also try to take an advantage of this feature and to adjust the poles in such a way that they correspond to the well established resonance states [24].

Let us note, that similar problems are generic for the description of the higher spin particles in terms of quantum field theory. As an example we can mention e.g. the problem with the renormalization of quantum gravity which is trying to be cured by imposing additional symmetry or by introducing a non-perturbative quantization believing that UV divergences are only artefact of a perturbative theory. In the context of the extensions of the χPT, this has been studied in connection with introducing of the spin-3/2 isospin-3/2 $\Delta(1232)$ resonance in the baryonic sector (for a review see [25] and references therein). The Rarita-Schwinger field commonly used for its description contains along with the spin-3/2 sector also spin-1/2 sector, which is frozen at the tree level due to the form of the free equations of motion. These provides the necessary constraints reducing the number of propagating spin degrees of freedom to four corresponding to spin 3/2 particles. However, these constraints are generally not present in the interacting theory and negative norm ghost [26] and/or tachyonic [27] poles might appear beyond the tree level. The appearance of these extra unphysical degrees of freedom can be avoided by means of the requirement of additional protective gauge symmetry under which the interaction Lagrangian has to be invariant. Such a symmetry, which is also a symmetry of the kinetic term (but not of the

mass term), is an analog of the $U(1)$ gauge symmetry of the electromagnetic field and its role is also similar. As it has been shown by means of path integral formalism, it leads to the same constraints as in the noninteracting theory and prevent therefore the extra spin-1/2 states from propagating.

On the other hand, it has been proved, that the most general interaction Lagrangian at most bilinear in Rarita-Schwinger field (*i.e.* without the protective gauge symmetry) is on shell equivalent to the gauge invariant one [28] . The latter is, however, nonlocal (or equivalently it contains an infinite number of terms). Also the above protective gauge symmetry is, as a rule, in a conflict with chiral symmetry, and has therefore to be implemented with a care. Though there are efficient methods how to handle this obstacles in concrete loop calculations [25], [28], the problem still has not been solved completely.

In the following, we would like to discuss these problems in more detail. As an explicit example we use the one-loop renormalization of the propagator corresponding to the fields which originally describe 1^{--} vector resonance (ρ meson) at the tree level within the Proca field, the antisymmetric tensor field and within the first order formalism in the chiral limit. The situation here is quite similar to the case of spin-3/2 resonances discussed above. In addition, to the spin-1 degrees of freedom, there are extra sectors that are frozen at the tree level. There exists a protective gauge symmetry which prevents these modes from propagation. The kinetic term is invariant with respect to this symmetry while the mass term is not.

By means of an explicit calculation we will show that (unlike the ordinary χPT) the one-loop corrections to the self-energy need counterterms with a number of derivatives ranging from zero up to six and also that a new kinetic counterterm with two derivatives (which was not present in the tree level Lagrangian) is necessary. We will also demonstrate that the corresponding propagator obtained by means of Dyson re-summation of the one-particle irreducible self-energy insertions has unavoidably additional poles. Due to the unusual higher order growth of the self-energy in the UV region some of them are inevitably pathological (with a negative norm or a negative mass squared). Though these additional poles are decoupled in the limit $N_C \to \infty$, for reasonable concrete values of the parameters of the Lagrangian they might appear near or even inside the region for which $R\chi T$ was originally designed. We also discuss briefly within the antisymmetric tensor formalism a possible interpretation of some of the non-pathological poles as a manifestation of the dynamical generation of various types of additional 1+- states. We will also show that the appropriate adjustment of coupling constants in the antisymmetric tensor case allows us (at least in principle) to generate in this way the one which could be identified *e.g.* with the $b_1(1235)$ meson [24]. Such a mechanism is analogous to the model [29] for the dynamical generation of the scalar resonances from the bare quark-antiquark "seed", the propagator of which develops (after dressing with pseudoscalar meson loops) additional poles identified *e.g.* as $a_0(980)$ (cf. also [30],[31]).

The paper organized as follows. In Section 2 we remind the basic facts about the propagators and briefly discuss the issue of the additional degrees of freedom in all three formalisms for the description of spin-one resonances. We use the path integral formulation where the protective symmetry analogous to the Rarita-Schwinger case is manifest. In Section 3 we discuss the power counting. We try to formulate here a formal self-consistent organization of the counterterms and one-particle irreducible graphs, which sorts the operators in the Lagrangian according to the number of derivatives as well as number of the resonance fields and which is useful for the proof of renormalizability of the $R\chi T$ as an effective theory. In Section 4 we present the results of the explicit calculation of the self-energies. Then we give a list of counterterms and briefly discuss the

renormalization prescription. Section 5 is devoted to the construction of the propagators and to the discussion of their poles. Because the basic ideas are similar within all three formalisms, we concentrate here on the antisymmetric tensor case. Section 6 contains summary and conclusions. Some of the long formulae are postponed to the appendices: the explicit form of the renormalization scale independent parameters of the self-energies are collected in Appendix D, namely for the Proca field in D.1, for the antisymmetric tensor field in D.2 and for the first order formalism in D.3. In Appendix E we give a proof of the positivity of the spectral functions for the antisymmetric tensor propagator.

2 Propagators and poles

In this section, we collect the basic properties of the propagators and the corresponding self-energies within the Proca field, the antisymmetric tensor field and the first order formalisms. The discussion will be as general as possible without explicit references to $R\chi T$, which can be assumed as the special example of the general case.

2.1 Proca formalism

2.1.1 General properties of the propagator

We start our discussion with a standard textbook example of the interacting Proca field. Let us write the Lagrangian in the form

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}, \tag{1}$$

where the free part of the Lagrangian is

$$\mathcal{L}_0 = -\frac{1}{4}\widehat{V}_{\mu\nu}\widehat{V}^{\mu\nu} + \frac{1}{2}M^2 V_\mu V^\mu \tag{2}$$

with

$$\widehat{V}_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu. \tag{3}$$

Without any additional assumptions on the form and symmetries of the interaction part of the Lagrangian $\mathcal{L}_{int}$, we can expect the following general structure of the full two-point one-particle irreducible (1PI) Green function

$$\Gamma^{(2)}_{\mu\nu}(p) = (M^2 - p^2 + \Sigma^T(p^2))P^T_{\mu\nu} + (M^2 + \Sigma^L(p^2))P^L_{\mu\nu}. \tag{4}$$

Here

$$P^L_{\mu\nu} = \frac{p_\mu p_\nu}{p^2} \tag{5}$$

$$P^T_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \tag{6}$$

are the usual longitudinal and transverse projectors and $\Sigma^{T,L}$ are the corresponding transverse and longitudinal self-energies, which vanish in the free field limit. Inverting (4) we get for the full propagator

$$\Delta_{\mu\nu}(p) = -\frac{1}{p^2 - M^2 - \Sigma^T(p^2)}P^T_{\mu\nu} + \frac{1}{M^2 + \Sigma^L(p^2)}P^L_{\mu\nu}. \tag{7}$$

The possible (generally complex) poles of such a propagator are of two types; either at $p^2 = s_V$, where s_V is given by the solutions of

$$s_V - M^2 - \Sigma^T(s_V) = 0, \tag{8}$$

or at $p^2 = s_S$ where s_S is the solution of

$$M^2 + \Sigma^L(s_S) = 0. \tag{9}$$

Let us first discuss the poles of the first type. Assuming that (8) is satisfied for $s_V = M_V^2 > 0$, then for $p^2 \to M_V^2$

$$\begin{aligned} \Delta_{\mu\nu}(p) &= \frac{Z_V}{p^2 - M_V^2}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2}\right) + O(1) \\ &= \frac{Z_V}{p^2 - M_V^2}\sum_\lambda \varepsilon_\mu^{(\lambda)}(p)\varepsilon_\nu^{(\lambda)*}(p) + O(1) \end{aligned} \tag{10}$$

where

$$Z_V = \frac{1}{1 - \Sigma'^T(M_V^2)} \tag{11}$$

and where $\varepsilon_\mu^{(\lambda)}(p)$ are the usual spin-one polarization vectors. Under the condition $Z_V > 0$ the poles of this type correspond to spin-one one particle states $|p, \lambda, V\rangle$ which couple to the Proca field as

$$\langle 0|V_\mu(0)|p, \lambda, V\rangle = {Z_V}^{1/2}\varepsilon_\mu^{(\lambda)}(p). \tag{12}$$

At least one of these states is expected to be perturbative in the sense that its mass and coupling to V_μ can be written as

$$M_V^2 = M^2 + \delta M_V^2 \tag{13}$$

$$Z_V = 1 + \delta Z_V, \tag{14}$$

where δM_V^2 and δZ_V are small corrections vanishing in the free field limit. This solution corresponds to the original degree of freedom described by the free part of the Lagrangian $\mathcal{L}_0$. The additional one particle states corresponding to the other possible (non-perturbative) solutions of (8) decouple in the free field limit.

The second type of poles is given by (intrinsically nonperturbative) solutions of (9). Suppose that this condition is satisfied by $s_S = M_S^2 > 0$. For $p^2 \to M_S^2$

$$\Delta_{\mu\nu}(p) = \frac{Z_S}{p^2 - M_S^2}\frac{p_\mu p_\nu}{M_S^2} + O(1) \tag{15}$$

where

$$Z_S = \frac{1}{\Sigma'^L(M_S^2)}. \tag{16}$$

Assuming $Z_S > 0$ this pole corresponds to the spin-zero one particle state $|p, S\rangle$ which couples to V_μ as

$$\langle 0|V_\mu(0)|p, S\rangle = \mathrm{i}p_\mu\frac{{Z_S}^{1/2}}{M_S}. \tag{17}$$

For the free field this scalar mode is frozen and does not propagate according to the special form of the Proca field Lagrangian. Therefore, in the limit of vanishing interaction the extra scalar state decouples.

Without any additional assumptions on the symmetries of the interaction Lagrangian we can therefore expect the appearance of additional dynamically generated degrees of freedom.

The general picture is, however, more subtle. Note that, the interpretation of the above additional spin-one and spin-zero poles as physical one-particle asymptotic states depends on the proper positive sign of the corresponding residues Z_V, $Z_S > 0$, otherwise the norm of these states is negative and the poles correspond to the negative norm ghosts. Similarly, also poles with $M^2_{V,S} < 0$ can be generated, which correspond to the tachyonic states. Let us illustrate this feature using a toy example. Suppose, that the only interaction terms are of the form

$$\mathcal{L}_{int} \equiv \mathcal{L}_{ct} = -\frac{\alpha}{4}\widehat{V}_{\mu\nu}\widehat{V}^{\mu\nu} - \frac{\beta}{2}(\partial_\mu V^\mu)^2 + \frac{\gamma}{2M^2}(\partial_\mu \widehat{V}^{\mu\nu})(\partial^\rho \widehat{V}_{\rho\nu}) + \frac{\delta}{2M^2}(\partial_\mu\partial_\rho V^\rho)(\partial^\mu\partial_\sigma V^\sigma). \quad (18)$$

Such a Lagrangian can be typically produced by radiative corrections in an effective field theory with Proca field, which does not couple to other fields in a $U(1)$ gauge invariant way, and can provide us with counterterms necessary to renormalize the loops contributing to the V field self-energy. $\mathcal{L}_{ct}$ gives rise to the following contributions to $\Sigma^T(p^2)$ and $\Sigma^L(p^2)$

$$\Sigma^T(p^2) = -\alpha p^2 + \gamma\frac{p^4}{M^2} \quad (19)$$

$$\Sigma^L(p^2) = -\beta p^2 + \delta\frac{p^4}{M^2}. \quad (20)$$

As a result, we have two spin-one and two spin-zero one-particle states. The masses and residue of the spin-one states are then

$$M^2_{V\pm} = M^2\left(1 + \frac{1+\alpha-2\gamma \mp \sqrt{(1+\alpha)^2-4\gamma}}{2\gamma}\right) \quad (21)$$

$$1 - \Sigma'^T(M^2_{V\pm}) = \pm\sqrt{(1+\alpha)^2-4\gamma}, \quad (22)$$

which are real for for $(1+\alpha)^2 - 4\gamma > 0$. In the limit α, $\gamma \to 0$, $\alpha/\gamma = const$ we get either the perturbative solution with mass M_{V+} or (for $\gamma > 0$) an additional spin-one ghost with mass M_{V-} (for $1+\alpha > 0$ and $\gamma < 0$ this pole is tachyonic). Similarly for the spin-zero states

$$M^2_{S\pm} = M^2\left(\frac{\beta \mp \sqrt{\beta^2-4\delta}}{2\delta}\right) \quad (23)$$

$$\Sigma'^L(M^2_{S\pm}) = \mp\sqrt{\beta^2-4\delta}. \quad (24)$$

The poles are real for $\beta^2 > 4\delta$ and *e.g.* for $\beta, \delta > 0$ one of the poles is spin-zero ghost. In both cases for appropriate values of the parameters we can get also two tachyons or even the complex Lee-Wick pair of ghosts. These features are of course well known in the connection with the higher derivative regularization (as well as with the properties of the gauge-fixing term).

2.1.2 Additional degrees of freedom in the path integral formalism

The additional degrees of freedom discussed in the previous subsection can be made manifest in the path integral formalism. Let us start with the generating functional for the interacting Proca field

$$Z[J] = \int \mathcal{D}V \exp\left(\mathrm{i}\int \mathrm{d}^4x \left(-\frac{1}{4}\widehat{V}_{\mu\nu}\widehat{V}^{\mu\nu} + \frac{1}{2}M^2 V_\mu V^\mu + \mathcal{L}_{int}(V, J, \ldots)\right)\right), \tag{25}$$

where the external sources are denoted collectively by J. In order to separate the transverse and longitudinal degrees of freedom of the field V_μ within the path integral we can use the standard Faddeev-Popov trick with respect to the $U(1)$ gauge transformation of the field V_μ

$$V_\mu \to V_\mu + \partial_\mu \Lambda. \tag{26}$$

As a result, we get the generating functional in the form

$$Z[J] = \int \mathcal{D}V_\perp \mathcal{D}\Lambda \exp\left(\mathrm{i}\int \mathrm{d}^4x \left(\frac{1}{2}V_\perp^\mu \Box V_{\perp\mu} + \frac{1}{2}M^2 V_\perp^\mu V_{\perp\mu} + \frac{1}{2}M^2 \partial_\mu\Lambda\partial^\mu\Lambda + \mathcal{L}_{int}(V_\perp - \partial\Lambda, J, \ldots)\right)\right). \tag{27}$$

Here $\mathcal{D}V_\perp = \mathcal{D}V\delta(\partial_\mu V^\mu)$ and

$$V_\perp^\mu = \left(g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{\Box}\right)V_\nu$$

is the transverse part of the vector field V^μ, the longitudinal part of which corresponds to the scalar field Λ, *i.e.*

$$V^\mu = V_\perp^\mu + \partial^\mu\Lambda. \tag{28}$$

The free propagators of the fields $V_\perp^\mu$ and Λ are

$$\Delta_\perp^{\mu\nu}(p) = -\frac{P^{T\,\mu\nu}}{p^2 - M^2} \tag{29}$$

$$\Delta_\Lambda(p) = \frac{1}{M^2}\frac{1}{p^2}. \tag{30}$$

Both these propagators have spurious poles at $p^2 = 0$, however, the only necessary combination which matters in the Feynman graphs is

$$\Delta_0^{\mu\nu}(p) = \Delta_\perp^{\mu\nu}(p) + p^\mu p^\nu \Delta_\Lambda(p), \tag{31}$$

which coincides with the original free propagator of the field V^μ and the spurious poles cancel each other.

Note that, provided the interaction Lagrangian $\mathcal{L}_{int}$ is symmetric under the $U(1)$ gauge transformation (26), the spin-zero field Λ completely decouples and can be integrated out . The theory can then be formulated solely in terms of the field $V_\perp^\mu$. The $U(1)$ invariant form of the interaction allows to simplify the propagator $\Delta_\perp^{\mu\nu}(p)$

$$\Delta_\perp^{\mu\nu}(p) \to -\frac{g_{\mu\nu}}{p^2 - M^2} \tag{32}$$

within the Feynman graphs and the spurious pole $p^2 = 0$ in (29) becomes harmless. In this case, the scalar one-particle states cannot be dynamically generated. On the other hand, in the case

when $\mathcal{L}_{int}$ is not invariant with respect to (26), we cannot forget the longitudinal component of V^μ which has now nontrivial interactions and, as a result, contributions to Σ^L can be generated.

Let us now return to the illustrative example discussed in the previous subsection. Suppose that the interaction Lagrangian has the form

$$\mathcal{L}_{int} = \mathcal{L}_{ct} + \mathcal{L}^{'}_{int} \tag{33}$$

where $\mathcal{L}_{ct}$ is the toy interaction Lagrangian (18) and we assume $\alpha > -1$ and $\delta > 0$ in what follows. Then it is possible to transform $Z[J]$ to the form of the path integral with all the additional degrees of freedom represented explicitly in the Lagrangian and the integration measure. In terms of the transverse and longitudinal degrees of freedom we get

$$\begin{aligned}\mathcal{L}_{int}(V_\perp - \partial\Lambda, J, \ldots) &= \mathcal{L}_{ct}(V_\perp - \partial\Lambda, J, \ldots) + \mathcal{L}^{'}_{int}(V_\perp - \partial\Lambda, J, \ldots) \\ &= \frac{\alpha}{2}V^\mu_\perp \Box V_{\perp\mu} - \frac{\beta}{2}(\Box\Lambda)^2 + \frac{\gamma}{2M^2}(\Box V^\mu_\perp)(\Box V_{\perp\mu}) + \frac{\delta}{2M^2}(\partial_\mu\Box\Lambda)(\partial^\mu\Box\Lambda) \\ &\quad + \mathcal{L}^{'}_{int}(V_\perp - \partial\Lambda, J\ldots).\end{aligned} \tag{34}$$

In order to lower the number of derivatives in the kinetic terms we integrate in auxiliary scalar fields χ, ρ, π, σ and auxiliary transverse vector field $B_{\perp\mu}$ writing *e.g.*

$$\exp\left(-i\int \mathrm{d}^4x \frac{\beta}{2}(\Box\Lambda)^2\right) = \int \mathcal{D}\chi \exp\left(i\int \mathrm{d}^4x \left(\frac{1}{2\beta}\chi^2 - \partial_\mu\chi\partial^\mu\Lambda\right)\right) \tag{35}$$

and similarly for other higher derivative terms. After the superfluous degrees of freedom are identified and integrated out, the fields are re-scaled and then the resulting mass matrix can be diagonalized by means of two symplectic rotations with angles θ_V and θ_S (the technical details are postponed to the Appendix A). Finally we get (under the conditions $(1+\alpha)^2 > 4\gamma$ and $\beta^2 > 4\delta$)

$$Z[J] = \int \mathcal{D}V_\perp \mathcal{D}B_\perp \mathcal{D}\Lambda \mathcal{D}\chi \mathcal{D}\sigma \exp\left(i\int \mathrm{d}^4x \mathcal{L}(V_\perp, B_\perp, \Lambda, \chi, \sigma, J, \ldots)\right) \tag{36}$$

where

$$\begin{aligned}\mathcal{L}(V_\perp, B_\perp, \Lambda, \chi, \sigma, J, \ldots) &= \frac{1}{2}V^\mu_\perp \Box V_{\perp\mu} + \frac{1}{2}M^2_{V+}V^\mu_\perp V_{\perp\mu} - \frac{1}{2}B^\mu_\perp \Box B^\mu_\perp + \frac{1}{2}M^2_{V-}B^\mu_\perp B_{\perp\mu} \\ &\quad + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}M^2_{S+}\sigma^2 - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}M^2_{S-}\chi^2 + \frac{1}{2}M^2\partial_\mu\Lambda\partial^\mu\Lambda \\ &\quad + \mathcal{L}^{'}_{int}(\overline{V}^{(\theta)}, J, \ldots).\end{aligned} \tag{37}$$

and

$$\overline{V}^{(\theta)} = \frac{\exp\theta_V}{(1+\alpha)^{1/2}}(V_\perp + B_\perp) - \partial\chi\cosh\theta_S - \partial\sigma\sinh\theta_S - \partial\Lambda \tag{38}$$

and where $M^2_{V\pm}$, $M^2_{S\pm}$ are the mass eigenvalues (21) and (23). The theory is now formulated in terms of two spin one and two spin zero fields, whereas two of them, namely $B^\mu_\perp$ and χ have a wrong sign of the kinetic terms and are therefore negative norm ghosts. As above, the field Λ does not correspond to any dynamical degree of freedom, its role is merely to cancel the spurious poles of the free propagators of the transverse fields $V_\perp$ and $B_\perp$ at $p^2 = 0$.

2.2 Antisymmetric tensor formalism

For the antisymmetric tensor field in the formalism [7, 8] the situation is quite analogous to the Proca field case so our discussion will be parallel to the previous subsection. Let us write the Lagrangian in the form

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}. \tag{39}$$

where the free part is

$$\mathcal{L}_0 = -\frac{1}{2}(\partial_\mu R^{\mu\nu})(\partial^\rho R_{\rho\nu}) + \frac{1}{4}M^2 R_{\mu\nu}R^{\mu\nu}, \tag{40}$$

and introduce the transverse and longitudinal projectors

$$\Pi^T_{\mu\nu\alpha\beta} = \frac{1}{2}\left(P^T_{\mu\alpha}P^T_{\nu\beta} - P^T_{\nu\alpha}P^T_{\mu\beta}\right) \tag{41}$$

$$\Pi^L_{\mu\nu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta}\right) - \Pi^T_{\mu\nu\alpha\beta} \tag{42}$$

with $P^T_{\mu\alpha}$ given by (6). Again, in analogy with (4), for completely general $\mathcal{L}_{int}$ we can expect the following general form of the full two-point 1PI Green function

$$\Gamma^{(2)}_{\mu\nu\alpha\beta}(p) = \frac{1}{2}(M^2 + \Sigma^T(p^2))\Pi^T_{\mu\nu\alpha\beta} + \frac{1}{2}(M^2 - p^2 + \Sigma^L(p^2))\Pi^L_{\mu\nu\alpha\beta} \tag{43}$$

where $\Sigma^{T,L}$ are the corresponding self-energies. The full propagator is then obtained by means of the inversion of $\Gamma^{(2)}_{\mu\nu\alpha\beta}$ in the form

$$\Delta_{\mu\nu\alpha\beta}(p) = -\frac{2}{p^2 - M^2 - \Sigma^L(p^2)}\Pi^L_{\mu\nu\alpha\beta} + \frac{2}{M^2 + \Sigma^T(p^2)}\Pi^T_{\mu\nu\alpha\beta}. \tag{44}$$

This propagator has two types of poles analogous to (8) and (9), either at $p^2 = s_V$, satisfying

$$s_V - M^2 - \Sigma^L(s_V) = 0, \tag{45}$$

or at $p^2 = s_{\widetilde{V}}$ where

$$M^2 + \Sigma^T(s_{\widetilde{V}}) = 0. \tag{46}$$

Assuming that the solution of (45) satisfies $s_V = M_V^2 > 0$, the propagator behaves at this pole as

$$\begin{aligned}\Delta_{\mu\nu\alpha\beta}(p) &= \frac{Z_V}{p^2 - M_V^2}\frac{p_\mu g_{\nu\alpha}p_\beta - p_\nu g_{\mu\alpha}p_\beta - (\alpha \leftrightarrow \beta)}{M_V^2} + O(1) \\ &= \frac{Z_V}{p^2 - M_V^2}\sum_\lambda u^{(\lambda)}_{\mu\nu}(p)u^{(\lambda)}_{\alpha\beta}(p)^* + O(1)\end{aligned} \tag{47}$$

where

$$Z_V = \frac{1}{1 - \Sigma'^L(M_V^2)} \tag{48}$$

and the wave function $u^{(\lambda)}_{\mu\nu}(p)$ can be expressed in terms of the spin-one polarization vectors $\varepsilon^{(\lambda)}_\nu(p)$ as

$$u^{(\lambda)}_{\mu\nu}(p) = \frac{\mathrm{i}}{M_V}\left(p_\mu \varepsilon^{(\lambda)}_\nu(p) - p_\nu \varepsilon^{(\lambda)}_\mu(p)\right). \tag{49}$$

For $Z_V > 0$ the pole of this type corresponds therefore to the spin-one state $|p,\lambda,V\rangle$ which couples to $R_{\mu\nu}$ as

$$\langle 0|R_{\mu\nu}(0)|p,\lambda,V\rangle = {Z_V}^{1/2} u^{(\lambda)}_{\mu\nu}(p). \tag{50}$$

Analogously to the Proca case, at least one of these poles is expected to be perturbative and corresponds to the original degree of freedom described by the free Lagrangian $\mathcal{L}_0$. This means

$$M_V^2 = M^2 + \delta M_V^2 \tag{51}$$

$$Z_V = 1 + \delta Z_V \tag{52}$$

with small corrections δM_V^2 and δZ_V vanishing in the free field limit. The other possible nonperturbative solutions of (45) decouple in this limit.

Provided there exists a solution of (46) for which $s_{\widetilde{V}} = M^2_{\widetilde{V}} > 0$, we get at this pole

$$\begin{aligned}\Delta_{\mu\nu\alpha\beta}(p) &= \frac{Z_{\widetilde{V}}}{p^2 - M^2_{\widetilde{V}}}\left(g_{\mu\alpha}g_{\nu\beta} + \frac{p_\mu g_{\nu\alpha}p_\beta - p_\mu g_{\nu\beta}p_\alpha}{M_A^2} - (\mu \leftrightarrow \nu)\right) + O(1) \\ &= \frac{Z_{\widetilde{V}}}{p^2 - M^2_{\widetilde{V}}}\sum_\lambda w^{(\lambda)}_{\mu\nu}(p) w^{(\lambda)}_{\alpha\beta}(p)^* + O(1)\end{aligned} \tag{53}$$

where

$$Z_{\widetilde{V}} = \frac{1}{\Sigma'^T(M^2_{\widetilde{V}})} \tag{54}$$

and the wave function is dual to the wave function (49)

$$w^{(\lambda)}_{\mu\nu}(p) = \widetilde{u}^{(\lambda)}_{\mu\nu}(p) = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}u^{(\lambda)\alpha\beta}(p). \tag{55}$$

Provided $Z_{\widetilde{V}} > 0$, the poles of this type correspond to the spin-one particle states $|p,\lambda,\widetilde{V}\rangle$ with the opposite intrinsic parity in comparison with $|p,\lambda,V\rangle$, which couple to the antisymmetric tensor field as

$$\langle 0|R_{\mu\nu}(0)|p,\lambda,\widetilde{V}\rangle = {Z_{\widetilde{V}}}^{1/2} w^{(\lambda)}_{\mu\nu}(p). \tag{56}$$

This degree of freedom is frozen in the free propagator due to the specific form of the free Lagrangian and it decouples in the limit of the vanishing interaction.

As in the Proca field case, we can therefore generally expect dynamically generated additional degrees of freedom, which can be either regular asymptotic states ($M^2_{V,\widetilde{V}}$, $Z_{V,\widetilde{V}} > 0$) or negative norm ghosts ($M^2_{V,\widetilde{V}} > 0$, $Z_{V,\widetilde{V}} < 0$) or tachyons ($M^2_{V,\widetilde{V}} < 0$). Complex poles on the unphysical sheets can be then interpreted as resonances.

As the toy illustration of these possibilities, let us take the interaction Lagrangian similar to (18) in the Proca field case *e.g.* in the form

$$\begin{aligned}\mathcal{L}_{int} \equiv \mathcal{L}_{ct} = &-\frac{\alpha-\beta}{2}(\partial_\mu R^{\mu\nu})(\partial^\rho R_{\rho\nu}) - \frac{\beta}{4}(\partial_\mu R^{\alpha\beta})(\partial^\mu R_{\alpha\beta}) \\ &+\frac{\gamma-\delta}{2M^2}(\partial_\alpha\partial_\mu R^{\mu\nu})(\partial^\alpha\partial^\rho R_{\rho\nu}) + \frac{\delta}{4M^2}(\partial_\rho\partial_\mu R^{\alpha\beta})(\partial^\rho\partial^\mu R_{\alpha\beta}).\end{aligned} \tag{57}$$

We get then the following contributions to the longitudinal and transverse self-energies

$$\Sigma^L(p^2) = -\alpha p^2 + \gamma\frac{p^4}{M^2} \tag{58}$$

$$\Sigma^T(p^2) \;\; = \;\; -\beta p^2 + \delta \frac{p^4}{M^2}. \tag{59}$$

These are exactly the same as (20) and (19) (with the identification $\Sigma^{T,L} \leftrightarrow \Sigma^{L,T}$). Therefore, provided we further identify $M^2_{S\pm} \leftrightarrow M^2_{\tilde{V}\pm}$, the properties of the poles and residues are the same as in the previous subsection (see the discussion after (20) and (19)), with the only exception that instead of the extra spin-zero states with the mass (23) we have now extra spin-one states with the same mass (23) but with the opposite parity in comparison with the original degrees of freedom described by the free lagrangian $\mathcal{L}_0$.

2.2.1 Path integral formulation

We can again made the additional degrees of freedom manifest within the path integral approach in the way parallel to subsection 2.1.2. An analog of the $U(1)$ gauge symmetry used in the case of the Proca field formalism in order to separate the transverse and longitudinal components of the field V_μ is here the following transformation with a pseudovector[1] parameter Λ_α

$$R^{\mu\nu} \to R^{\mu\nu} + \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}, \tag{60}$$

where

$$\widehat{\Lambda}_{\alpha\beta} = \partial_\alpha \Lambda_\beta - \partial_\beta \Lambda_\alpha. \tag{61}$$

This leaves the kinetic term invariant, while the mass term is changed. Note, that the transformation with the parameters Λ_α and Λ^λ_α where

$$\Lambda^\lambda_\alpha = \Lambda_\alpha + \partial_\alpha \lambda \tag{62}$$

are the same. This residual gauge invariance has to be taken into account when using the Faddeev-Popov trick in order to isolate the longitudinal and transverse degrees of freedom of the field $R_{\mu\nu}$. Analog of the formula (28) is now

$$R^{\mu\nu} = R^{\mu\nu}_{\parallel} + \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta} \tag{63}$$

where $R^{\mu\nu}_{\parallel}$ is the longitudinal component of $R_{\mu\nu}$. Its transverse component is described with the transverse component $\Lambda^\mu_\perp$ of the field Λ^μ where

$$\Lambda^\mu = \Lambda^\mu_\perp + \partial^\mu \lambda. \tag{64}$$

Starting with the path integral representation of the generating functional[2]

$$Z[J] = \int \mathcal{D}R \exp\left(\mathrm{i}\int \mathrm{d}^4x \left(-\frac{1}{2}(\partial_\mu R^{\mu\nu})(\partial^\rho R_{\rho\nu}) + \frac{1}{4}M^2 R_{\mu\nu}R^{\mu\nu} + \mathcal{L}_{int}(R^{\mu\nu}, J, \ldots)\right)\right) \tag{65}$$

[1]This is of course true only in the case of the proper tensor field $R_{\mu\nu}$. Provided $R_{\mu\nu}$ is a pseudotensor, the parameter of the transformation is vectorial.

[2]Here J are the external sources, cf. previous subsecrion.

and using the Faddeev-Popov trick twice with respect to the transformations (60) and (62) we finally find for $Z[J]$ the following representation

$$Z[J] = \int \mathcal{D}R_{\parallel}\mathcal{D}\Lambda_{\perp} \exp\left(\mathrm{i}\int \mathrm{d}^4x \mathcal{L}(R^{\mu\nu}_{\parallel}, \Lambda^{\mu}_{\perp}, \ldots)\right) \tag{66}$$

where the integral measure is

$$\mathcal{D}R_{\parallel}\mathcal{D}\Lambda_{\perp} = \mathcal{D}R\mathcal{D}\Lambda\delta(\partial_{\alpha}R_{\mu\nu} + \partial_{\nu}R_{\alpha\mu} + \partial_{\mu}R_{\nu\alpha})\delta(\partial_{\mu}\Lambda^{\mu}) \tag{67}$$

and

$$R^{\mu\nu}_{\parallel} = -\frac{1}{2\Box}(\partial^{\mu}g^{\nu\alpha}\partial^{\beta} + \partial^{\nu}g^{\mu\beta}\partial^{\alpha} - (\mu \leftrightarrow \nu))R_{\alpha\beta} \tag{68}$$

$$\Lambda^{\mu}_{\perp} = \left(g^{\mu\nu} - \frac{\partial^{\mu}\partial^{\nu}}{\Box}\right)\Lambda_{\nu}. \tag{69}$$

are the longitudinal part of the tensor field $R^{\mu\nu}$and the transverse part of the vector field Λ^{μ} (describing the transverse part of the tensor field $R^{\mu\nu}$) respectively[3]. The Lagrangian expressed in these variables reads

$$\mathcal{L}(R^{\mu\nu}_{\parallel}, \Lambda^{\mu}_{\perp}, J, \ldots) = \frac{1}{4}R^{\mu\nu}_{\parallel}\Box R_{\parallel\,\mu\nu} + \frac{1}{4}M^2R^{\mu\nu}_{\parallel}R_{\parallel\,\mu\nu} + \frac{1}{2}M^2\Lambda^{\mu}_{\perp}\Box\Lambda_{\perp\mu} + \mathcal{L}_{int}(R^{\mu\nu}_{\parallel} - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}, J, \ldots). \tag{70}$$

The free propagators of the fields $R^{\mu\nu}_{\parallel}$ and $\Lambda^{\mu}_{\perp}$ are therefore

$$\Delta^{\mu\nu\alpha\beta}_{\parallel}(p) = -\frac{2}{p^2 - M^2}\Pi^{L\,\mu\nu\alpha\beta} \tag{71}$$

$$\Delta^{\mu\nu}_{\perp}(p) = -\frac{1}{M^2}\frac{1}{p^2}P^{T\,\mu\nu} \tag{72}$$

and, similarly to the case of the Proca field, they have spurious poles at $p^2 = 0$. Due to the form of the interaction, however, only the combination

$$\begin{aligned}\Delta^{\mu\nu\alpha\beta}_{0}(p) &= \Delta^{\mu\nu\alpha\beta}_{\parallel}(p) + \varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\kappa\lambda}p_{\rho}p_{\kappa}\Delta_{\perp\,\sigma\lambda}(p) \\ &= -\frac{2}{p^2 - M^2}\Pi^{L\,\mu\nu\alpha\beta} + \frac{2}{M^2}\Pi^{T\,\mu\nu\alpha\beta}\end{aligned} \tag{73}$$

corresponding to the free propagator of the original tensor field $R^{\mu\nu}$ is relevant within the Feynman graphs and the spurious poles cancel. By analogy with the Proca field case, for the interaction Lagrangian invariant with respect to the transformation (60) the field $\Lambda^{\mu}_{\perp}$ completely decouples and can be integrated out. Such a form of the interaction also allows to modify the propagator $\Delta^{\mu\nu\alpha\beta}_{\parallel}(p)$ within the Feynman graphs

$$\Delta^{\mu\nu\alpha\beta}_{\parallel}(p) \to -\frac{g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}}{p^2 - M^2} \tag{74}$$

[3]Note again that, the field Λ_{μ} has opposite parity than the field $R_{\mu\nu}$ (being pseudovector for proper tensor field $R_{\mu\nu}$ and vice versa).

and no spurious pole at $p^2 = 0$ effectively appears. In this case the opposite parity spin-one states discussed in the previous subsection cannot be dynamically generated.

In order to illustrate the appearance of the additional degrees of freedom connected with the interaction Lagrangian (57) within the path integral formalism, we can make the same exercise with the interaction Lagrangian (57) as we did in the previous subsection with (18). Our aim is again to make the additional degrees of freedom explicit in the path integral representation of $Z[J]$. The procedure is almost one-to-one to the case of the Proca fields so that we will be more concise. The technical details can be found in the Appendix B.

We assume the interaction Lagrangian to be of the form

$$\mathcal{L}_{int} = \mathcal{L}_{ct} + \mathcal{L}^{'}_{int}, \tag{75}$$

where $\mathcal{L}_{ct}$ is given by (57) and we assume $\alpha > -1$ and $\delta > 0$ as above. $\mathcal{L}_{int}$ can be then re-express it in terms of the longitudinal and transverse components of the original field $R_{\mu\nu}$

$$\mathcal{L}_{int}(R^{\mu\nu}_{\parallel} - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}, J, \ldots) = \mathcal{L}_{ct}(R^{\mu\nu}_{\parallel} - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}, J, \ldots) + \mathcal{L}^{'}_{int}(R^{\mu\nu}_{\parallel} - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}, J, \ldots) \tag{76}$$

where

$$\begin{aligned}\mathcal{L}_{ct}(R^{\mu\nu} - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}J, \ldots) &= \frac{\alpha}{4}R^{\mu\nu}_{\parallel}\Box R_{\parallel\,\mu\nu} + \frac{\gamma}{4M^2}(\Box R^{\mu\nu}_{\parallel})(\Box R_{\parallel\,\mu\nu}) \\ &+\frac{\beta}{2}(\Box\Lambda^{\mu}_{\perp})(\Box\Lambda_{\perp\mu}) - \frac{\delta}{2M^2}(\partial^{\alpha}\Box\Lambda^{\mu}_{\perp})(\partial_{\alpha}\Box\Lambda_{\perp\mu}).\end{aligned} \tag{77}$$

We then introduce the auxiliary (longitudinal) antisymmetric tensor field $B^{\mu\nu}_{\parallel}$ and (transverse) vector fields $\chi^{\mu}_{\perp}$, $\rho^{\mu}_{\perp}$, $\sigma^{\mu}_{\perp}$ and $\pi^{\mu}_{\perp}$ in order to avoid the higher derivative terms in a complete analogy with the Proca field case. Again, not all the fields correspond to propagating degrees of freedom and such redundant fields can be integrated out. After rescaling the fields and diagonalization of the resulting mass terms by means of two symplectic rotations with angles θ_V and $\theta_{\widetilde{V}}$ exactly as in the case of the Proca fields (see the Appendix B for details) we end up with

$$Z[J] = \int \mathcal{D}R_{\parallel}\mathcal{D}B_{\parallel}\mathcal{D}\Lambda_{\perp}\mathcal{D}\chi_{\perp}\mathcal{D}\rho_{\perp}\mathcal{D}\sigma_{\perp}\mathcal{D}\pi_{\perp}\exp\left(\mathrm{i}\int \mathrm{d}^4x\mathcal{L}(R_{\parallel}, B_{\parallel}, \Lambda_{\perp}, \chi_{\perp}, \rho_{\perp}, \sigma_{\perp}, \pi_{\perp}, J, \ldots)\right) \tag{78}$$

with (cf. (187))

$$\begin{aligned}\mathcal{L} &= \frac{1}{4}R^{\mu\nu}_{\parallel}\Box R_{\parallel\,\mu\nu} + \frac{1}{4}M^2_{V+}R^{\mu\nu}_{\parallel}R_{\parallel\,\mu\nu} \\ &-\frac{1}{4}B^{\mu\nu}_{\parallel}\Box B_{\parallel\,\mu\nu} + \frac{1}{4}M^2_{V-}B^{\mu\nu}_{\parallel}B_{\parallel\,\mu\nu} \\ &+\frac{1}{2}M^2\Lambda^{\mu}_{\perp}\Box\Lambda_{\perp\mu} \\ &-\frac{1}{2}\chi^{\mu}_{\perp}\Box\chi_{\perp\mu} + \frac{1}{2}M^2_{\widetilde{V}-}\chi^{\mu}_{\perp}\chi_{\perp\mu} + \frac{1}{2}\sigma^{\mu}_{\perp}\Box\sigma_{\perp\mu} + \frac{1}{2}M^2_{\widetilde{V}+}\sigma^{\mu}_{\perp}\sigma_{\perp\mu} \\ &+\mathcal{L}_{int}(\overline{R}^{(\theta)}, J, \ldots)\end{aligned} \tag{79}$$

where

$$\overline{R}^{(\theta)\mu\nu} = \frac{\exp\theta_V}{(1+\alpha)^{1/2}}(R^{\mu\nu}_{\parallel} + B^{\mu\nu}_{\parallel}) - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\left(\widehat{\Lambda}_{\alpha\beta} + \widehat{\sigma}_{\perp\alpha\beta}\sinh\theta_{\widetilde{V}} + \widehat{\chi}_{\perp\alpha\beta}\cosh\theta_{\widetilde{V}}\right)$$

and with the diagonal mass terms corresponding to the eigenvalues (21, 23) (with identification $M^2_{\widetilde{V}\pm} \to M^2_{S\pm}$). Again we have two pairs of fields with the opposite signs of the kinetic terms, namely $(R^{\mu\nu}_{\parallel}, B^{\mu\nu}_{\parallel})$ and $(\chi^{\mu}_{\perp}, \sigma^{\mu}_{\perp})$ respectively. As a result we have found four spin-one states, two of them being negative norm ghosts, namely $B^{\mu\nu}_{\parallel}$ and $\sigma^{\mu}_{\perp}$ and two of them with the opposite parity, namely $\chi^{\mu}_{\perp}$ and $\sigma^{\mu}_{\perp}$. As in the Proca field case, the field $\Lambda^{\mu}_{\perp}$ effectively compensates the spurious $p^2 = 0$ poles in the $R^{\mu\nu}_{\parallel}$ and $B^{\mu\nu}_{\parallel}$ propagators within Feynman graphs.

2.3 First order formalism

The first order formalism is a natural alternative to the previous two (for the motivation and details of the quantization see [12], cf. also [11]). It introduces both vector and antisymmetric tensor fields into the Lagrangian, therefore the analysis is a little bit more complex in comparison with previous two cases. In this case, the Lagrangian is of the form

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} \tag{80}$$

where now the free part is

$$\mathcal{L}_0 = MV_\nu \partial_\mu R^{\mu\nu} + \frac{1}{2}M^2 V_\mu V^\mu + \frac{1}{4}M^2 R_{\mu\nu}R^{\mu\nu}. \tag{81}$$

Instead of just one one-particle irreducible two point Green function we have a matrix

$$\Gamma^{(2)}(p) = \begin{pmatrix} \Gamma^{(2)}_{VV}(p)_{\mu\nu} & \Gamma^{(2)}_{VR}(p)_{\alpha\mu\nu} \\ \Gamma^{(2)}_{RV}(p)_{\mu\nu\alpha} & \Gamma^{(2)}_{RR}(p)_{\mu\nu\alpha\beta} \end{pmatrix} \tag{82}$$

where (without any additional assumptions on the form of $\mathcal{L}_{int}$) the matrix elements have the following general form (cf. (4) and (43))

$$\Gamma^{(2)}_{RR}(p)_{\mu\nu\alpha\beta} = \frac{1}{2}(M^2 + \Sigma^T_{RR}(p^2))\Pi^T_{\mu\nu\alpha\beta} + \frac{1}{2}(M^2 + \Sigma^L_{RR}(p^2))\Pi^L_{\mu\nu\alpha\beta} \tag{83}$$

$$\Gamma^{(2)}_{VV}(p)_{\mu\nu} = (M^2 + \Sigma^T_{VV}(p^2))P^T_{\mu\nu} + (M^2 + \Sigma^L_{VV}(p^2))P^L_{\mu\nu} \tag{84}$$

$$\Gamma^{(2)}_{RV}(p)_{\mu\nu\alpha} = \frac{\mathrm{i}}{2}\left(M + \Sigma_{RV}(p^2)\right)\Lambda_{\mu\nu\alpha} \tag{85}$$

$$\Gamma^{(2)}_{VR}(p)_{\alpha\mu\nu} = \frac{\mathrm{i}}{2}\left(M + \Sigma_{VR}(p^2)\right)\Lambda^t_{\alpha\mu\nu}. \tag{86}$$

Here $\Sigma^{T,L}_{RR}(p^2)$, $\Sigma^{T,L}_{VV}(p^2)$ and $\Sigma_{RV}(p^2) = \Sigma_{VR}(p^2)$ are corresponding self-energies and the off-diagonal tensor structures are

$$\Lambda_{\mu\nu\alpha} = -\Lambda^t_{\alpha\mu\nu} = p_\mu g_{\nu\alpha} - p_\nu g_{\mu\alpha}. \tag{87}$$

This matrix of propagators

$$\Delta(p) = \begin{pmatrix} \Delta_{VV}(p)_{\mu\nu} & \Delta_{VR}(p)_{\alpha\mu\nu} \\ \Delta_{RV}(p)_{\mu\nu\alpha} & \Delta_{RR}(p)_{\mu\nu\alpha\beta} \end{pmatrix} \tag{88}$$

can be obtained by means of the inversion of the matrix (82) with the result

$$\Delta_{RR}(p)_{\mu\nu\alpha\beta} = \frac{2}{M^2+\Sigma^T_{RR}(p^2)}\Pi^T_{\mu\nu\alpha\beta} + 2\frac{M^2+\Sigma^T_{VV}(p^2)}{D(p^2)}\Pi^L_{\mu\nu\alpha\beta} \quad (89)$$

$$\Delta_{VV}(p)_{\mu\nu} = \frac{1}{M^2+\Sigma^L_{VV}(p^2)}P^L_{\mu\nu} + \frac{M^2+\Sigma^L_{RR}(p^2)}{D(p^2)}P^T_{\mu\nu} \quad (90)$$

$$\Delta_{RV}(p)_{\mu\nu\alpha} = -\mathrm{i}\frac{M+\Sigma_{RV}(p^2)}{D(p^2)}\Lambda_{\mu\nu\alpha} \quad (91)$$

$$\Delta_{VR}(p)_{\alpha\mu\nu} = -\mathrm{i}\frac{M+\Sigma_{VR}(p^2)}{D(p^2)}\Lambda^t_{\alpha\mu\nu}, \quad (92)$$

where

$$D(p^2) = (M^2+\Sigma^L_{RR}(p^2))(M^2+\Sigma^T_{VV}(p^2)) - p^2(M+\Sigma_{RV}(p^2))(M+\Sigma_{VR}(p^2)). \quad (93)$$

Let us now discuss the structure of the poles, which is now richer than in previous two cases. We have three possible types of poles, namely s_V, $s_{\widetilde{V}}$ and s_S, being solutions of

$$\begin{aligned} D(s_V) &= 0 \\ M^2+\Sigma^T_{RR}(s_{\widetilde{V}}) &= 0 \\ M^2+\Sigma^L_{VV}(s_S) &= 0 \end{aligned} \quad (94)$$

respectively. As far as the pole s_V is concerned, let us assume $s_V = M_V^2 > 0$. We get then at this pole (see also previous two subsections)

$$\Delta_{RR}(p)_{\mu\nu\alpha\beta} = \frac{Z_{RR}}{p^2-M_V^2}\sum_\lambda u^{(\lambda)}_{\mu\nu}(p)u^{(\lambda)}_{\alpha\beta}(p)^* + O(1) \quad (95)$$

$$\Delta_{VV}(p)_{\mu\nu} = \frac{Z_{VV}}{p^2-M_V^2}\sum_\lambda \varepsilon^{(\lambda)}_{\mu}(p)\varepsilon^{(\lambda)*}_{\nu}(p) + O(1) \quad (96)$$

$$\Delta_{RV}(p)_{\mu\nu\alpha} = \frac{Z_{RV}}{p^2-M_V^2}\sum_\lambda u^{(\lambda)}_{\mu\nu}(p)\varepsilon^{(\lambda)}_{\alpha}(p)^* + O(1) \quad (97)$$

$$\Delta_{VR}(p)_{\alpha\mu\nu} = \frac{Z_{VR}}{p^2-M_V^2}\sum_\lambda \varepsilon^{(\lambda)}_{\alpha}(p)u^{(\lambda)*}_{\mu\nu}(p) + O(1) \quad (98)$$

where $u^{(\lambda)}_{\mu\nu}(p)$ is given by (49) and the residue are

$$Z_{RR} = \frac{M^2+\Sigma^T_{VV}(M_V^2)}{D'(M_V^2)} \quad (99)$$

$$Z_{VV} = \frac{M^2+\Sigma^L_{RR}(M_V^2)}{D'(M_V^2)} \quad (100)$$

$$Z_{RV} = \frac{M+\Sigma_{RV}(M_V^2)}{D'(M_V^2)}M_V = Z_{VR} = \frac{M+\Sigma_{VR}(M_V^2)}{D'(M_V^2)}M_V. \quad (101)$$

Note that, as a consequence of (94) we get the following relation

$$Z_{RR}Z_{VV} = Z^2_{RV} = Z^2_{VR}, \quad (102)$$

(remember $\Sigma_{RV}(p^2) = \Sigma_{VR}(p^2)$), therefore assuming $Z_{RR}, Z_{VV} > 0$ the pole $p^2 = M_V^2 > 0$ corresponds to the spin-one one-particle state $|p, \lambda, V\rangle$ which couples to the fields as

$$\begin{aligned} \langle 0|R_{\mu\nu}(0)|p,\lambda,V\rangle &= {Z_{RR}}^{1/2} u^{(\lambda)}_{\mu\nu}(p) \quad (103) \\ \langle 0|V_{\mu}(0)|p,\lambda,V\rangle &= {Z_{VV}}^{1/2} \varepsilon^{(\lambda)}_{\mu}(p). \quad (104) \end{aligned}$$

Again at least one of such states is expected to be perturbative as above and it correspond to the original degree of freedom described by $\mathcal{L}_0$; the others decouple when the interactions is switched off. The other possible poles, $s_S = M_S^2$ and $s_{\widetilde{V}} = M_{\widetilde{V}}^2$ are analogical to the spin-zero and spin-one (opposite parity) states discussed in detail in the previous two subsections; they correspond to the modes which are frozen at the leading order and decouple in the free field limit. As we have already discussed, without further restriction on the form of the interaction, all the additional states can be also negative norm ghosts or tachyons.

Let us illustrate the general case using a toy interaction Lagrangian of the form

$$\begin{aligned} \mathcal{L}_{ct} &= -\frac{\alpha_V}{4}\widehat{V}_{\mu\nu}\widehat{V}^{\mu\nu} - \frac{\beta_V}{2}(\partial_\mu V^\mu)^2 \\ &\quad -\frac{\alpha_R - \beta_R}{2}(\partial_\mu R^{\mu\nu})(\partial^\rho R_{\rho\nu}) - \frac{\beta_R}{4}(\partial_\mu R^{\alpha\beta})(\partial^\mu R_{\alpha\beta}). \end{aligned} \quad (105)$$

This gives

$$\begin{aligned} \Sigma^L_{RR}(p^2) &= -\alpha_R p^2 \\ \Sigma^T_{RR}(p^2) &= -\beta_R p^2 \\ \Sigma^T_{VV}(p^2) &= -\alpha_V p^2 \\ \Sigma^L_{VV}(p^2) &= -\beta_V p^2 \\ \Sigma_{RV}(p^2) &= \Sigma_{VR}(p^2) = 0 \end{aligned} \quad (106)$$

and for $\beta_{V,R} > 0$ the spectrum of one-particle states consists of one spin-zero ghost, one spin-one ghost with opposite parity. Their masses and residue are

$$M_S^2 = \frac{M^2}{\beta_V}, \quad Z_S = -\frac{1}{\beta_V} \quad (107)$$

$$M_{\widetilde{V}}^2 = \frac{M^2}{\beta_R}, \quad Z_{\widetilde{V}} = -\frac{1}{\beta_R} \quad (108)$$

(provided $\beta_R < 0$ or $\beta_V < 0$ the corresponding states are tachyons) and two spin-one states with masses

$$\begin{aligned} M^2_{V\pm} &= M^2 \frac{1 + \alpha_R + \alpha_V \pm \sqrt{\mathcal{D}}}{2\alpha_R\alpha_V} \\ \mathcal{D} &= (1 + \alpha_R + \alpha_V)^2 - 4\alpha_R\alpha_V. \end{aligned} \quad (109)$$

To get both $M^2_{V\pm} > 0$ we need $\mathcal{D} > 0$, $\alpha_V\alpha_R > 0$ and $1 + \alpha_R + \alpha_V > 0$; in this case we get for the residue $Z^{(\pm)}_{RR}$ and $Z^{(\pm)}_{VV}$ at poles $M^2_{V\pm}$

$$\alpha_R Z^{(+)}_{RR} Z^{(-)}_{RR} = \alpha_V Z^{(+)}_{VV} Z^{(-)}_{VV} = \frac{1}{\mathcal{D}} > 0 \quad (110)$$

Assuming $Z_{RR}^{(-)}$, $Z_{VV}^{(-)} > 0$ (note that, for small couplings $M_{V-}^2 = M^2(1 + O(\alpha_R, \alpha_V))$ with $Z_{RR}^{(-)}, Z_{VV}^{(-)} = 1+O(\alpha_R, \alpha_V)$ corresponds to the perturbative solution), the additional spin one-state is either positive norm state for $\alpha_{V,R} > 0$ or ghost for $\alpha_{V,R} < 0$ (in this latter case the extra kinetic terms in $\mathcal{L}_{ct}$ have wrong signs).

Also in this case the propagating degrees of freedom can be made manifest within the path integral formalism. The corresponding discussion is in a sense synthesis of subsections 2.1.2 and 2.2.1 and is postponed to Appendix C.

3 Organization of the counterterms

Let us now return to the concrete case of $R\chi T$. Our aim is to calculate the one loop self-energies defined in the previous section in all three formalisms discussed there. In the process of the loop calculation we are lead to the problem of performing a classification of the countertems, which have to be introduced in order to renormalize infinities. For this purpose, it is convenient to have a scheme, which allows us to assign to each operator in the Lagrangian and to each Feynman graph an appropriate expansion index. Indices of the counterterms, which are necessary in order to cancel the divergences of the given Feynman graph, should be then correlated with the indices of the vertices of the graph as well as with the number of the loops. When we restrict ourselves to the (one-particle irreducible) graphs with a given index, the number of the allowed operators contributing to the graph as well as that of necessary counterterms should be finite.

There are several possibilities how to do it, some of them being quite efficient but purely formal and unphysical, some of them having good physical meaning, but not very useful in practise.

In the literature, several attempts to organize the individual terms of the $R\chi T$ Lagrangian can be found. Let us briefly comment on some of them from the point of view of its applicability to our purpose.

The first one is intimately connected with the effective chiral Lagrangian $\mathcal{L}_{\chi,\mathrm{res}}$ which appears as a result of the (tree-level) integrating out of the resonances from the $R\chi T$. Such a counting assigns to each operator of the resonance part of the $R\chi T$ Lagrangian $\mathcal{L}_{\mathrm{res}}$ a chiral order according to the minimal chiral order of the coupling (LEC) of the effective chiral Lagrangian $\mathcal{L}_{\chi,\mathrm{res}}$ to which the corresponding operator contributes [10], [9]. More generally, in this scheme the chiral order of the operators from $\mathcal{L}_{\mathrm{res}}$ refers to the minimal chiral order of its contribution to the generating functional of the currents $Z[v, a, p, s] = \sum_n Z^{(2n)}[v, a, p, s]$. The loop expansion of $Z[v, a, p, s]$ formally corresponds to the expansion around the classical fields which are solutions of the classical equation of motion. The formal chiral order of the resonance fields corresponds then to the chiral order of the leading term of the expansion of the classical resonance fields in powers of p and external sources according to the standard chiral power counting, *i.e.*

$$V^\mu = O(p^3), \quad R^{\mu\nu} = O(p^2). \tag{111}$$

At the same time, for the resonance mass (which plays a role of the hadronic scale within the standard power counting) we take

$$M = O(1), \tag{112}$$

and for the external sources as usual

$$v^\mu, a^\mu = O(p), \quad \chi, \chi^+ = O(p^2). \tag{113}$$

The resonance propagators are then of the (minimal) order $O(1)$ and the order of the operators which contain the resonance fields is at least $O(p^4)$. This formal power counting therefore restricts both the number of the resonance fields in the generic operator as well as the number of the derivatives. When combined with the large N_C arguments, it allows for the construction of the complete operator basis necessary for the saturation of the LEC's in the chiral Lagrangian at a given chiral order and a leading order in the $1/N_C$ expansion [9].

Originally this type of power counting was designed for the leading order (tree-level) matching of $R\chi T$ and χPT within the large N_C expansion and there is no straightforward extension to the general graph Γ with L loops. The reason is that the above power counting of the resonance propagators inside the loops does not reproduce correctly the standard chiral order of the graph. As a result, the loop graphs violate the naive chiral power counting in a way analogous to the χPT with baryons [13] .

The second possibility applicable to loops is to generalize the Weinberg [1] power counting scheme and *formally* arrange the computation as an expansion in the power of the momenta *and* the resonance masses [32] (though there is no mass gap and no natural scale which would give to such a formal power counting a reasonable physical meaning[4]). Nevertheless, provided we make a following assignment to the resonance field and to the resonance mass M

$$V^\mu, R^{\mu\nu} = O(1), \quad M = O(p) \tag{114}$$

we get for the kinetic and mass term of the resonance field

$$\mathcal{L}_{kin}, \mathcal{L}_{mass} = O(p^2) \tag{115}$$

i.e. the same order as for the lowest order chiral Lagrangian, which allows the same power counting of the resonance propagators as for PGB within the pure χPT. As a result, the Weinberg formula for the order D_Γ of a given graph Γ with L loops built from the vertices with the order D_V,

$$D_\Gamma = 2 + 2L + \sum_V (D_V - 2), \tag{116}$$

remains valid also within $R\chi T$. Note however, that now $p^2/M^2 = O(1)$ and therefore the counterterms needed for renormalization of the graph with chiral order D_Γ might contain more than D_Γ derivatives (this feature is typical for graphs with resonances inside the loops because of the nontrivial numerator of the resonance propagator). Therefore this type of power counting is less useful for the classification of the counterterms than in the case of the pure χPT, where D_Γ gives an upper bound on the number of derivatives of the counterterms needed to renormalize Γ.

There are also some other complications, which depreciate this counting in the case of $R\chi T$. First note that the interaction vertices with the resonance fields can carry a chiral order smaller than two. This applies *e.g.* to the trilinear vertex in the antisymmetric tensor representation

$$\mathcal{O}^{RRR} = \mathrm{i} g_{\rho\sigma} \langle R_{\mu\nu} R^{\mu\rho} R^{\nu\sigma} \rangle \tag{117}$$

[4]Sometimes it is argued [32],[33], that such a counting can be used within the large N_C limit, due to the fact that the natural χPT scale $\Lambda_{\chi PT} = 4\pi F = O(\sqrt{N_C})$ grows with N_C while the masses of the resonances behave as $O(1)$. In fact this results only in the suppression of the loops but generally not in the suppression of the counterterm contributions. In the latter case the expansion is rather controlled by the scale $\Lambda_H \sim M_R = O(1)$, where M_R is the typical mass of the higher resonance in the considered channel not included in truncated Lagrangian corresponding to minimal hadronic ansatz.

or to the odd intrinsic parity vertex mixing the vector and rge antisymmetric tensor field in the first order formalism

$$\mathcal{O}^{RV} = \varepsilon_{\alpha\beta\mu\nu}\langle\{V^\alpha, R^{\mu\nu}\}u^\beta\rangle. \tag{118}$$

Therefore, increasing number of such vertices will decrease the formal chiral order causing again a mismatch between the chiral counting and the loop expansion. Furthermore, such a naive scheme unlike the previous one does not restrict the number of the resonance fields in a general operator because only the number of derivatives, the resonance masses and the external sources score.

The former drawback can be *formally* cured by adding an artificial power of M in front of such operators[5] (or equivalently counting the corresponding couplings as $O(p^2)$ and $O(p)$ respectively) in order to increase artificially their chiral order and preserve the validity of the Weinberg formula, which now can serve as a *formal* tool for the classification of the counterterms. How to treat the latter drawback we will discuss further bellow. Let us, however, stress once again, that there is *no* physical content in such a classification scheme, though it might be technically useful.

Third possibility how to assign an index to the given interaction terms and to the general graphs, independent of the previous two, is offered by the large N_C expansion. In the $N_C \to \infty$ limit, the amplitude of the interaction of the n mesonic resonances is suppressed at least by the factor $O(N_C^{1-n/2})$ and, more generally, the matrix element of arbitrary number of quark currents and n mesons in the initial and final states has the same leading order behavior; *e.g.* for the GB decay constant we get $F = O(N_C^{1/2})$. Because within the chiral building blocks the GB fields always go with the factor $1/F$, we can treat the coupling $c_\mathcal{O}$ corresponding to the operator $\mathcal{O}$ of the $R\chi T$ Lagrangian as $c_\mathcal{O} = O(N_C^{\omega_\mathcal{O}})$, where

$$\omega_\mathcal{O} = 1 - \frac{n_R^\mathcal{O}}{2} - s_\mathcal{O}, \tag{119}$$

$n_R^\mathcal{O}$ is the number of the resonance fields contained in $\mathcal{O}$ and $s_\mathcal{O}$ is a possible additional suppression coming *e.g.* from multiple flavor traces or from the fact, that this coupling appears as a counterterm renomalizing the loop divergences[6]. From such an operator, generally the infinite number of vertices V with increasing number n_{GB}^V of GB legs can be derived, each accompanied with a factor $c_\mathcal{O} F^{-n_{GB}/2}$ and therefore, suppressed as $O(N_C^{\omega_V})$, where the index ω_V is given by[7]

$$\omega_V = 1 - \frac{n_R^\mathcal{O}}{2} - \frac{n_{GB}^V}{2} - s_\mathcal{O}. \tag{120}$$

For a given graph, we have the large N_C behavior $O(N_C^{\omega_\Gamma})$ where[8]

$$\omega_\Gamma = \sum_V \omega_V = 1 - \frac{1}{2}E - L - \sum_\mathcal{O} s_\mathcal{O}, \tag{121}$$

where L is number of the loops, E is the number of external mesonic lines and we have used the identities

$$\sum_V (n_R^V + n_{GB}^V) \;=\; 2I_R + 2I_{GB} + E$$

[5] In the case of $\mathcal{O}^{RV}$ it seems to be natural from the dimensional reason.

[6] Note that, each additional mesonic loop yields a further suppression $1/N_C$, see also bellow.

[7] Here and in what follows we use subscript $\mathcal{O}$ when referring to the operator, while the superscript V corresponds to the concrete vertex derived from the operator $\mathcal{O}$.

[8] Here and in what follows, the sum over $\mathcal{O}$ include all the operators from which the individual vertices entering the graph Γ are derived with necessary multiplicity.

$$I_R + I_{GB} \;=\; L + V - 1 \tag{122}$$

relating L and E with the number of resonance and GB internal lines I_R and I_{GB}. The loop expansion is therefore correlated with the large N_C expansion; higher loops need additionally N_C-suppressed counterterms $\mathcal{O}_{ct}$ with higher $s_{\mathcal{O}_{ct}}$:

$$s_{\mathcal{O}_{ct}} = \left(1 - \frac{1}{2}E\right) - \omega_\Gamma = L + \sum_{\mathcal{O}} s_{\mathcal{O}} \tag{123}$$

Though the formula (121) refers seemingly to individual vertices, reformulated in in the form (123) it points to the members of the chiral symmetric operator basis of the $R\chi T$ Lagrangian. However, as it stays, it does not suit for our purpose because the large N_C counting rules give no restriction for the number of derivatives as well as to the number of resonance fields (once the couplings respect the leading order large N_C behavior described above). The formula (123) expresses merely the the fact that the large N_C expansion coincide with the loop one.

Let us now describe another useful technical way how to classify the couterterms, which could overcome the problems with the above schemes and is in a sense a combination of them. Let us start with the familiar formula for the degree of superficial divergence d_Γ of a given *one particle irreducible* graph Γ, which provides us with the upper bound on the number of derivatives $d_{\mathcal{O}_{ct}}$ in a counterterm $\mathcal{O}_{ct}$ needed for the renormalization of Γ. Because in the Proca and antisymmetric tensor formalisms the spin 1 resonance propagator behaves as[9] $O(1)$ for $p \to \infty$, we get

$$d_{\mathcal{O}_{ct}} \leq d_\Gamma = 4L - 2I_{GB} + \sum_{\mathcal{O}} d_{\mathcal{O}} \tag{124}$$

where $d_{\mathcal{O}}$ means the number of derivatives of the vertex V derived from the operator $\mathcal{O}$. Eliminating I_{GB} in favour of L and I_R and using the identity

$$\sum_{\mathcal{O}} n_R^{\mathcal{O}} = 2I_R + E_R,$$

relating I_R with the number of external resonance lines E_R, we get eventually

$$d_{\mathcal{O}_{ct}} \leq d_\Gamma = 2 + 2L + \sum_{\mathcal{O}} (d_{\mathcal{O}} + n_R^{\mathcal{O}} - 2) - E_R.$$

Adding further to both sides $\sum_{\mathcal{O}}(2n_s^{\mathcal{O}} + 2n_p^{\mathcal{O}} + n_v^{\mathcal{O}} + n_a^{\mathcal{O}})$, the total number of insertions of the external v, a, p and s sources weighted with its chiral order, we have

$$D_{\mathcal{O}_{ct}} + n_R^{ct} - 2 \leq 2L + \sum_{\mathcal{O}} (D_{\mathcal{O}} + n_R^{\mathcal{O}} - 2)$$

where $D_{\mathcal{O}}$ is the usual chiral order (as in pure χPT) of generic operator $\mathcal{O}$. Therefore, introducing an index $i_{\mathcal{O}}$ of a general operator $\mathcal{O}$ as follows[10]

$$i_{\mathcal{O}} = D_{\mathcal{O}} + n_R^{\mathcal{O}} - 2 \tag{125}$$

[9] In the case of the first order formalism, the mixed propagator behaves as $O(p^{-1})$. In this case, $d_\Gamma = 4L - 2I_{GB} - I_{RV} + \sum_{\mathcal{O}} d_{\mathcal{O}}$ where I_{RV} is number of the internal mixed lines. In the following considerations we can take the r.h.s. of (124) as an upper bound on d_Γ with the conclusions unchanged.

[10] Analogous assignment of the chiral order to the interaction terms with at least two resonance fields is proposed in [32], note however, that in this reference it is used by means of substitution $D_V \to i_{\mathcal{O}}$ in the Weinberg formula (116) with counting $M = O(p)$.

we get analog of the Weinberg formula[11], now in the form of an upper bound

$$i_{\mathcal{O}_{ct}} \leq i_\Gamma = 2L + \sum_{\mathcal{O}} i_{\mathcal{O}}. \tag{126}$$

Let us now discuss its properties more closely. First, the number of operators with given $i_{\mathcal{O}} \leq i_{\max}$ is finite, because this requirement limits both the number of derivatives as well as the number of resonance fields. Second, note that, for general operator $\mathcal{O}$ the index $i_{\mathcal{O}} \geq 0$. We have $i_{\mathcal{O}} = 0$ for the leading order χPT Lagrangian, for the resonance mass (counter)terms as well as for the resonance-GB mixing term $\langle A^\mu u_\mu \rangle$ possible for 1^{+-} resonances in the Proca field formalism[12]. The usual interaction terms with one resonance field and $O(p^2)$ building blocks correspond to the sector $i_{\mathcal{O}} = 1$, the same is true for the trilinear resonance vertex (117) as well as for the "mixed" vertex (118), while the two resonance vertices with $O(p^2)$ building blocks correspond to the sector $i_{\mathcal{O}} = 2$, *etc.*

Therefore, according to the formula (126), the loop expansion is correlated with the organization of the operators and loop graphs according to the indices $i_{\mathcal{O}}$ and i_Γ respectively analogously to the pure χPT, with the only exception that also lower sectors of the Lagrangian w.r.t. $i_{\mathcal{O}}$ are renormalized at each step. Therefore, we get the renormalizability provided we limit ourselves to the graphs composed from one-particle ireducible building blocs for which the RHS of (126) is smaller or equal to $i_{\max}$.

The counting rules can be summarized as follows

$$R_{\mu\nu},\, V_\mu = O(p),\, M = O(1) \tag{127}$$

and for the external sources as usual

$$v^\mu, a^\mu = O(p),\ \chi, \chi^+ = O(p^2). \tag{128}$$

Note also that, the index $i_{\mathcal{O}}$ can be rewritten as

$$i_{\mathcal{O}} = D_{\mathcal{O}} - 2\left(1 - \frac{n_R^{\mathcal{O}}}{2}\right) \tag{129}$$

and in the last bracket we recognize the exponent controlling the leading large N_C behavior of the coupling constant in front of the operator $\mathcal{O}$. Remember, however, that the loop induced counterterms have an additional $1/N_C$ suppression for each loop (cf. (121)). Therefore it is natural to modify the index $i_{\mathcal{O}}$ and i_Γ as follows (the coefficient $1/2$ is a matter of convenience, see bellow)

$$\begin{aligned} \widehat{i}_{\mathcal{O}} &= \frac{i_{\mathcal{O}}}{2} + s_{\mathcal{O}} = \frac{1}{2}D_{\mathcal{O}} - \left(1 - \frac{n_R^{\mathcal{O}}}{2} - s_{\mathcal{O}}\right) = \frac{1}{2}D_{\mathcal{O}} - \omega_{\mathcal{O}} \\ \widehat{i}_\Gamma &= \frac{i_\Gamma}{2} + s_\Gamma = L + \sum_{\mathcal{O}} \frac{i_{\mathcal{O}}}{2} + s_\Gamma = 2L + \sum_{\mathcal{O}} \widehat{i}_{\mathcal{O}} \end{aligned} \tag{130}$$

[11] This can be recovered for $n_R^{\mathcal{O}} = 0$, when the inequality changes to the equality.

[12] Note however, that this term can be removed by means of the field redefinition.

where $\omega_{\mathcal{O}}$ is given by (119) and we have used (123) in the last line. With such a modified indices $\widehat{i}_{\mathcal{O}}$, $\widehat{i}_\Gamma$ the formula (126) has the form

$$\widehat{i}_{\mathcal{O}_{ct}} \leq \widehat{i}_\Gamma = 2L + \sum_{\mathcal{O}} \widehat{i}_{\mathcal{O}} \tag{131}$$

The content of this redefinition of $i_{\mathcal{O}}$ is evident: the operators are now classified according to the combined derivative and large N_C expansion according to the counting rules (for pure χPT introduced in [34], [35], [36])

$$p = O(\delta^{1/2}),\ v, a = O(\delta^{1/2}),\ \chi, \chi^+ = O(\delta),\ \frac{1}{N_C} = O(\delta) \tag{132}$$

In what follows we shall use for the classification of the counterterms and for the organization of our calculation the index $i_{\mathcal{O}}$ given by (125) and (126). Note however, that these formulae similarly to the previous cases, do not have much of physical content and serve only as a *formal* tool for the proof of the renormalizability and for the ordering of the counterterms. Namely, the index i_Γ which is by construction related to the superficial degree of the divergence (and which applies to one-particle irreducible graphs only) does not reflect the infrared behavior of the (one-particle irreducible) graph Γ, rather it refers to its ultraviolet properties.

Note also, that the hierarchy of the contributions to the GF by means of fixing i_Γ for *one-particle irreducible building blocks*[13] might appear to be unusual. For instance, let us assume the antisymmetric tensor formalism. Taking then $i_\Gamma = 0$ allows only the tree graphs with vertices from pure $O(p^2)$ chiral Lagrangian with resonaces completely decoupled (the only $i_{\mathcal{O}} = 0$ relevant term with resonance fields is the resonance mass terms) and such a case is therefore equivalent to the LO χPT. When fixing $i_\Gamma \leq 1$, also the terms linear in the resonance fields (at least in the antisymmetric tensor formalism, where the linear sources start at $O(p^2)$) can be used as the one-particle irreducible building blocks and again only the tree graphs are in the game. However, the resonance propagator is still derived from the mass terms only. Therefore, summing up all the tree graphs with resonance internal lines leads then effectively to the contributions equivalent to those of the pure $O(p^4)$ χPT operators with $O(p^4)$ LEC saturated with the resonances in the usual way[14]. Because the resonance kinetic term has $i_{\mathcal{O}} = 2$, the resonances start to propagate only when we take $i_\Gamma \leq 2$. At this level we recover the complete NLO χPT as a part of the theory (including the loop graphs) supplemented with tree graphs built from the free resonance propagators and vertices with $i_{\mathcal{O}} \leq 2$. As far as the resonance part of the Lagrangian is concerned, these vertices coincide with the $O(p^6)$ vertices in the first type of power counting we have considered in the beginning of this section (where we assumed $R_{\mu\nu} = O(p^2)$, see (111)) but also the four resonance term without derivatives is allowed. The resonance loops start to contribute at $i_\Gamma \leq 3$ (with the resonance tadpoles) and $i_\Gamma \leq 4$ (with the pure resonance bubbles). In order to renormalize the corresponding divergences, plethora of new counterterms with increasing number of resonances as well as increasing order of the chiral building blocks is needed. In what follows we will encounter graphs with $i_\Gamma = 6$ (the mixed GB and resonance bubbles) for which we will need counterterms up to the index $i_{\mathcal{O}} \leq 6$.

[13] That means at a given level $i_{\max}$ we allow for all the graphs with one-particle irreducible building blocks satisfying $i_\Gamma \leq i_{\max}$. This point of view is crucial in order to preserve the symmetric properties of the corresponding GF.

[14] Here we tacitly assume that the trilinear term without derivatives has been removed by means of field redefinition, cf. [9, 10].

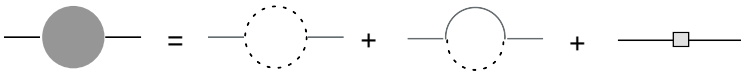

Figure 1: The one-loop graphs contributing to the self-energy of the Proca field. The dotted and full lines corresponds to the Goldstone boson and resonance propagators respectively. Both one-loop graphs have $i_\Gamma = 6$

4 The self-energies at one loop

In this section we present the main result of our paper, namely the one-loop self-energies within all three formalisms discussed in the Section 2 in the chiral limit. In what follows, the loops are calculated within the dimensional regularization scheme. In order to avoid complications with the $d-$dimensional Levi-Civita tensor, we use its simplest variant known as Dimensional reduction, *i.e.* we perform the four-dimensional tensor algebra first in order to reduce the tensor integrals to scalar ones and only then we continue to d dimensions.

4.1 The Proca field case

Our starting point is the following Lagrangian for 1^{--} resonances [37] (see also [38])

$$\begin{aligned}\mathcal{L}_V &= -\frac{1}{4}\langle \widehat{V}_{\mu\nu}\widehat{V}^{\mu\nu}\rangle + \frac{1}{2}M^2\langle V_\mu V^\mu\rangle \\ &\quad -\frac{\mathrm{i}}{2\sqrt{2}}g_V\langle \widehat{V}^{\mu\nu}[u_\mu,u_\nu]\rangle + \frac{1}{2}\sigma_V\varepsilon_{\alpha\beta\mu\nu}\langle\{V^\alpha,\widehat{V}^{\mu\nu}\}u^\beta\rangle + \ldots \end{aligned} \tag{133}$$

where we have written down explicitly only the terms contributing to the self-energy. Originally it was constructed to encompass terms up to the order $O(p^6)$ within the chiral power-counting (111, 112). In the large N_C limit the couplings behave as $g_V = O(N_C^{1/2})$ and $\sigma_V = O(N_C^{-1/2})$. This suggests that the odd intrinsic parity terms are of higher order, however the vertices relevant for our calculations have the same order $O(N_C^{-1})$ in both cases due to the presence of the factor $1/F = O(N_C^{-1/2})$ which accompanies each Goldstone bosons field. In the above Lagrangian the operators shown explicitly have no more than two derivatives and two resonance fields. Therefore, because the interaction terms are $O(p^2)$ we would expect (by analogy with the χPT power counting) the counterterms necessary to cancel the divergencies of the one-loop graphs to have four derivatives at most. However, the nontrivial structure of the free resonance propagator (namely the presence of the P_L part) results in the failure of this naive expectation. In fact, according to (125) and (126), the operators in (133) have index up to $i_\mathcal{O} \leq 2$, whereas the Feynman graphs corresponding to the self-energies $\Sigma_{L,T}$ (depicted in Fig. 1) $i_\Gamma = 6$. In order to cancel the infinite part of the loops we have therefore to introduce a set of counterterms with two resonance fields and indices[15] $i_\mathcal{O} \leq 6$, namely

$$\begin{aligned}\mathcal{L}_V^{ct} &= \frac{1}{2}M^2 Z_M\langle V_\mu V^\mu\rangle + \frac{Z_V}{4}\langle \hat{V}_{\mu\nu}\hat{V}^{\mu\nu}\rangle - \frac{Y_V}{2}\langle (D_\mu V^\mu)^2\rangle \\ &\quad + \frac{X_{V1}}{4}\langle\{D_\alpha,D_\beta\}V_\mu\{D^\alpha,D^\beta\}V^\mu\rangle + \frac{X_{V2}}{4}\langle\{D_\alpha,D_\beta\}V_\mu\{D^\alpha,D^\mu\}V^\beta\rangle\end{aligned}$$

[15]Note that, for these counterterms the index $i_\mathcal{O}$ coincides with the usual chiral order $D_\mathcal{O}$.

$$+\frac{X_{V3}}{4}\langle\{D_\alpha,D_\beta\}V^\beta\{D^\alpha,D^\mu\}V_\mu\rangle+\frac{X_{V4}}{2}\langle D^2V_\mu\{D^\mu,D^\beta\}V_\beta\rangle+X_{V5}\langle D^2V_\mu D^2V^\mu\rangle$$
$$+\mathcal{L}_V^{ct(6)}. \tag{134}$$

Here the last term accumulates the operators with six derivatives ($i_\mathcal{O}=6$), which we do not write down explicitly. The bare couplings are split into a finite part renormalized at a scale μ and a divergent part. The infinite parts of the bare couplings are fixed according to

$$\begin{aligned}
Z_M &= Z_M^r(\mu)\\
Z_V &= Z_V^r(\mu)+\frac{80}{3}\left(\frac{M}{F}\right)^2\sigma_V^2\lambda_\infty\\
X_V &= X_V^r(\mu)-\frac{80}{9}\left(\frac{M}{F}\right)^2\sigma_V^2\frac{1}{M^2}\lambda_\infty\\
Y_V &= Y_V^r(\mu)\\
X_V^{'} &= X_V^{'r}(\mu)
\end{aligned}$$

where

$$\begin{aligned}
X_V^r(\mu) &= X_{V1}^r(\mu)+X_{V5}^r(\mu)\\
X_V^{'r}(\mu) &= X_{V1}^r(\mu)+X_{V2}^r(\mu)+X_{V3}^r(\mu)+X_{V4}^r(\mu)+X_{V5}^r(\mu),
\end{aligned}$$

and

$$\lambda_\infty=\frac{\mu^{d-4}}{(4\pi)^2}\left(\frac{1}{d-4}-\frac{1}{2}(\ln 4\pi-\gamma+1)\right).$$

The result can be written in the form (in the following formulae $x=s/M^2$)

$$\begin{aligned}
\Sigma_T^r(s) &= M^2\left(\frac{M}{4\pi F}\right)^2\left[\sum_{i=0}^3\alpha_i x^i-\frac{1}{2}g_V^2\left(\frac{M}{F}\right)^2x^3\widehat{B}(x)-\frac{40}{9}\sigma_V^2(x-1)^2x\widehat{J}(x)\right]\\
\Sigma_L^r(s) &= M^2\left(\frac{M}{4\pi F}\right)^2\sum_{i=0}^3\beta_i x^i
\end{aligned}$$

In the above formulae α_i and β_i can be expressed in terms of the renormalization scale independent combinations of the counterterm couplings and χlogs. The explicit formulae are collected in the Appendix D.1. The functions $\widehat{B}(x)$ and $\widehat{J}(x)$ correspond to the vacuum bubbles with two Goldstone boson lines or with one Goldstone boson and one resonance line respectively. On the first (physical) sheet,

$$\begin{aligned}
\widehat{B}(x) &= \widehat{B}^I(x)=1-\ln(-x)\\
\widehat{J}(x) &= \widehat{J}^I(x)=\frac{1}{x}\left[1-\left(1-\frac{1}{x}\right)\ln(1-x)\right],
\end{aligned} \tag{135}$$

where we take the principal branch of the logarithm ($-\pi<\mathrm{Im}\ln x\leq\pi$) with cut for $x<0$. On the second sheet we have then $\widehat{B}^{II}(x-\mathrm{i}0)=\widehat{B}^I(x+\mathrm{i}0)=\widehat{B}^I(x-\mathrm{i}0)+2\pi\mathrm{i}$ and similarly for $\widehat{J}(x)$, therefore

$$\widehat{B}^{II}(x) = \widehat{B}^I(x)+2\pi\mathrm{i}$$

$$\widehat{J}^{II}(x) \;=\; \widehat{J}^{I}(x) + \frac{2\pi \mathrm{i}}{x}\left(1-\frac{1}{x}\right). \tag{136}$$

The equation for the pole in the 1^{--} channel

$$s - M^2 - \Sigma_T(s) = 0$$

has a perturbative solution corresponding to the original 1^{--} vector resonance, which develops a mass correction and a finite width of the order $O(1/N_C)$ due to the loops. This solution can be written in the form $\overline{s} = M^2_{\rm phys} - \mathrm{i} M_{\rm phys}\Gamma_{\rm phys}$ where

$$\begin{aligned} M^2_{\rm phys} &= M^2 + \mathrm{Re}\Sigma_T(M^2) = M^2\left[1+\left(\frac{M}{4\pi F}\right)^2\left(\sum_{i=0}^{3}\alpha_i - \frac{1}{2}g_V^2\left(\frac{M}{F}\right)^2\right)\right] \\ M_{\rm phys}\Gamma_{\rm phys} &= -\mathrm{Im}\Sigma_T(M^2) = M^2\left(\frac{M}{4\pi F}\right)^2\frac{1}{2}g_V^2\left(\frac{M}{F}\right)^2\pi \end{aligned}$$

which gives a constraint on the values of α_i's

$$M^2_{\rm phys} + \frac{1}{\pi}M_{\rm phys}\Gamma_{\rm phys} = M^2\left[1+\left(\frac{M}{4\pi F}\right)^2\sum_{i=0}^{3}\alpha_i\right]$$

and in terms of the physical mass and the width we have then

$$\begin{aligned} \Sigma_T^r(s) &= M^2_{\rm phys}\left(\frac{M_{\rm phys}}{4\pi F}\right)^2\left[\sum_{i=0}^{3}\alpha_i x^i - \frac{40}{9}\sigma_V^2(x-1)^2 x\widehat{J}(x)\right] - \frac{1}{\pi}M_{\rm phys}\Gamma_{\rm phys}x^3\widehat{B}(x) \\ \Sigma_L^r(s) &= M^2_{\rm phys}\left(\frac{M_{\rm phys}}{4\pi F}\right)^2\sum_{i=0}^{3}\beta_i x^i. \end{aligned}$$

For further numerical estimates it is convenient to adopt the on shell renormalization prescription demanding $M^2 = M^2_{\rm phys}$ and also to identify F with F_π (because $F = F_\pi$ at the leading order). This gives

$$\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}} = \left(\frac{M}{4\pi F}\right)^2\sum_{i=0}^{3}\alpha_i$$

and, introducing parameters a_i, b_i with natural size $O(1)$

$$\begin{aligned} a_i &= \pi\frac{M_{\rm phys}}{\Gamma_{\rm phys}}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\alpha_i \sim O(1) \\ b_i &= \pi\frac{M_{\rm phys}}{\Gamma_{\rm phys}}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\beta_i \sim O(1) \end{aligned}$$

we get in this scheme for $\sigma^r_{T,L}(x) = M^{-2}_{\rm phys}\Sigma^r_{T,L}(M^2_{\rm phys}x)$

$$\begin{aligned} \sigma_T^r(x) &= \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left(1+\sum_{i=1}^{3}a_i(x^i-1) - x^3\widehat{B}(x)\right) - \frac{40}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\sigma_V^2(x-1)^2x\widehat{J}(x) \\ \sigma_L^r(x) &= \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\sum_{i=0}^{3}b_i x^i. \end{aligned}$$

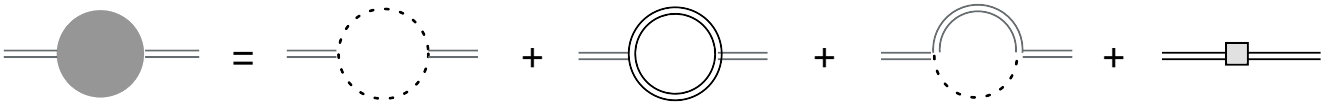

Figure 2: The one-loop graphs contributing to the self-energy of the antisymmetric tensor field. The dotted and double lines correspond to the Goldstone boson and resonance propagators respectively. The GB and pure resonance bubbles have $i_\Gamma = 4$, while the "mixed" one has $i_\Gamma = 6$

4.2 The antisymmetric tensor case

We start with the following Lagrangian for 1^{--} resonances (here only the terms relevant for the one-loop selfenergy are shown explicitly)

$$\begin{aligned}
\mathcal{L}_R &= -\frac{1}{2}\langle D_\mu R^{\mu\nu} D^\alpha R_{\alpha\nu}\rangle + \frac{1}{4}M^2\langle R^{\mu\nu}R_{\mu\nu}\rangle \\
&+ \frac{iG_V}{2\sqrt{2}}\langle R^{\mu\nu}[u_\mu, u_\nu]\rangle + d_1\varepsilon_{\mu\nu\alpha\sigma}\langle D_\beta u^\sigma \{R^{\mu\nu}, R^{\alpha\beta}\}\rangle \\
&+ d_3\varepsilon_{\rho\sigma\mu\lambda}\langle u^\lambda\{D_\nu R^{\mu\nu}, R^{\rho\sigma}\}\rangle + d_4\varepsilon_{\rho\sigma\mu\alpha}\langle u_\nu\{D^\alpha R^{\mu\nu}, R^{\rho\sigma}\}\rangle \\
&+ \mathrm{i}\lambda^{VVV}\langle R_{\mu\nu}R^{\mu\rho}R^{\nu\sigma}\rangle + \ldots
\end{aligned} \tag{137}$$

Note that, in the large N_C limit the coupling G_V behaves as $G_V = O(N_C^{1/2})$, whereas $d_i = O(1)$ and $\lambda^{VVV} = O(N_C^{-1/2})$. Apparently the intrinsic parity odd part and the trilinear resonance coupling are thus of higher order. However, the trilinear vertices contributing to the one-loop self-energies are $O(N_C^{-1/2})$ in both cases due to the appropriate power of $1/F = O(N_C^{-1/2})$ accompanying u_α. Therefore, the operators with two and three resonance fields cannot be got rid of using the large N_C arguments. Also nonzero d_i are required in order to satisfy the OPE constraints for VVP GF at the LO; especially for d_3 we get [39]

$$d_3 = -\frac{N_C}{64\pi^2}\left(\frac{M}{F_V}\right)^2 + \frac{1}{8}\left(\frac{F}{F_V}\right)^2 \tag{138}$$

where F_V is the strength of the resonance coupling to the vector current.

The Lagrangian (137) includes terms up to the index $i_\mathcal{O} \leq 2$. The one-loop Feynman graphs contributing to the self-energy are depicted in Fig. 2. The first two bubbles include only interaction vertices with $i_\mathcal{O} = 1$ and therefore they have indices $i_\Gamma = 4$ while the third one is built from vertices with $i_\mathcal{O} = 2$ and has the index $i_\Gamma = 6$. In order to cancel the infinite part of the loops we have then to add counterterms with indices $i_\mathcal{O} \leq 6$, namely the following set

$$\begin{aligned}
\mathcal{L}_R^{ct} &= \frac{1}{4}M^2 Z_M\langle R^{\mu\nu}R_{\mu\nu}\rangle + \frac{1}{2}Z_R\langle D_\alpha R^{\alpha\mu}D^\beta R_{\beta\mu}\rangle + \frac{1}{4}Y_R\langle D_\alpha R^{\mu\nu}D^\alpha R_{\mu\nu}\rangle \\
&+\frac{1}{4}X_{R1}\langle D^2 R^{\mu\nu}\{D_\nu, D^\sigma\}R_{\mu\sigma}\rangle + \frac{1}{8}X_{R2}\langle\{D_\nu, D_\alpha\}R^{\mu\nu}\{D^\sigma, D^\alpha\}R_{\mu\sigma}\rangle \\
&+\frac{1}{8}X_{R3}\langle\{D^\sigma, D^\alpha\}R^{\mu\nu}\{D_\nu, D_\alpha\}R_{\mu\sigma}\rangle \\
&+\frac{1}{4}W_{R1}\langle D^2 R^{\mu\nu}D^2 R_{\mu\nu}\rangle + \frac{1}{16}W_{R2}\langle\{D^\alpha, D^\beta\}R^{\mu\nu}\{D_\alpha, D_\beta\}R_{\mu\nu}\rangle
\end{aligned}$$

$$+\mathcal{L}_R^{ct(6)}, \tag{139}$$

where the last term accumulates the operators with six derivatives ($i_{\mathcal{O}} = 6$), which we do not write down explicitly. The infinite parts of the bare couplings are fixed as

$$\begin{aligned}
Z_M &= Z_M^r(\mu) + \frac{80}{3}\left(\frac{M}{F}\right)^2 d_1^2\lambda_\infty - 60\left(\frac{\lambda^{VVV}}{M}\right)^2\lambda_\infty \\
Z_R &= Z_R^r(\mu) + \frac{40}{9}\left(\frac{M}{F}\right)^2 \frac{1}{M^2}(12d_1(d_3+d_4) - d_3^2 - 9d_4^2 + 6d_3d_4)\lambda_\infty \\
&\quad +80\left(\frac{\lambda^{VVV}}{M}\right)^2 \frac{1}{M^2}\lambda_\infty \\
Y_R &= Y_R^r(\mu) + \frac{40}{9}\left(\frac{M}{F}\right)^2 \frac{1}{M^2}(6d_1^2 - 12d_1(d_3+d_4) + 5d_3^2 + 9d_4^2 - 6d_3d_4)\lambda_\infty \\
&\quad -40\left(\frac{\lambda^{VVV}}{M}\right)^2 \frac{1}{M^2}\lambda_\infty \\
X_R &= X_R^r(\mu) + \frac{40}{9}\left(\frac{M}{F}\right)^2 \frac{1}{M^2}(d_3^2 - 6d_3d_4 + 5d_4^2)\lambda_\infty - \left(\frac{G_V}{F}\right)^2 \frac{1}{M^2}\lambda_\infty \\
W_R &= W_R^r(\mu) + \frac{40}{9}\left(\frac{M}{F}\right)^2 \frac{1}{M^4}(d_3^2 + 6d_3d_4 - 5d_4^2)\lambda_\infty - 10\left(\frac{\lambda^{VVV}}{M}\right)^2 \frac{1}{M^4}\lambda_\infty
\end{aligned}$$

where

$$\begin{aligned}
X_R^r(\mu) &= X_{R1}^r(\mu) + X_{R2}^r(\mu) + X_{R3}^r(\mu) \\
W_R^r(\mu) &= W_{R1}^r(\mu) + W_{R2}^r(\mu).
\end{aligned}$$

An explicit calculation gives for the renormalized self-energies (in the following formulae $x = s/M^2$)

$$\begin{aligned}
\Sigma_L^r(s) &= M^2\left(\frac{M}{4\pi F}\right)^2\left[\sum_{i=0}^{3}\alpha_i x^i - \left(\frac{1}{2}\left(\frac{G_V}{F}\right)^2 x^2\widehat{B}(x) + \frac{40}{9}d_3^2(x^2-1)^2\widehat{J}(x)\right)\right] \\
&\quad -5\left(\frac{\lambda^{VVV}}{4\pi}\right)^2 (x-4)(x+2)\overline{J}(x) \\
\Sigma_T^r(s) &= M^2\left(\frac{M}{4\pi F}\right)^2\left[\sum_{i=0}^{3}\beta_i x^i + \frac{20}{9}\left(2d_3^2 + (d_3^2 + 6d_3d_4 + d_4^2)x + 2d_4^2x^2\right)(x-1)^2\widehat{J}(x)\right] \\
&\quad +5\left(\frac{\lambda^{VVV}}{4\pi}\right)^2 (x^2 - 2x + 4)\overline{J}(x).
\end{aligned}$$

Here the functions $\widehat{B}(x)$ and $\widehat{J}(x)$ are same as in the previous subsection and $\overline{J}(x)$ is given on the physical sheet by

$$\overline{J}(x) = \overline{J}^I(x) = 2 + \sqrt{1-\frac{4}{x}}\ln\frac{\sqrt{1-\frac{4}{x}}-1}{\sqrt{1-\frac{4}{x}}+1}.$$

with the same branch of the logarithm as before. On the second sheet we have $\overline{J}^{II}(x-\mathrm{i}0) = \overline{J}^{I}(x+\mathrm{i}0) = \overline{J}^{I}(x-\mathrm{i}0) + 2\mathrm{i}\pi\sqrt{1-4/x}$ and therefore

$$\overline{J}^{II}(x) = \overline{J}^{I}(x) + 2\mathrm{i}\pi\sqrt{1-\frac{4}{x}}.$$

The explicit dependence of the renormalization scale invariant polynomial parameters α_i and β_i on the counterterm couplings and χlogs are given in the Appendix D.2.

In order to simplify the following discussion we put $\lambda^{VVV} = 0$ in the rest of this subsection. This is in accord with the fact, that the corresponding trilinear interaction term can be effectively removed by resonance field redefinition [9]. Also, the two-resonance cut starts at $x = 4$ which is far from the region we are interested in. Here the effect of the resonance bubble can be effectively absorbed to the polynomial part of the self-energies.

The equation for the propagator poles in the 1^{--} channel

$$s - M^2 - \Sigma_L(s) = 0$$

has an approximative perturbative solution corresponding to the original 1^{--} vector resonance, which develops a mass correction and a finite width of the order $O(1/N_C)$ due to the loops. This solution can be written in the form $\overline{s} = M^2_{\rm phys} - \mathrm{i}M_{\rm phys}\Gamma_{\rm phys}$ where

$$\begin{aligned} M^2_{\rm phys} &= M^2 + \mathrm{Re}\Sigma_L(M^2) = M^2\left[1 + \left(\frac{M}{4\pi F}\right)^2\left(\sum_{i=0}^{3}\alpha_i - \frac{1}{2}\left(\frac{G_V}{F}\right)^2\right)\right] \\ M_{\rm phys}\Gamma_{\rm phys} &= -\mathrm{Im}\Sigma_L(M^2) = M^2\left(\frac{M}{4\pi F}\right)^2\frac{1}{2}\left(\frac{G_V}{F}\right)^2\pi, \end{aligned}$$

which gives a constraint on the values of α_i's

$$M^2_{\rm phys} + \frac{1}{\pi}M_{\rm phys}\Gamma_{\rm phys} = M^2\left(1 + \frac{1}{(4\pi)^2}\left(\frac{M}{F}\right)^2\sum_{i=0}^{3}\alpha_i\right).$$

This allows us to re-parameterize perturbatively $\Sigma_L(s)$ in terms of $M_{\rm phys}$ and $\Gamma_{\rm phys}$ as

$$\begin{aligned} \Sigma^r_L(s) &= M^2_{\rm phys}\left(\frac{M_{\rm phys}}{4\pi F}\right)^2\left[\sum_{i=0}^{3}\alpha_i x^i - \frac{40}{9}d_3^2(x^2-1)^2\widehat{J}(x)\right] - \frac{1}{\pi}\Gamma_{\rm phys}M_{\rm phys}x^2\widehat{B}(x) \\ \Sigma^r_T(s) &= M^2_{\rm phys}\left(\frac{M_{\rm phys}}{4\pi F}\right)^2\left[\sum_{i=0}^{3}\beta_i x^i + \frac{20}{9}\left(2d_3^2 + (d_3^2 + 6d_3d_4 + d_4^2)x + 2d_4^2x^2\right)(x-1)^2\widehat{J}(x)\right]. \end{aligned}$$

As for the Proca field case, within the on shell renormalization prescription $M^2 = M^2_{\rm phys}$ and we get a constraint

$$\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}} = \left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\sum_{i=0}^{3}\alpha_i. \tag{140}$$

As a result, we can re-write the self-energy (in the units of $M^2_{\rm phys}$, *i.e.* as in the previous section $\sigma^r_{T,L}(x) = M^{-2}_{\rm phys}\Sigma^r_{T,L}(M^2_{\rm phys}x)$ in what follows) in the form

$$\sigma^r_L(x) = \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left[1 - x^2\widehat{B}(x) + \sum_{i=1}^{3} a_i(x^i - 1)\right] - \frac{40}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 d_3^2(x^2-1)^2\widehat{J}(x)$$

using the re-scaled parameters a_i with a natural size $O(1)$

$$a_i = \pi\frac{M_{\rm phys}}{\Gamma_{\rm phys}}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\alpha_i \sim O(1).$$

So that the $\Sigma^r_L(s)$ has four independent parameters α_i, $i = 1, 2, 3$ and d_3. Similarly, $\Sigma^r_T(s)$ can be written in this scheme in terms of six independent dimensionless parameters b_i , d_3 and γ

$$\begin{aligned} b_i &= \pi\frac{M_{\rm phys}}{\Gamma_{\rm phys}}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\beta_i \sim O(1) \\ \gamma &= d_4/d_3 \sim O(1) \end{aligned}$$

as

$$\sigma^r_T(x) = \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\sum_{i=0}^{3} b_i x^i + \frac{20}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 d_3^2\left(2 + (1 + 6\gamma + \gamma^2)x + 2\gamma^2x^2\right)(x-1)^2\widehat{J}(x).$$

In order to satisfy the OPE constraints for VVP correlator [39], we have to put further (according to (138))

$$d_3 = -\frac{3}{4}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\left(\frac{F_\pi}{F_V}\right)^2\left[1 - \frac{1}{6}\left(\frac{4\pi F_\pi}{M_{\rm phys}}\right)^2\right] \tag{141}$$

which reduces the number of the independent parameters for $\sigma^r_L(x)$ and $\sigma^r_T(x)$ to three and five respectively.

4.3 The first order formalism

In this case, the interaction part of the Lagrangian describing 1^{--} resonances collects all the terms from the previous two formalisms. It contains also one extra term which mixes the the fields $R_{\mu\nu}$ and V_α

$$\begin{aligned} \mathcal{L}_{RV} &= \frac{1}{2}M^2\langle V_\mu V^\mu\rangle + \frac{1}{4}M^2\langle R^{\mu\nu}R_{\mu\nu}\rangle - \frac{1}{2}\langle R^{\mu\nu}\widehat{V}_{\mu\nu}\rangle \\ &\quad -\frac{\rm i}{2\sqrt{2}}g_V\langle\widehat{V}^{\mu\nu}[u_\mu, u_\nu]\rangle + \frac{1}{2}\sigma_V\varepsilon_{\alpha\beta\mu\nu}\langle\{V^\alpha, \widehat{V}^{\mu\nu}\}u^\beta\rangle \\ &\quad +\frac{iG_V}{2\sqrt{2}}\langle R^{\mu\nu}[u_\mu, u_\nu]\rangle + d_1\varepsilon_{\mu\nu\alpha\sigma}\langle D_\beta u^\sigma\{R^{\mu\nu}, R^{\alpha\beta}\}\rangle \\ &\quad +d_3\varepsilon_{\rho\sigma\mu\lambda}\langle u^\lambda\{D_\nu R^{\mu\nu}, R^{\rho\sigma}\}\rangle + d_4\varepsilon_{\rho\sigma\mu\alpha}\langle u_\nu\{D^\alpha R^{\mu\nu}, R^{\rho\sigma}\}\rangle \\ &\quad +\frac{1}{2}M\sigma_{RV}\varepsilon_{\alpha\beta\mu\nu}\langle\{V^\alpha, R^{\mu\nu}\}u^\beta\rangle + {\rm i}\lambda^{VVV}\langle R_{\mu\nu}R^{\mu\rho}R^{\nu\sigma}\rangle + \dots \end{aligned}$$

Figure 3: The extra one-loop graphs contributing to the vector field self-energy of in the first order formalism. The dotted and double lines corresponds to the Goldstone boson and antisymmetric tensor field propagators respectively, the thick line stay symbolically for the "mixed" propagator.

Because the free diagonal propagators are the same as in the pure Proca or antisymmetric tensor cases, all the graphs depicted in the Figs. 1, 2 contribute also here to the diagonal self-energies Σ_{RR} and Σ_{VV}. The mixed vertex and mixed propagator generate additional graphs contributing to Σ_{RR}, Σ_{VV} and Σ_{RV} which are depicted in the Figs. 3, 4 and 5 respectively (in the latter case also the GB bubble contributes).

Figure 4: The extra one-loop graphs contributing to the antisymmetric tensor field self-energy in the first order formalism. The meaning of the various types of lines is the same as in the previous figures.

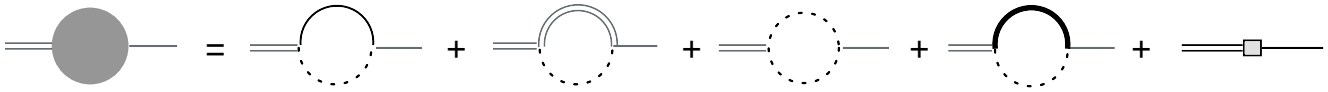

Figure 5: The one-loop graphs contributing to the "mixed" self-energy in the first order formalism.

Similarly, the set of counterterms necessary to renormalize the infinities includes all the terms (134) and (139) and additional mixed terms

$$
\begin{aligned}
\mathcal{L}^{ct}_{RV} \;=\; & \frac{1}{2}M^2 Z_{MV}\langle V_\mu V^\mu\rangle + \frac{Z_V}{4}\langle \hat{V}_{\mu\nu}\hat{V}^{\mu\nu}\rangle - \frac{Y_V}{2}\langle (D_\mu V^\mu)^2\rangle \\
& + \frac{X_{V1}}{4}\langle \{D_\alpha, D_\beta\}V_\mu \{D^\alpha, D^\beta\}V^\mu\rangle + \frac{X_{V2}}{4}\langle \{D_\alpha, D_\beta\}V_\mu \{D^\alpha, D^\mu\}V^\beta\rangle \\
& + \frac{X_{V3}}{4}\langle \{D_\alpha, D_\beta\}V^\beta \{D^\alpha, D^\mu\}V_\mu\rangle + \frac{X_{V4}}{2}\langle D^2 V_\mu \{D^\mu, D^\beta\}V_\beta\rangle + X_{V5}\langle D^2 V_\mu D^2 V^\mu\rangle \\
& + \frac{1}{4}M^2 Z_{MR}\langle R^{\mu\nu}R_{\mu\nu}\rangle + \frac{1}{2}Z_R\langle D_\alpha R^{\alpha\mu}D^\beta R_{\beta\mu}\rangle + \frac{1}{4}Y_R\langle D_\alpha R^{\mu\nu}D^\alpha R_{\mu\nu}\rangle \\
& + \frac{1}{4}X_{R1}\langle D^2 R^{\mu\nu}\{D_\nu, D^\sigma\}R_{\mu\sigma}\rangle + \frac{1}{8}X_{R2}\langle \{D_\nu, D_\alpha\}R^{\mu\nu}\{D^\sigma, D^\alpha\}R_{\mu\sigma}\rangle \\
& + \frac{1}{8}X_{R3}\langle \{D^\sigma, D^\alpha\}R^{\mu\nu}\{D_\nu, D_\alpha\}R_{\mu\sigma}\rangle \\
& + \frac{1}{4}W_{R1}\langle D^2 R^{\mu\nu}D^2 R_{\mu\nu}\rangle + \frac{1}{16}W_{R2}\langle \{D^\alpha, D^\beta\}R^{\mu\nu}\{D_\alpha, D_\beta\}R_{\mu\nu}
\end{aligned}
$$

$$-\frac{1}{2}Z_{RV}M\langle R^{\mu\nu}\widehat{V}_{\mu\nu}\rangle+\frac{1}{2}X_{RV1}M\langle D^{\alpha}R^{\mu\nu}D_{\alpha}\widehat{V}_{\mu\nu}\rangle+\frac{1}{2}X_{RV2}M\langle D_{\mu}R^{\mu\nu}D^{\sigma}\widehat{V}_{\sigma\nu}$$
$$+\mathcal{L}_{RV}^{ct(6)}$$

Now the infinite parts of the bare couplings have to be fixed as follows

$$\begin{aligned}
Z_{RV} &= Z_{RV}^{r}(\mu)-\frac{20}{3}\left(\frac{M}{F}\right)^{2}(\sigma_{RV}+2\sigma_{V})(2d_{1}-\sigma_{RV})\lambda_{\infty}\\
X_{RV} &= X_{RV}^{r}(\mu)-\frac{20}{9}\left(\frac{M}{F}\right)^{2}\frac{1}{M^{2}}(\sigma_{RV}+2\sigma_{V})(4d_{3}+\sigma_{RV})\lambda_{\infty}\\
Z_{MV} &= Z_{MV}^{r}(\mu)\\
Z_{V} &= Z_{V}^{r}(\mu)+\frac{20}{3}\left(\frac{M}{F}\right)^{2}\left(\sigma_{RV}(\sigma_{RV}+2\sigma_{V})+4\sigma_{V}^{2}\right)\lambda_{\infty}\\
X_{V} &= X_{V}^{r}(\mu)-\frac{20}{9}\left(\frac{M}{F}\right)^{2}\frac{1}{M^{2}}\left(\sigma_{RV}(\sigma_{RV}+2\sigma_{V})+4\sigma_{V}^{2}\right)\lambda_{\infty}\\
Y_{V} &= Y_{V}^{r}(\mu)\\
X_{V}^{'} &= X_{V}^{'r}(\mu)\\
Z_{MR} &= Z_{MR}^{r}(\mu)+\frac{20}{3}\left(\frac{M}{F}\right)^{2}(4d_{1}^{2}-\sigma_{RV}(\sigma_{RV}-2d_{1}))\lambda_{\infty}\\
Z_{R} &= Z_{R}^{r}(\mu)+\frac{40}{9}\left(\frac{M}{F}\right)^{2}(12d_{1}(d_{3}+d_{4})-d_{3}^{2}-9d_{4}^{2}+6d_{3}d_{4})\lambda_{\infty}+\frac{10}{9}\sigma_{RV}(10d_{3}+18d_{4}+\sigma_{RV})\lambda_{\infty}\\
Y_{R} &= Y_{R}^{r}(\mu)+\frac{10}{9}\left(\frac{M}{F}\right)^{2}\frac{1}{M^{2}}(24d_{1}^{2}-48d_{1}(d_{3}+d_{4})+20d_{3}^{2}+36d_{4}^{2}\\
&\quad -24d_{3}d_{4}-\sigma_{RV}^{2}+2\sigma_{RV}(d_{3}+3d_{4}))\lambda_{\infty}\\
X_{R} &= X_{R}^{r}(\mu)+\frac{40}{9}\left(\frac{M}{F}\right)^{2}\frac{1}{M^{2}}(d_{3}^{2}-6d_{3}d_{4}+5d_{4}^{2})\lambda_{\infty}\\
&\quad -\frac{10}{9}\frac{1}{M^{2}}\sigma_{RV}(6(d_{3}+d_{4})-\sigma_{RV})\lambda_{\infty}-\left(\frac{G_{V}}{F}\right)^{2}\frac{1}{M^{2}}\lambda_{\infty}\\
W_{R} &= W_{R}^{r}(\mu)+\frac{40}{9}\left(\frac{M}{F}\right)^{2}\frac{1}{M^{4}}(d_{3}^{2}+6d_{3}d_{4}-5d_{4}^{2})\lambda_{\infty}+\frac{10}{9}\sigma_{RV}(\sigma_{RV}-2(d_{3}+3d_{4}))\lambda_{\infty}
\end{aligned}$$

where

$$\begin{aligned}
X_{V}^{r}(\mu) &= X_{V1}^{r}(\mu)+X_{V5}^{r}(\mu)\\
X_{V}^{'r}(\mu) &= X_{V1}^{r}(\mu)+X_{V2}^{r}(\mu)+X_{V3}^{r}(\mu)+X_{V4}^{r}(\mu)+X_{V5}^{r}(\mu)\\
X_{R}^{r}(\mu) &= X_{R1}^{r}(\mu)+X_{R2}^{r}(\mu)+X_{R3}^{r}(\mu)\\
W_{R}^{r}(\mu) &= W_{R1}^{r}(\mu)+W_{R2}^{r}(\mu)\\
X_{RV}^{r}(\mu) &= X_{RV1}^{r}(\mu)+X_{RV2}^{r}(\mu)
\end{aligned}$$

The renormalized self-energies can be then written in the form

$$\Sigma_{RV}(s)^{r} = M\left(\frac{M}{4\pi F}\right)^{2}\left[\sum_{i=0}^{2}\alpha_{i}^{RV}x^{i}+\frac{1}{2}\frac{g_{V}G_{V}}{M}\left(\frac{M}{F}\right)^{2}x^{2}\widehat{B}(x)\right.$$

$$
\begin{aligned}
&+\frac{10}{9}(\sigma_{RV}+2\sigma_V)(2d_3x+2d_3-\sigma_{RV})(x-1)^2\widehat{J}(x)\Big] = \Sigma_{VR}(s)^r\\
\Sigma^T_{VV}(s)^r &= M^2\left(\frac{M}{4\pi F}\right)^2\left[\sum_{i=0}^{3}\alpha_i^{VV}x^i-\frac{1}{2}g_V^2\left(\frac{M}{F}\right)^2x^3\widehat{B}(x)\right.\\
&\left.-\frac{10}{9}\left(\sigma_{RV}(\sigma_{RV}+2\sigma_V)+4\sigma_V^2\right)(x-1)^2x\widehat{J}(x)\right]\\
\Sigma^L_{VV}(s)^r &= M^2\left(\frac{M}{4\pi F}\right)^2\sum_{i=0}^{3}\beta_i^{VV}x^i\\
\Sigma^L_{RR}(s)^r &= M^2\left(\frac{M}{4\pi F}\right)^2\left[\sum_{i=0}^{3}\alpha_i^{RR}x^i-\frac{1}{2}\left(\frac{G_V}{F}\right)^2x^2\widehat{B}(x)\right.\\
&\left.-\frac{10}{9}(4d_3^2(x+1)^2-2d_3\sigma_{RV}(x+1)+\sigma_{RV}^2)(x-1)^2\widehat{J}(x)\right]\\
\Sigma^T_{RR}(s)^r &= M^2\left(\frac{M}{4\pi F}\right)^2\left[\sum_{i=0}^{3}\beta_i^{RR}x^i+\frac{5}{9}(8d_3^2-4\sigma_{RV}d_3+2\sigma_{RV}^2\right.\\
&\left.+(4d_3^2-2\sigma_{RV}d_3+\sigma_{RV}^2+24d_3d_4+4d_4^2-6\sigma_{RV}d_4)x+8d_4^2x^2)(x-1)^2\widehat{J}(x)\right].
\end{aligned}
$$

Here again the renormalization scale independent coefficients of the polynomial parts of the self-energies are expressed in terms of the couplings and chiral logs; the explicit formulae can be found in the Appendix D.3.

The equation for the poles in the 1^{--} channel

$$D(s) = (M^2+\Sigma^L_{RR}(s))(M^2+\Sigma^T_{VV}(s)) - s(M+\Sigma_{RV}(s))(M+\Sigma_{VR}(s)) = 0$$

can be solved perturbatively writing the solution in the form $\overline{s} = M^2_{\rm phys} - {\rm i}M_{\rm phys}\Gamma_{\rm phys} = M^2+\Delta$. To the first order in Δ and the self-energies we get then

$$\overline{s} = M^2+\Sigma^L_{RR}(M^2)+\Sigma^T_{VV}(M^2) - M(\Sigma_{RV}(M^2)+\Sigma_{VR}(M^2))$$

and therefore

$$
\begin{aligned}
M^2_{\rm phys} &= M^2+{\rm Re}\left[\Sigma^L_{RR}(M^2)+\Sigma^T_{VV}(M^2)-M(\Sigma_{RV}(M^2)+\Sigma_{VR}(M^2))\right]\\
&= M^2\left[1+\left(\frac{M}{4\pi F}\right)^2\left(\sum_{i=0}^{3}(\alpha_i^{RR}+\alpha_i^{VV})-2\sum_{i=0}^{2}\alpha_i^{RV}-\frac{1}{2}\left(\frac{M}{F}\right)^2\left(g_V+\frac{G_V}{M}\right)^2\right)\right]\\
M_{\rm phys}\Gamma_{\rm phys} &= -{\rm Im}\left[\Sigma^L_{RR}(M^2)+\Sigma^T_{VV}(M^2)-M(\Sigma_{RV}(M^2)+\Sigma_{VR}(M^2))\right]\\
&= \pi M^2\left(\frac{M}{4\pi F}\right)^2\frac{1}{2}\left(\frac{M}{F}\right)^2\left(g_V+\frac{G_V}{M}\right)^2
\end{aligned}
$$

which yield the constraint

$$M^2_{\rm phys}+\frac{1}{\pi}M_{\rm phys}\Gamma_{\rm phys} = M^2\left(1+\left(\frac{M}{4\pi F}\right)^2\left(\sum_{i=0}^{3}(\alpha_i^{RR}+\alpha_i^{VV})-2\sum_{i=0}^{2}\alpha_i^{RV}\right)\right).$$

In the on-shell scheme $M^2 = M^2_{\rm phys}$we get further

$$\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}} = \left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 \left(\sum_{i=0}^{3}(\alpha_i^{RR}+\alpha_i^{VV}) - 2\sum_{i=0}^{2}\alpha_i^{RV}\right)$$

On the contrary to the previous two cases, this allows to exclude both the constants g_V and G_V in favor of the physical observables only for the combination

$$\begin{aligned}
\sigma(x) &\equiv x\sigma^L_{RR}(x) + \sigma^T_{VV}(x) - x(\sigma_{RV}(x)+\sigma_{VR}(x)) \\
&= \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left(1+\sum_{i=0}^{4}a_i(x-1)^i - x^3\widehat{B}(x)\right) \\
&\quad -\frac{20}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 x(x-1)^2\widehat{J}(x)\left[d_3(x+1)(2d_3(x+1)+\sigma_{RV}+4\sigma_V)+\sigma_V(\sigma_{RV}-2\sigma_V)\right]
\end{aligned}$$

(here $\Sigma^{Lr}_{RR} = M^2\sigma^L_{RR}$, $\Sigma^r_{RV} = M\sigma_{RV}$ etc.), where

$$a_i = \pi\frac{M_{\rm phys}}{\Gamma_{\rm phys}}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\left(\alpha^{RR}_{i-1}+\alpha^{VV}_i - 2\alpha^{RV}_{i-1}\right)$$

with $\alpha^{RR}_{-1} = \alpha^{RV}_{-1} = 0$ are parameters of order $O(1)$,

From the OPE constraints applied to VVP correlator within the first order formalism we get further

$$\begin{aligned}
d_3 &= -\frac{N_C}{64\pi^2}\left(\frac{M}{F_V}\right)^2 + \frac{1}{8}\left(\frac{F}{F_V}\right)^2 + \frac{1}{2}(\sigma_{RV}+\sigma_V) \\
&= -\frac{3}{4}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\left(\frac{F_\pi}{F_V}\right)^2\left[1-\frac{1}{6}\left(\frac{4\pi F_\pi}{M_{\rm phys}}\right)^2\right] + \frac{1}{2}(\sigma_{RV}+\sigma_V)
\end{aligned}$$

Using dimensionless variables, we can write the condition for the poles in the form

$$(1+\sigma^L_{RR}(x))(1+\sigma^T_{VV}(x)) - x(1+\sigma_{RV}(x))(1+\sigma_{VR}(x)) = 0$$

in the 1^{--} channel and

$$\begin{aligned}
1+\sigma^T_{RR}(x) &= 0 \\
1+\sigma^L_{VV}(x) &= 0
\end{aligned}$$

in the 1^{+-} and 0^{+-} channels respectively. Within the on-shell scheme

$$\begin{aligned}
\sigma_{RV}(s)^r &= \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left(\sum_{i=0}^{2}a^{RV}_i x^i - (1-C)Cx^2\widehat{B}(x)\right) \\
&\quad +\frac{10}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\left[(\sigma_{RV}+2\sigma_V)(2d_3x+2d_3-\sigma_{RV})(x-1)^2\widehat{J}(x)\right] \\
\sigma^T_{VV}(s)^r &= \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left(\sum_{i=0}^{3}a^{VV}_i x^i + C^2x^3\widehat{B}(x)\right)
\end{aligned}$$

$$
\begin{aligned}
&-\frac{10}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\left[\left(\sigma_{RV}(\sigma_{RV}+2\sigma_V)+4\sigma_V^2\right)(x-1)^2x\widehat{J}(x)\right]\\
\sigma^L_{VV}(s)^r &= \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\sum_{i=0}^{3}b_i^{VV}x^i\\
\sigma^L_{RR}(s)^r &= \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left(\sum_{i=0}^{3}a_i^{VV}x^i+(1-C)^2x^3\widehat{B}(x)\right)\\
&\quad-\frac{10}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\left[(4d_3^2(x+1)^2-2d_3\sigma_{RV}(x+1)+\sigma_{RV}^2)(x-1)^2\widehat{J}(x)\right]\\
\sigma^T_{RR}(s)^r &= \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\sum_{i=0}^{3}b_i^{RR}x^i+\frac{5}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\left[(8d_3^2-4\sigma_{RV}d_3+2\sigma_{RV}^2\right.\\
&\quad\left.+(4d_3^2-2\sigma_{RV}d_3+\sigma_{RV}^2+24d_3d_4+4d_4^2-6\sigma_{RV}d_4)x+8d_4^2x^2)(x-1)^2\widehat{J}(x)\right].
\end{aligned}
$$

and

$$
\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}C^2=\frac{1}{2}g_V^2\left(\frac{M_{\rm phys}}{F_\pi}\right)^2M_{\rm phys}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2
$$

and the other parameters are of natural size $O(1)$ with the constraint

$$
\sum_{i=0}^{3}(a_i^{RR}+a_i^{VV})-2\sum_{i=0}^{2}a_i^{RV}=\sum_{i=0}^{4}a_i=1.
$$

4.4 Note on the counterterms

Let us note, that the counterterm Lagrangians (134), (139) and (142) might be further simplified using the leading order equations of motion (EOM) in order to eliminate the terms with more then two derivatives as it has been done *e.g.* in [16]. However, this does not mean, that we do not need to introduce such counterterms at all. As we have proved by means of the above explicit calculations, without the higher derivative counterterms (or equivalently without the couterterms proportional to the EOM) we would not have the off-shell self-energies finite.

In fact, the infinities originating in the missing EOM-proportional counterterms are not always dangerous. Note *e.g.*, that such infinities are in fact harmless, provided we restrict our treatment to strict one-loop contribution to the GF of quark bilinears or to the corresponding on-shell S-matrix elements. Namely, in this case, the one-loop generating functional of the GF is obtained by means of the Gaussian functional integration of the quantum fluctuations around the solution of the lowest order EOM. As a result, the EOM can be safely used to simplify the infinite part of the one-loop generating functional. On the strict one-loop level the infinite parts of the self-energy subgraphs corresponding to the missing EOM-proportional counterterms cancel with similar infinities stemming from the vertex corrections.

Nevertheless, already at the one-loop level these counterterms might be necessary under some conditions. Namely, near the resonance poles we can (and in fact have to) go beyond the strict one-loop expansion *e.g.* by means of the Dyson resumation of the one-loop self-energy contributions to the propagator. This will generally destroy such a compensation of infinities. This is the reason why we keep the counterterm Lagrangian in the general form (134), (139) and (142).

5 From self-energies to propagators

In the previous sections we have given the explicit form of the self-energies in a given approximation within all three formalisms for the description of the spin-1 resonances. Here we would like to discuss interpretation of these results and the construction of the corresponding propagators. We will concentrate on the most frequently used antisymmetric tensor representation, where all the characteristic features of other approaches are visible without unsubstantial technical complications. The remaining two cases can be discussed along the same lines with similar results.

Let us remind the form of the self-energies for the antisymmetric tensor case

$$\sigma_L^r(x) = \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left[1 - x^2\widehat{B}(x) + \sum_{i=1}^{3} a_i(x^i - 1)\right] - \frac{40}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 d_3^2(x^2-1)^2\widehat{J}(x) \tag{142}$$

$$\sigma_T^r(x) = \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\sum_{i=0}^{3} b_i x^i + \frac{20}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 d_3^2\left(2 + (1+6\gamma+\gamma^2)x + 2\gamma^2x^2\right)(x-1)^2\widehat{J}(x), \tag{143}$$

where d_3 is given by (138) and where we have already re-parametrized the general result in terms of the parameters of the perturbative solution of the pole equation in the 1^{--} channel (which we have identified with the original degree of freedom). In doing that we have tacitly assumed the validity of the general relation between the self-energies and the propagator (44). The equations determining the additional poles of the propagators are then

$$f_L(x) \equiv x - 1 - \sigma_L^r(x) = 0 \tag{144}$$

$$f_T(x) \equiv 1 + \sigma_T^r(x) = 0. \tag{145}$$

In what follows we shall discuss these equations in more detail. We will find a lower and upper bound on the number of their solutions and give a proof, that the corresponding lover bounds are greater than one on both sheets. We will also briefly discuss the compatibility of the relation (44) with the Källén-Lehman representation and show, that at least one of the roots of (6) and (145) corresponds inevitably either to the negative norm ghost or the tachyon.

5.1 The number of poles using Argument principle

Let us first briefly discuss a determination of the number of solution of the equations (6) and (145). This can be made using the theorem known as Argument principle (see *e.g.* [40]). According to this theorem, for a meromorphic function $f(z)$ with no zeros or poles on a simple closed contour C, the difference between the number of zeros N and poles P (counted according to their multiplicity) inside C is given as

$$N - P = \frac{1}{2\pi}[\arg f(z)]_C. \tag{146}$$

Here $[\arg f(z)]_C$ is the change of the argument of $f(z)$ along C. Using this theorem we will show, that in both cases (6) and (145) there is a nonzero lower bound on the number of solutions on

the first and the second sheet, which correspond to the poles of the propagator (44). We will also give conditions for the saturation of these lower bounds.

Let us start with (145). The left hand side of the pole equation $f_T(z) = 1 + \sigma_T^r(z)$ is analytic on the first sheet (and meromorphic on the second sheet) of the cut complex plane with cut from $z = 1$ to $z = +\infty$. Let us choose contour $C = C_+ + C_R - C_- + C_\varepsilon$ which is usually used for the proof of the dispersive representation for the self-energy, namely the one consisting of the infinitesimal circle C_ε encircling the point $z = 1$ clockwise, two straight lines $C_\pm$ infinitesimally above and bellow the real axis going from $z = 1$ to $z = R$ and a circle C_R corresponding to $z = R\,\mathrm{e}^{\mathrm{i}\theta}$, $0 < \theta < 2\pi$, and take the limit with $\varepsilon \to 0$, $R \to \infty$ in the end. According to the argument principle, the total change of the phase of the function $f_T^{I,II}(z)$ along this contour gives the number of zeros (with their multiplicities) n^I of $f(z)$ on the first sheet and $n^{II} - 2$, where n^{II} is the number of zeros of $f(z)$ on the second sheet (note that $f_T^{II}(z)$ has pole of the second order at $z = 0$) lying inside the contour C, i.e.

$$\begin{aligned} n^I &= \frac{1}{2\pi}[\arg f_T^I(z)]_C \\ n^{II} &= \frac{1}{2\pi}[\arg f_T^{II}(z)]_C + 2. \end{aligned}$$

Let us assume the contour C_ε first. Suppose that $x = 1$ is not a solution of the equation $f_T(z) = 0$. As a consequence, $[\arg f_T^{I,II}(z)]_{C_\varepsilon}$ vanishes [16].

On the contour C_R, *i.e.* for $z = R\,\mathrm{e}^{\mathrm{i}\theta}$ we get for $b_3 \neq 0$

$$f_T^{I,II}(R\,\mathrm{e}^{\mathrm{i}\theta}) = R^3\mathrm{e}^{3\mathrm{i}\theta}\left(\frac{1}{\pi}\frac{\Gamma_{\mathrm{phys}}}{M_{\mathrm{phys}}}b_3 + \frac{20}{9}\left(\frac{M_{\mathrm{phys}}}{4\pi F_\pi}\right)^2 d_3^2 2\gamma^2\left[1 - \ln R + \mathrm{i}(2\pi - \theta \mp \pi)\right] + O\left(\frac{1}{R}, \frac{\ln R}{R}\right)\right)$$

and therefore, for $R \to \infty$, $[\arg f_T^{I,II}(z)]_{C_R} \to 6\pi$. The same is valid also for $b_3 = 0$ with $\gamma \neq 0$. However, for $b_3 = \gamma = 0$ we get

$$f_T^{I,II}(R\,\mathrm{e}^{\mathrm{i}\theta}) = R^2\mathrm{e}^{2\mathrm{i}\theta}\left(\frac{1}{\pi}\frac{\Gamma_{\mathrm{phys}}}{M_{\mathrm{phys}}}b_2 + \frac{20}{9}\left(\frac{M_{\mathrm{phys}}}{4\pi F_\pi}\right)^2 d_3^2\left[1 - \ln R + \mathrm{i}(2\pi - \theta \mp \pi)\right] + O\left(\frac{1}{R}, \frac{\ln R}{R}\right)\right).$$

In this case $[\arg f_T^{I,II}(z)]_{C_R} \to 4\pi$ and because $d_3 \neq 0$ (unless we are in a conflict with OPE for the tree level VVP correlator[17]), this gives also the lower bound for $[\arg f_T^{I,II}(z)]_{C_R}$.

Finally let us discuss the lines $C_\pm$. Because $\mathrm{Im}\ f_T^I(x \pm \mathrm{i}0) = \mathrm{Im}\sigma_T^r(x \pm \mathrm{i}0) \gtrless 0$ (and f_T^I is real analytic), $\mathrm{Im}\ f_T^{II}(x \pm \mathrm{i}0) > 0$ for $x > 1$, and $\mathrm{Re} f_T^{I,II}(R \pm \mathrm{i}0) \to -\infty$ for $R \to \infty$, we can easily conclude that in this limit $[\arg f_T^{I,II}(z)]_{C_+} = 0$ unless $f_T^{I,II}(1) > 0$, in the latter case $[\arg f_T^{I,II}(z)]_{C_+} = \pi$ and in both cases $[\arg f_T^{I,II}(z)]_{C_-} = \pm[\arg f_T^{I,II}(z)]_{C_+}$.

Putting all pieces together we get under the assumption $f_T^{I,II}(1) \neq 0$ the following bound

$$[\arg f_T^{I,II}(z)]_C \geq 4\pi$$

[16] In the case $f_T^{I,II}(x) \to 0$ for $x \to 1$ when $f_T^{I,II}(x) = (x-1)^k\ g_T^{I,II}(x)$ where $k \leq 3$ and when $g_T^{I,II}(x)$ (which has the branching point at $x = 1$) has a finite nonzero limit at $x = 1$) we get $[\arg f_T^{I,II}(z)]_{C_\varepsilon} = -2\pi k$.

[17] Note however, that the requirement that the tree level conditions for OPE are satisfied might be modified by loop corrections.

and therefore for the number of zeros in the cut complex plane we get

$$2 \leq n^I \leq 4 \tag{147}$$
$$4 \leq n^{II} \leq 5 \tag{148}$$

where the lower bound is saturated for $f_T^{I,II}(1) < 0$, $b_3 = \gamma = 0$ and the upper bound for $f_T^{I,II}(1) > 0$ and either $b_3 \neq 0$ or $\gamma \neq 0$. For $f_T^{I,II}(1) = 0$ (provided we include also this zero with its multiplicity into $n^{I,II}$) the these bounds are valid too[18].

An analogous simple analysis for $f_L(z) = z - 1 - \sigma_L^r(z)$ in the cut complex plane with the cut from $z = 0$ to $z = +\infty$ gives [19] for $f_L^{I,II}(0) \neq 0$

$$3 \leq n^I \leq 4 \tag{149}$$
$$n^{II} = 5 \tag{150}$$

where the lower bounds are saturated for $f_L^{I,II}(0) < 0$ otherwise n^I equals to the upper bound.

We can not therefore avoid in any way the generation of the additional poles (some of them might even be of the higher order) in both 1^{--} and 1^{+-} channels of the propagator only by means of an appropriate choice of the free parameters a_i, b_i and γ. The minimal number of the additional poles (with their orders) on the second sheet is the same for both channels (note that, one pole in 1^{--} channel has to correspond to the perturbative solution describing the original degrees of freedom we have started with). The conditions for the saturation of the lower bounds in the 1^{--} and 1^{+-} channels are

$$-\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left[1 - \sum_{i=1}^{3} a_i\right] + \frac{20}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 d_3^2 < 1 \tag{151}$$

and

$$b_3 = \gamma = 0 \tag{152}$$
$$-\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\sum_{i=0}^{3} b_i > 1 \tag{153}$$

respectively. Note that, while the first condition is in accord with the large N_C counting, the last one is not. Let us now discuss the physical relevance of such additional poles.

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[18] In this case the point $x = 1$ is solution of $f_T^{I,II}(x) = 0$ and provided $f_T^{I,II}(x) = (x-1)^k\ g_T^{I,II}(x)$ (zero with multiplicity $k \leq 3$) we have according to the footnote 16 the phase deficit $-2\pi k$ (*i.e.* the number of the poles different from $z = 1$ is then reduced by k) in comparison with the case $f_T^{I,II}(x) \neq 0$.

[19] Note, that in this case,

$$f_L^{I,II}(R\,\mathrm{e}^{\mathrm{i}\theta}) = R^3\mathrm{e}^{3\mathrm{i}\theta}\left(-\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}a_3 + \frac{40}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 d_3^2\left[1 - \ln R + \mathrm{i}(2\pi - \theta \mp \pi)\right] + O\left(\frac{1}{R}, \frac{\ln R}{R}\right)\right)$$

and therefore $[\arg f_L^{I,II}(z)]_{C_R} = 6\pi$.

5.2 The Källén-Lehman representation and nature of the poles

In this subsection, we will show that the the propagator (44) with self-energies (142) and (143) is incompatible with the Källén-Lehman representation with the positive spectral function. Moreover, at least one of the solutions of both equations (6) and (145) is pathological and corresponds to the negative norm ghost or the tachyonic pole.

Let us first briefly remind the Källén-Lehman representation of the antisymmetric tensor field propagator. According to the Lorentz structure we can write the following spectral representation of the full propagator (modulo generally non-covariant contact terms)

$$\Delta_{\mu\nu\alpha\beta}(p) = p^2\Pi^T_{\mu\nu\alpha\beta}(p)\Delta_T(p^2) - p^2\Pi^L_{\mu\nu\alpha\beta}(p)\Delta_L(p^2) + \Delta^{\rm contact}_{\mu\nu\alpha\beta}(p)$$

where (up to the necessary subtractions)

$$\Delta_{L,T}(p^2) = \int_0^\infty {\rm d}\mu^2 \frac{\rho_{L,T}(\mu^2)}{p^2-\mu^2+{\rm i}0} \tag{154}$$

and where the spectral functions $\rho_{T,L}(p^2)$ are given in terms of the sum over the intermediate states as

$$(2\pi)^{-3}\theta(p^0)\left[\rho_T(p^2)p^2\Pi^T_{\mu\nu\alpha\beta}(p) - \rho_L(p^2)p^2\Pi^L_{\mu\nu\alpha\beta}(p)\right] = \sum_N \delta^{(4)}(p-p_N)\langle 0|R_{\mu\nu}(0)|N\rangle\langle N|R_{\alpha\beta}(0)|0\rangle. \tag{155}$$

Note that, in the above formula we assume all the states $|N\rangle$ to have a positive norm; the spectral functions $\rho_{L,T}(p^2)$ are then positive (for the proof see the Appendix E). For the one particle spin-one bound stated states $|p,\lambda\rangle$ with mass M either

$$\langle 0|R_{\mu\nu}(0)|p,\lambda\rangle = Z_L^{1/2} u^{(\lambda)}_{\mu\nu}(p) \tag{156}$$

or

$$\langle 0|R_{\mu\nu}(0)|p,\lambda\rangle = Z_T^{1/2} w^{(\lambda)}_{\mu\nu}(p) \tag{157}$$

according to its parity (cf. (50) and (56)). Therefore (using the formulae from the Appendix E), the corresponding one particle contribution to $\rho_{L,T}(\mu^2)$ is

$$\rho^{\rm one\text{-}particle}_{L,T}(\mu^2) = \frac{2}{M^2} Z_{L,T}\delta(\mu^2-M^2). \tag{158}$$

Positivity $\rho_{L,T}(\mu^2)$ implies $Z_{L,T} > 0$ in the above one-particle contributions.

For free fields with mass M we get

$$\begin{aligned}\rho^{free}_L(\mu^2) &= \frac{2}{M^2}\left(\delta(\mu^2-M^2) - \delta(\mu^2)\right)\\ \rho^{free}_T(\mu^2) &= \frac{2}{M^2}\delta(\mu^2).\end{aligned} \tag{159}$$

Note the kinematical poles in $\Delta_{L,T}(p^2)$ at $p^2 = 0$, which do not correspond to any one-particle intermediate state and which sum up to the contact terms of the form

$$\Delta^{free,{\rm contact}}_{\mu\nu\alpha\beta}(p) = \frac{1}{M^2}\left(g_{\mu\alpha}g_{\beta\nu} - g_{\mu\beta}g_{\nu\alpha}\right). \tag{160}$$

Let us now define for complex z by means of the analytic continuation (up to the possible subtractions)

$$\Delta_{L,T}(z) = \int_0^\infty \mathrm{d}s \frac{\rho_{L,T}(s)}{s-z} \ , \tag{161}$$

Within the perturbation theory however, the primary quantities are the self-energies, which we define as (cf. (44))

$$\begin{aligned} \Delta_T(s) &= \frac{1}{s}\frac{2}{M^2+\Sigma_T(s)} \\ \Delta_L(s) &= \frac{1}{s}\frac{2}{s-M^2-\Sigma_L(s^2)}. \end{aligned} \tag{162}$$

The poles at $s=0$ are of the kinematical origin and in analogy with the free propagator they sum up into the contact terms provided $\Sigma_T(0)=\Sigma_L(0)$. The formulae (162) can be understood as the Dyson re-summation of the $1PI$ self-energy insertions to the propagator or as an inversion of the $1PI$ two-point function. Due to the positivity of $\rho_{L,T}(s)$, we get for the imaginary parts of $\Sigma_{L,T}$ the following positivity (negativity) constraints:

$$\begin{aligned} \mathrm{Im}\Sigma_L(s+i0) &= \frac{1}{2}\theta(s)s\ \mathrm{Im}\Delta_L(s+i0)|s-M^2-\Sigma_L(s+i0)|^2 \le 0 \\ \mathrm{Im}\Sigma_T(s+i0) &= -\frac{1}{2}\theta(s)s\ \mathrm{Im}\Delta_T(s+i0)|M^2+\Sigma_L(s+i0)|^2 \ge 0. \end{aligned} \tag{163}$$

Let us now turn to the $R\chi T$ -like effective theories and try to demonstrate their possible limitations. In such a framework the self-energies $\Sigma_{L,T}$ are given by a sum of the 1PI graphs organized according to some counting rule (for $R\chi T$ *e.g.* by the index i_Γ , cf. (126)). Up to a fixed given order (which we assume to be fixed from now on) we have the asymptotic behavior $\Sigma_{L,T}(z) = O(z^n \ln^k z))$ for $z \to -\infty$ according to the Weinberg theorem. Here n corresponds to the maximal degree of divergence of the contributing (sub)graphs and therefore, it grows with the number of loops as well as with the index of the vertices (cf. (125)).

Such a grow of the inverse propagator is known to lead to problems. Suppose *e.g.*, that we can organize the result of the calculation of the 1PI graphs in the form of a dispersive representation for the functions $\Sigma_{L,T}(z)$ on the first sheet[20]

$$\Sigma^I_{L,T}(z) = P^{L,T}_n(z) + \frac{Q^{L,T}_{n+1}(z)}{\pi}\int_{x_t}^\infty \frac{dx}{Q^{L,T}_{n+1}(x)}\frac{\mathrm{Im}\Sigma_{L,T}(x+i0)}{x-z} \tag{164}$$

where $x_t \ge 0$ is the lowest multi-particle threshold, $P^{L,T}_n(z)$ and $Q^{L,T}_{n+1}(z)$ (we suppose $Q^{L,T}_{n+1}(x) > 0$ for $x > 0$) are renormalization scale independent real polynomials of the order n and $n+1$ respectively and $\mathrm{Im}\Sigma_{L,T}(x+i0)$ can be obtained using the Cutkosky rules. The contributions to

[20]Here we do not assume the existence of any CDD poles [41] for simplicity. In general case, provided the spectral representation of $\Delta_{L,T}$ is valid in the form (161), and $\mathrm{Im}\Delta^{-1}_{L,T}(s) = O(s^n)$ for $s \to \infty$ we formally get

$$\Delta^{-1}_{L,T}(z) = P_n(z) + Q^{L,T}_{n+1}(z)\left(\frac{1}{\pi}\int_{x_t}^\infty \frac{dx}{Q^{L,T}_{n+1}(x)}\frac{\mathrm{Im}\Delta^{-1}_{L,T}(x)}{x-z} - \sum_i \frac{C_i}{z-z_{0i}}\right)$$

where $C_i > 0$ and $0 < z_{0i} < x_t$ correspond to the CDD poles.

$P_n^{L,T}(z)$ stem from the counterterms necessary to renormalize the superficial divergences of the contributing 1PI graphs as well as from the loops (χlogs)[21].

As a consequence, the functions $z^k\Delta_{L,T}(z)$ where $0 \leq k \leq n$ and where $\Delta_{L,T}(s)$ is naively defined by (162) are analytic (up to the finite number of complex poles z_j generally different for Δ_L and Δ_T and a kinematical pole at $z = 0$ - see bellow) in the cut complex plane. As far as the number of poles z_j are concerned, provided $\mathrm{Im}\Sigma_{L,T}(x + i0) \lesseqgtr 0$ as suggested by (163), we can almost literally repeat the analysis from the previous subsection based on the argument principle. The change of a phase of the inverse propagator along the path C_R is now $[\arg \Delta_{L,T}^{-1}(z)]_{C_R} \to 2\pi n$ (for $R \to \infty$), while the absolute value of the $[\arg \Delta_{L,T}^{-1}(z)]_{C_\pm}$ is bounded by π due to the positivity (negativity) of $\mathrm{Im}\Sigma_{L,T}(x \pm i0)$. Provided $\Delta_{L,T}^{-1}(x_t) \neq 0$, we can therefore conclude

$$n - 1 \leq n^I \tag{165}$$

$$n \leq n^{II} - p^{II} \tag{166}$$

where $n^{I,II}$ is the number of the solutions of the equation $\Delta_{L,T}^{-1}(z) = 0$ on the first and second sheet respectively and p^{II} is the number of the poles (weighted with their order) of $\Sigma_{L,T}(z)$ on the second sheet[22].

Therefore, because $z^k\Delta_{L,T}(z) = O(z^{k-n-1})$, we can write for $0 < k \leq n$ an unsubtracted dispersion relation (cf. (161), we will omit the subscript L, T in the following formulae for brevity and write simply $\Delta(z)$, $\rho(s)$ *etc.*)

$$z^k\Delta(z) = \sum_{j>0} \frac{R_j z_j^k}{z - z_j} + \frac{1}{\pi}\int_{x_t}^{\infty} \mathrm{d}x \frac{x^k \mathrm{disc}\Delta(x)}{x - z}$$

or

$$\Delta(z) = \frac{1}{z^k}\sum_{j>0} \frac{R_j z_j^k}{z - z_j} + \frac{1}{\pi z^k}\int_{x_t}^{\infty} \mathrm{d}x \frac{x^k \mathrm{disc}\Delta(x)}{x - z}. \tag{167}$$

and for $k = 0$ (note the kinematical pole at $z = 0$)

$$\Delta(z) = \frac{R_0}{z} + \sum_{j>0} \frac{R_j}{z - z_j} + \frac{1}{\pi}\int_{x_t}^{\infty} \mathrm{d}x \frac{\mathrm{disc}\Delta(x)}{x - z} \tag{168}$$

Due to the asymptotic fall off $\Delta(z) = O(z^{-n-1})$ the discontinuity $\mathrm{disc}\Delta(x)$ has to satisfy the following sum rules

$$-\frac{1}{\pi}\int_{x_t}^{\infty} \mathrm{d}x x^k \mathrm{disc}\Delta(x) + \sum_j R_j z_j^k = 0, \quad 0 < k \leq n - 1. \tag{169}$$

$$-\frac{1}{\pi}\int_{x_t}^{\infty} \mathrm{d}x \mathrm{disc}\Delta(x) + \sum_j R_j + R_0 = 0 \tag{170}$$

[21] In what follows we give such an representation of our one-loop $i_\Gamma \leq 6$ result explicitly.

[22] Note that, the case $n = 1$ is in some sense exceptional. In this case it is possible to get a realistic resonance propagator compatible with the Källén-Lehman representation with no pole on the first sheet and one pole on the unphysical sheet. Such a propagator has been obtained in [42] for scalar resonances. Cf. also [43].

Suppose on the other hand validity of the dispersive representation (161). Then all the poles have to be real, and we can identify

$$\rho(s) = -\frac{1}{\pi}\mathrm{disc}\Delta(s) + \sum_j R_j \delta(s - z_j) + R_0\delta(s). \tag{171}$$

However, the sum rules (169) are generally inconsistent with the spectral representation (161). The validity of some of them might require either an appearance of the states with the negative norm in the spectrum, *i.e.* we are in a conflict with the positivity of the spectral function $\rho(s) \geq 0$ or an appearance of physically non-acceptable tachyon poles leading to the acausality. For instance, suppose $\mathrm{disc}\Delta(s) \leq 0$, then for $R_0 \geq 0$ at least one of the poles has to correspond to a negative norm one-particle state (ghost). On the other hand, for $\mathrm{disc}\Delta(s) \leq 0$, $R_j > 0$ we can still satisfy the $k = 0$ sum rule with negative R_0, however, from the $k = 1$ sum rule we need at least one pole to be negative (tachyon) (in this case, however, the sum rules with even k cannot be satisfied)[23]. These considerations illustrate the known fact that the representation of the propagator based on the formulas (162) has limited range of validity within the fixed order of the perturbation theory and has to be taken with some care.

One point of view might be that the range of applicability of the formulae (162) is $|z| < \Lambda_{\max} = \min\{|z_j|\}$ where $\{z_j\}$ is the set of unwanted poles. Provided there exists a genuine expansion parameter α applicable to the organization of the perturbative series, according to which $\Sigma_{L,T} = \sum_{i>0} \alpha^i \Sigma^{(i)}_{L,T}$ (*e.g.* expanding in powers of $\alpha = 1/N_C$ in $R\chi T$), one can expect the additional (generally pathological) poles of $\Delta_{L,T}(z)$ to decouple (*i.e.* $\Lambda_{\max} \to \infty$ for $\alpha \to 0$). In such a case we could argue that they are in fact harmless. However, the size of $\Lambda_{\max}$ for actual value of α need not to be far from M which could invalidate this approach to the theory in the region for which it was originally designed.

Alternatively, instead of using the (partial) Dyson re-summation, we can expand directly $\Delta_{\mu\nu\alpha\beta}(p)$ to the fixed finite order n which leads to

$$\begin{aligned}
\Delta_L(s) &= \frac{2}{s}\left(\frac{1}{s-M^2} + \alpha\frac{1}{s-M^2}\Sigma^{(1)}_L(s^2)\frac{1}{s-M^2} + \ldots + \alpha^n\frac{1}{s-M^2}\Sigma^{(n)}_L(s^2)\frac{1}{s-M^2}\right)\\
\Delta_T(s) &= \frac{2}{s}\left(\frac{1}{M^2} + \alpha\frac{1}{M^2}\Sigma^{(1)}_T(s^2)\frac{1}{M^2} + \ldots + \alpha^n\frac{1}{M^2}\Sigma^{(n)}_T(s^2)\frac{1}{M^2}\right).
\end{aligned}$$

This expansion (which does not give rise to the additional poles of the propagator) might be useful for $s \ll M^2$, however, in this case a higher-order pole at $s = M^2$ is generated, which is not correct physically in the resonance region $s \sim M^2$. Here we instead expect a single pole on the second sheet of $\Delta_L(z)$, where $z = M^2_{\mathrm{phys}} - iM_{\mathrm{phys}}\Gamma_{\mathrm{phys}}$ (where the mass $M^2_{\mathrm{phys}} = M^2 + O(\alpha)$ and the width $\Gamma_{\mathrm{phys}} = O(\alpha)$) corresponding to the original degree of freedom of the free Lagrangian. Therefore, the Dyson re-summation (*i.e.* the application of the formulae (162)) suplemented with some other more sophisticated approaches (*e.g.* the Redmond and Bogolyubov method [44, 45] consisting of the subtraction[24] of the additional unwanted poles from the propagator, or diagonal Padé approximation method [46]) seems to be inevitable for $s \sim M^2$.

[23] An analogous discussion can be done for the second sheet. Concrete examples of various types of poles will be given in the next section.

[24] Note that, in order to perform this on the lagrangian level, nonperturbative and nonlocal counterterms would have to be added to the theory. However the status of such a counterterms is not clear, cf. [47].

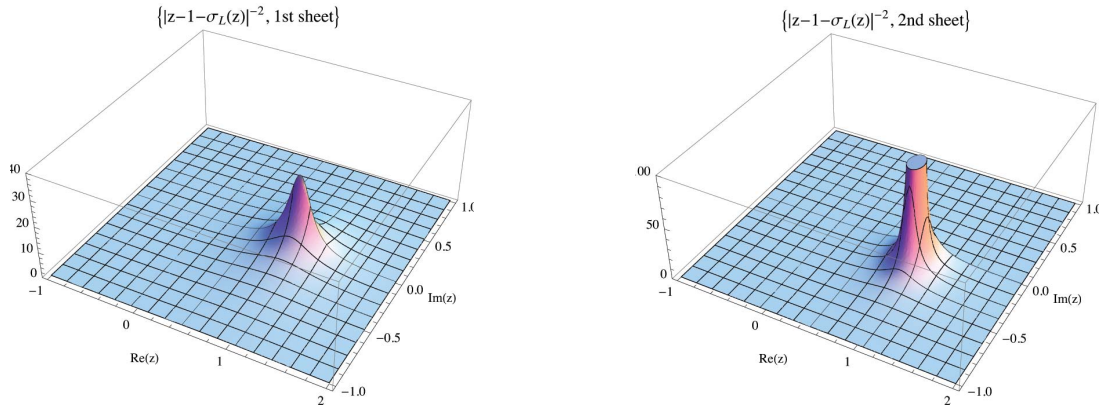

Figure 6: The plot of the the square of the modulus of the propagator function $|z-1-\sigma_L(z)|^{-2}$ on the first and the second sheet for $a_i = 0$. The pole on the second sheet and the peak on the first sheet correspond to the $\rho(770)$.

However, in the concrete case of our calculations of the antisymmetric tensor field propagator, the plain Dyson re-summation might produce various types of poles some of which we illustrate in the next subsection.

5.3 Examples of the poles

The additional poles of the propagator can have different nature. Let us assume the 1^{--} channel first. By construction for any values of the constants a_i we have one pole on the second sheet (which is directly accessible from the physical sheet by means of the crossing of the cut for $0 < z < 1$) which corresponds to the physical resonance (ρ meson) we have started with at the tree level. On the first sheet we get then a typical resonance peak. These two structures are illustrated in the Fig. 6, where the square of the modulus of the propagator function, namely *i.e.* $|z-1-\sigma_L(z)|^{-2}$, is plotted[25] on the first and the second sheet for $a_i = 0$. In this case, no additional pole appears in the region of assumed applicability of $R\chi T$. However, for another set of parameters we can get also pathological poles not far from this region (*e.g.* tachyon as it is illustrated in analogous Fig. 7, now for $a_0 = a_1 = a_2 = 10$, $a_3 = 0$).

In the 1^{+-} channel, there is no tree-level pole in the propagator. The structure of the poles of the Dyson resumed propagator is strongly dependent on the parameters b_i and γ in this case. Let us illustrate this briefly. Note *e.g.* that, the equation (145) can have (exact) solution $x = 1$ on the first sheet provided the parameters b_i satisfy the following constraint

$$\sum_{i=0}^{3} b_i = -\pi \frac{M_{\rm phys}}{\Gamma_{\rm phys}} \sim -16 \tag{172}$$

where the numerical estimate corresponds to $(M_{\rm phys}, \Gamma_{\rm phys}) \sim (M_\rho, \Gamma_\rho)$. In order to interpret this

[25] We have used the following numerical inputs: $M_{\rm phys} = 770MeV$, $\Gamma_{\rm phys} = 150MeV$, $F = 93.2MeV$, $F_V = 154MeV$.

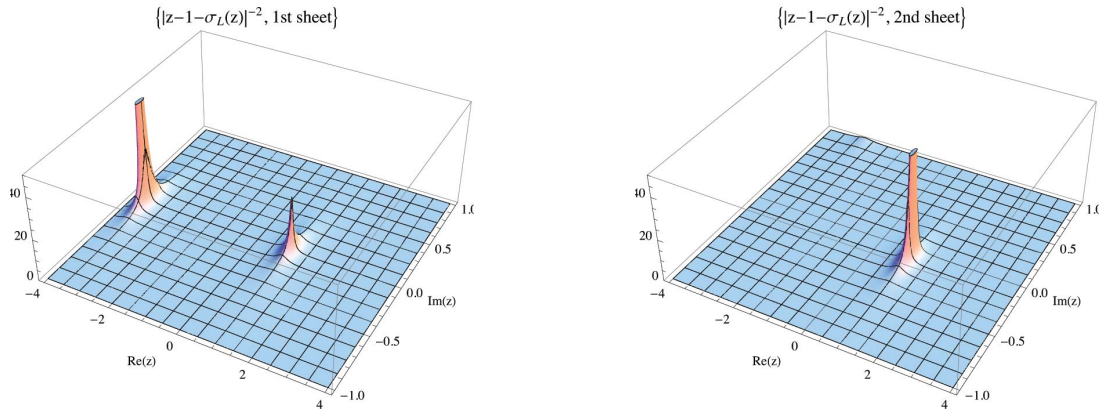

Figure 7: The plot of the the square of the modulus of the propagator function $|z-1-\sigma_L(z)|^{-2}$ on the first and the second sheet for $a_0 = a_1 = a_2 = 10$, $a_3 = 0$. The additional pole on the first sheet is a tachyon.

solution as a 1^{+-} bound state pole we need the residuum Z_A at this pole to be positive, *i.e.*

$$Z_A^{-1} = \sigma_T^r(x)'|_{x=1} = \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\sum_{j=1}^{3} j b_j > 0 \tag{173}$$

otherwise the pole is a negative norm ghost state. Of course, from the phenomenological point of view, both these possibilities are meaningless. Note also that, the constraints (172) and (173) require unnatural large values of the parameters b_i and it is also in a conflict with the large N_C counting[26].

For $\gamma = 0$, a pathological tachyonic solution of (145) exists for $x = -2$ provided

$$\sum_{i=0}^{3}(-2)^i b_i = -\pi\frac{M_{\rm phys}}{\Gamma_{\rm phys}}$$

which might be satisfied with more reasonable values of the parameters b_i than in the previous case. More generally, we can have pathological poles $x = x_\gamma$ where x_γ is a solution of

$$2 + (1 + 6\gamma + \gamma^2)x_\gamma + 2\gamma^2 x_\gamma^2 = 0.$$

This x_γ is a pole of the propagator on both physical and unphysical sheets under the conditions that the following constraint on the parameters b_i

$$\sum_{i=0}^{3} x_\gamma^i b_i = -\pi\frac{M_{\rm phys}}{\Gamma_{\rm phys}}$$

is satisfied. Here x_γ is real (and negative) for $|\gamma+5| > 2\sqrt{6}$ and it represents therefore a physically unacceptable tachyonic pole. Outside of this region of γ we get pair of complex conjugate poles on the physical sheet with ${\rm Re}x_\gamma > 0$ when $-3 + 2\sqrt{2} > \gamma > -3 - 2\sqrt{2}$.

[26]While $b_i = O(1)$ in the large N_C limit, the right hand side of (172) begaves as $O(N_C)$.

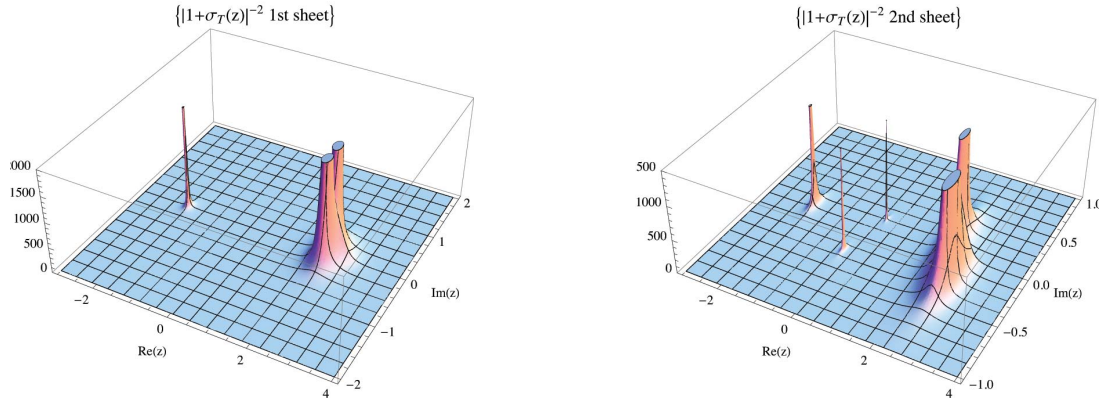

Figure 8: The plot of the the square of the modulus of the propagator function $|1+\sigma_T(z)|^{-2}$ on the first and the second sheet for $b_0 = -2.16$, $b_1 = -3.66$, $b_2 = -4.45$, $b_3 = 1.47$ and $\gamma = 0$. Along the desired $b_1(1235)$ pole on the 2nd sheet ($z = 2.552 - 0.295$i) and peak on the 1st sheet, additional structures appear.

However, we can easily get a more realistic situation and ensure that the position of the complex pole $z_R = x_R - \mathrm{i}y_R$ on the second sheet in the 1^{+-} channel corresponds *e.g.* to a resonance $b_1(1235)$. In this case, two conditions for b_i, and γ have to be satisfied, which correspond to the real and imaginary part of the pole equation $1 + \sigma_T^r(z_R) = 0$. This allows us to eliminate two of the five independent parameters in favor of the mass and the width of the desired resonance[27]. However, it might be difficult to eliminate additional pathological poles in the assumed region of applicability of $R\chi T$. We illustrate this in the Fig. 8, where the the square of the modulus of the propagator function $|1+\sigma_T(z)|^{-2}$ on the first and the second sheet for $b_0 = -2.16$, $b_1 = -3.66$, $b_2 = -4.45$, $b_3 = 1.47$ and $\gamma = 0$ is plotted on the first and the second sheet. In addition to the desired $b_1(1235)$ pole on the second sheet we get also four additional poles on the second sheet which is difficult to interpret physically as well as two additional structures the first sheet one of which can be interpreted as an tachyonic pole.

In general it is not so straightforward to formulate the conditions for a_i, b_i, and γ under which there are *no* additional poles on the real axis in the antisymmetric tensor field propagator. Because $\mathrm{Im}\sigma_L^r(x+i0)$ is negative for $x > 0$ (and similarly $\mathrm{Im}\sigma_T^r(x+i0)$ is positive for $x > 1$), we can clearly conclude, that there is no real pole in these regions on the first and the second sheet. As far as the regions of $x < 0$ (for σ_L^r) and $x < 1$ (for σ_T^r) are concerned, we can proceed as follows. Note, that we can write for the functions $\widehat{J}(x)$ and $\widehat{B}(x)$ the following dispersive representation

$$\begin{aligned}
\widehat{B}(x) &= 2 + x + (x+1)^2 \int_0^\infty \frac{dx'}{(x'+1)^2} \frac{1}{x'-x} \\
&\equiv 2 + x + b(x) \\
\widehat{J}(x) &= \int_1^\infty \frac{dx'}{x'} \left(1 - \frac{1}{x'}\right) \frac{1}{x'-x},
\end{aligned}$$

from which the representation (164) for Σ_L with desired properties easily follows. From this we can see that on the first sheet $b(x)$, $\widehat{J}(x) > 0$ for $x < 0$ and $x < 1$ respectively. Similarly, for Σ_T

[27]Similar conditions we get in the 1^{--} channel, provided we demand to generate *e.g.* $\rho(1450)$ dynamically.

we can write

$$\left(2+(1+6\gamma+\gamma^2)x+2\gamma^2x^2\right)\widehat{J}(x)=1+\frac{1}{6}\left(3\gamma^2+18\gamma+5\right)x+j(x)$$

where

$$j(x)=x^2\int_1^\infty\frac{dx^{'}}{x^{'3}}\left(1-\frac{1}{x^{'}}\right)\left(2+(1+6\gamma+\gamma^2)x^{'}+2\gamma^2x^{'2}\right)\frac{1}{x^{'}-x}$$

and $j(x)>0$ for $x<1$. The equations (5) and (145) have therefore the following structure

$$p_L(x) = -\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}x^2\left(b(x)+2\right)-\frac{40}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 d_3^2(x^2-1)^2\widehat{J}(x) \quad (174)$$

$$p_T(x) = -\frac{20}{9}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 d_3^2(x-1)^2\left(j(x)+1\right), \quad (175)$$

where $p_{L,T}(x)$ are the following polynomials of the third order

$$p_L(x) = x-1-\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left[1-x^3+\sum_{i=1}^3 a_i(x^i-1)\right]=(x-1)q_L(x)$$

$$p_T(x) = 1+\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\sum_{i=0}^3 b_i x^i+\frac{10}{27}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 d_3^2\left(3\gamma^2+18\gamma+5\right)x(x-1)^2.$$

where

$$q_L(x)=1-\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left((1+x+x^2)(a_3-1)+a_1+a_2(x+1)\right)$$

Because the right hand sides of the equations (174) and (175) are negative in the regions of interest, the sufficient (but not necessary) condition of the absence of the poles in these regions is $q_L(x)<0$ for $x<0$ and $p_T(x)>0$ for $x<1$. For $q_L(x)$ this can be achieved in many ways, *e.g.* for

$$a_3 \geq 1$$

$$q_L(0) = 1-\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}\left(a_1+a_2+a_3-1\right)<0$$

$$q_L^{'}(0) = -\frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}}(a_3+a_2-1)>0$$

i.e.

$$a_1>\pi\frac{M_{\rm phys}}{\Gamma_{\rm phys}},\ a_2<0,\ a_3\geq 1.$$

Note however, that such a condition for a_1 requires unnatural value for this parameter and is in a conflict with the large N_C counting. Similarly, the condition $p_T(x)>0$ can be ensured *e.g.* when the coefficients at the third power of x vanish identically, *i.e.*

$$b_3=-\frac{10}{27}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2\pi\frac{M_{\rm phys}}{\Gamma_{\rm phys}}d_3^2\left(3\gamma^2+18\gamma+5\right),$$

the coefficients at the second power of x are positive, *i.e*

$$b_2 > \frac{20}{27}\left(\frac{M_{\rm phys}}{4\pi F_\pi}\right)^2 \pi \frac{M_{\rm phys}}{\Gamma_{\rm phys}} d_3^2 \left(3\gamma^2 + 18\gamma + 5\right),$$

and

$$\begin{aligned} p_T(1) &= 1 + \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}} \sum_{i=0}^{3} b_i > 0 \\ p_T^{'}(1) &= \frac{1}{\pi}\frac{\Gamma_{\rm phys}}{M_{\rm phys}} \sum_{j=0}^{3} b_j j > 0. \end{aligned}$$

On the contrary to the previous case, these conditions respect the large N_C counting. Therefore without any detailed information about the actual value of the a_i and b_i it seems to be quite natural to have tachyonic pole in the 1^{--} channel and no bound states or tachyon poles in the 1^{+-} channel of the propagator.

6 Summary and discussion

In this paper we have studied and illustrated various aspects of the renormalization procedure of the Resonance Chiral Theory using the spin-one resonance self-energy and the corresponding propagator as a concrete example. The explicit calculation of the one-loop self-energies within three possible formalisms for the description of the spin-one resonances, namely the Proca filed, antisymmetric tensor field and the first order formalism is the main result of our article. Because the theory is non-renormalizable and the loop corrections break the ordinary chiral power counting, we had presumed an accurence of problems of several types which have proved to be true within our explicit example.

The first sort of problems concerned the technical aspects of the process of renormalization, namely the organization of the loop corrections and the counterterms and the mixing of the ordinary chiral orders by the loops. In order to organize our calculations we have proposed a self-consistent scheme for classification of the one-particle irreducible graphs Γ and corresponding counterterms $\mathcal{O}_i$ which renormalize its superficial divergences. The classification is according to the indices i_Γ and $i_{\mathcal{O}_i}$ assigned to graph Γ and operator $\mathcal{O}_i$ respectively. Though the scheme based on $i_\mathcal{O}$ restricts both the chiral order of the chiral building blocs (number of derivatives and external sources) as well as the number of resonance fields in the operators in the $R\chi T$ Lagrangian at each fixed order and can be understood as a combination of the chiral and $1/N_C$ counting, it is however not possible to assign to i_Γ a clear physical meaning connected with the infrared characteristics of the graphs Γ. Nevertheless the scheme works at least formally and can be used for the proof of the renormalizability of $R\chi T$ to given order $i_\Gamma, i_{\mathcal{O}_i} \leq i_{\rm max}$. We have used it at the level $i_{\rm max} \leq 6$ and proved that the complete set of counterterms from zero up to six derivatives is necessary to renormalize the divergences of the one-loop self-energies in the contrary to the naive expectations based on the usual chiral powercounting.

The last aspect, namely that the complete set of counterterms including also those with two derivatives (*i.e.* the kinetic terms) is necessary, is connected to the second sort of problems. The

tree level Lagrangian is constructed using just one of such a kinetic term in order to ensure the propagation of just three degrees of freedom corresponding to the spin-one particle state. If we would include all possible kinetic terms with two derivatives into the free Lagrangian, we would get (according to the formalism used) additional poles in the free propagator corresponding to the additional one-particle states some of them being necessarily either negative norm ghost or tachyon. This was the first signal of the problems with unphysical degrees of freedom connected with the one-loop corrections to the self-energies. The higher derivative kinetic terms further increase the number of these extra degrees of freedom. We have studied this feature also using the path integral representation and integrated in additional fields which appear to be responsible for the additional propagator poles.

The problems with additional degrees of freedom are also connected with the well known fact that the propagator obtained by means of the Dyson re-summation of the perturbative one-particle irreducible self-enery insertions might be incompatible with the Källén-Lehman spectral representation even in the case of the renormalizable theories [48]. As is well known, in this case tachyonic or negative norm ghost state can appear as an additional pole. Such an extra pole is usually harmless because it is very far from the energy range where the theory is applicable. In the power-counting non-renormalizable effective theories like $R\chi T$ such problems are much stronger either because of the worse UV behavior of the self-energies (which increases the number of additional poles) or because the additional pathological poles might lie near the region where the theory was assumed to be valid. The nontrivial Lorentz structure of the fields describing spin-one resonances further complicates this delineation because some of the additional poles might have different quantum numbers than the original tree-level degrees of freedom. As far as this type of poles is concerned, we have demonstrated using the path integral formalism that it can be eliminated by means of the requirement of additional protective symmetry of the interaction Lagrangian, which is an analog of $U(1)$ gauge transformation known for the Proca and Rarita-Schwinger fields. However, these symmetries are in general in conflict with chiral symmetry, though individual interaction vertices can posses such a symmetry accidentally.

The results of our calculations proved to fit this general picture. Using the explicit example of the one-loop antisymmetric tensor self-energy we have shown that the Dyson re-summed propagator has always (ie. irrespectively to the actual values of the couterterm couplings) at least three additional poles on the first sheet in the 1^{--} channel, just five such poles on the second sheet (one of them corresponding to the original degree of freedom) and at least two additional poles on the first sheet in the 1^{+-} channel and at least four such poles on the second sheet. As we have seen in explicit analysis of the pole equations, without any additional information about the size of the counterterm couplings and consequently about the actual values of the renormalization scale invariant parameters entering the polynomial part of the self-energies, a rich variety of poles in the propagator is possible. Some of the poles might be unphysical (complex conjugated pairs of poles on the first sheet and tachyonic or negative norm ghosts on both sheets) and some of them even can be situated near or inside the assumed applicability region of $R\chi T$.

It might be argued that the additional poles are just artifacts of the inappropriate treatment of the theory and that the one-loop one-particle irreducible self-energy insertion cannot be re-summed in order to construct a reliable approximation of the full resonance propagator. However, the mere truncation of the Dyson series keeping only first two terms (corresponding to tree-level contribution and to the strict one-loop correction to the propagator respectively) generates double poles at $s = M^2$ on both sheets and is therefore in contradiction with the expected

analytic structure of the full propagator. Though this might be an useful approximation of the full propagator for $s \ll M^2$, it cannot be correct in the resonance region. Therefore provided we would like to use $R\chi T$ at one-loop also for $s \sim M^2$, the construction the propagator using some sort of re-summation (*i.e.* the Dyson one or its modifications like *e.g.* the Redmond and Bogolyubov procedure or Padé approximation) might be inevitable. The actual position of the additional poles (if there are any within the chosen procedure) might be then understood as a bound limiting the range of applicability of the theory. In the most optimistic scenario all the additional poles are far form the region of interest and $R\chi T$ can be treated as a consistent effective theory describing just the degrees of freedom we start with at the tree level. The less satisfactory case when only the pathological poles are far-distant, we can either abandon the theory as inconsistent or alternatively we can try to interpret the non-pathological poles as a prediction of the theory corresponding to the dynamical generation of higher resonances. Such a treatment was used in the case of scalar resonances in [29] (see also [30, 31]). Eventually in the case when all the additional poles lie near $s \sim M^2$, either the approximative construction of the propagator or one-loop $R\chi T$ itself might be problematic. Which scenario actually turns up depends on the values of the couplings in the $R\chi T$ Lagrangian.

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A Additional degrees of freedom in the path integral - the Proca field

Suppose that the interaction Lagrangian has the form

$$\mathcal{L}_{int} = \mathcal{L}_{ct} + \mathcal{L}'_{int} \tag{176}$$

where $\mathcal{L}_{ct}$ is the toy interaction Lagrangian (18). Our aim will be to transform $Z[J]$ to the form of the path integral with all the additional degrees of freedom represented explicitly in the Lagrangian and the integration measure. In terms of the transverse and longitudinal degrees of freedom we get

$$\begin{aligned}\mathcal{L}_{int}(V_\perp - \partial\Lambda, J, \ldots) &= \mathcal{L}_{ct}(V_\perp - \partial\Lambda, J, \ldots) + \mathcal{L}'_{int}(V_\perp - \partial\Lambda, J, \ldots) \\ &= \frac{\alpha}{2} V_\perp^\mu \Box V_{\perp\mu} - \frac{\beta}{2}(\Box\Lambda)^2 + \frac{\gamma}{2M^2}(\Box V_\perp^\mu)(\Box V_{\perp\mu}) + \frac{\delta}{2M^2}(\partial_\mu\Box\Lambda)(\partial^\mu\Box\Lambda) \\ &\quad + \mathcal{L}'_{int}(V_\perp - \partial\Lambda, J \ldots).\end{aligned} \tag{177}$$

In order to lower the number of derivatives in the kinetic terms we integrate in auxiliary scalar fields χ, ρ, π, σ and auxiliary transverse vector field $B_{\perp\mu}$. Writing

$$\exp\left(-i\int \mathrm{d}^4x\frac{\beta}{2}(\Box\Lambda)^2\right)=\int\mathcal{D}\chi\exp\left(i\int\mathrm{d}^4x\left(\frac{1}{2\beta}\chi^2-\partial_\mu\chi\partial^\mu\Lambda\right)\right) \tag{178}$$

and similarly for other higher derivative terms we can finally formulate the theory as

$$Z[J]=\int\mathcal{D}V_\perp\mathcal{D}B_\perp\mathcal{D}\Lambda\mathcal{D}\chi\mathcal{D}\rho\mathcal{D}\sigma\mathcal{D}\pi\exp\left(i\int\mathrm{d}^4x\mathcal{L}(V_\perp,B_\perp,\Lambda,\chi,\rho,\sigma,\pi,J,\ldots)\right) \tag{179}$$

with

$$\begin{aligned}\mathcal{D}B_\perp &= \mathcal{D}B\delta(\partial_\mu B^\mu) &(180)\\ B^\mu_\perp &= \left(g^{\mu\nu}-\frac{\partial^\mu\partial^\nu}{\Box}\right)B_\nu. &(181)\end{aligned}$$

and

$$\begin{aligned}\mathcal{L}(V_\perp,B_\perp,\Lambda,\chi,\rho,\sigma,\pi,J,\ldots) &= \frac{1}{2}(1+\alpha)V^\mu_\perp\Box V_{\perp\mu}+\frac{1}{2}M^2V^\mu_\perp V_{\perp\mu}-\frac{1}{2\gamma}M^2B^\mu_\perp B_{\perp\mu}-B^\mu_\perp\Box V^\mu_\perp\\ &+\frac{1}{2}M^2\partial_\mu\Lambda\partial^\mu\Lambda+\frac{1}{2\beta}\chi^2-\partial_\mu\chi\partial^\mu\Lambda\\ &-\frac{1}{2\delta}M^2\partial_\mu\rho\partial^\mu\rho-\partial_\mu\rho\partial^\mu\sigma-\partial_\mu\pi\partial^\mu\Lambda-\pi\sigma\\ &+\mathcal{L}^{'}_{int}(V_\perp-\partial\Lambda,J,\ldots)\end{aligned} \tag{182}$$

In this formulation the kinetic terms have no more than two derivatives, however, the number of fields is higher than the actual number of degrees of freedom. We therefore have to integrate out the redundant variables. As a first step we diagonalize the kinetic terms performing the shifts

$$\begin{aligned}V^\mu_\perp &\to V^\mu_\perp+\frac{1}{1+\alpha}B^\mu_\perp\\ \Lambda &\to \Lambda+\frac{1}{M^2}\chi+\frac{1}{M^2}\pi\\ \rho &\to \rho-\frac{\delta}{M^2}\sigma\\ \chi &\to \chi-\pi\end{aligned} \tag{183}$$

respectively to the form

$$\begin{aligned}\mathcal{L}(V_\perp,B_\perp,\Lambda,\chi,\rho,\sigma,\pi,J,\ldots) &= \frac{1}{2}(1+\alpha)V^\mu_\perp\Box V_{\perp\mu}+\frac{1}{2}M^2V^\mu_\perp V_{\perp\mu}\\ &-\frac{1}{2}(1+\alpha)^{-1}B^\mu_\perp\Box B^\mu_\perp+\frac{1}{2}M^2\left((1+\alpha)^{-2}-\gamma^{-1}\right)B^\mu_\perp B_{\perp\mu}\\ &+M^2(1+\alpha)^{-1}V^\mu_\perp B_{\perp\mu}\\ &+\frac{1}{2}M^2\partial_\mu\Lambda\partial^\mu\Lambda-\frac{1}{2M^2}\partial_\mu\chi\partial^\mu\chi+\frac{1}{2\beta}(\chi-\pi)^2\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{2\delta}M^2\partial_\mu\rho\partial^\mu\rho+\frac{\delta}{2M^2}\partial_\mu\sigma\partial^\mu\sigma-\pi\sigma\\
&+\mathcal{L}^{'}_{int}(\overline{V},J,\ldots).
\end{aligned}
\tag{184}
$$

where

$$
\overline{V}=V_\perp+\frac{1}{1+\alpha}B_\perp-\partial\Lambda-\frac{1}{M^2}\partial\chi
\tag{185}
$$

Now the superfluous degrees of freedom are easily identified. Namely, the fields ρ and σ decouple and moreover π has no kinetic term. Both of them can be therefore easily integrated out. As a result of the gaussian integration we get

$$
Z[J]=\int\mathcal{D}V_\perp\mathcal{D}B_\perp\mathcal{D}\Lambda\mathcal{D}\chi\mathcal{D}\sigma\exp\left(i\int\mathrm{d}^4x\mathcal{L}(V_\perp,B_\perp,\Lambda,\chi,\sigma,J,\ldots)\right)
\tag{186}
$$

where

$$
\begin{aligned}
\mathcal{L}(V_\perp,B_\perp,\Lambda,\chi,\sigma,J,\ldots) &= \frac{1}{2}(1+\alpha)V^\mu_\perp\Box V_{\perp\mu}+\frac{1}{2}M^2V^\mu_\perp V_{\perp\mu}\\
&-\frac{1}{2}(1+\alpha)^{-1}B^\mu_\perp\Box B^\mu_\perp+\frac{1}{2}M^2\left((1+\alpha)^{-2}-\gamma^{-1}\right)B^\mu_\perp B_{\perp\mu}\\
&+M^2(1+\alpha)^{-1}V^\mu_\perp B_{\perp\mu}\\
&-\frac{1}{2M^2}\partial_\mu\chi\partial^\mu\chi+\frac{\delta}{2M^2}\partial_\mu\sigma\partial^\mu\sigma-\frac{1}{2}\beta\sigma^2-\chi\sigma\\
&+\frac{1}{2}M^2\partial_\mu\Lambda\partial^\mu\Lambda\\
&+\mathcal{L}^{'}_{int}(\overline{V},J,\ldots).
\end{aligned}
\tag{187}
$$

Let us assume $\alpha>-1$ and $\delta>0$ in what follows. Note that, in this case the fields $B^\mu_\perp$ and χ have opposite minus sign at their kinetic terms. This is a signal of the appearence of the negative norm ghosts in the spectrum of the theory. The "dangerous" fields $B^\mu_\perp$ and χ mix with the fields $V^\mu_\perp$ and σ respectively. In order to identify the mass eigenstates we further rescale the fields

$$
\begin{aligned}
V^\mu_\perp &\rightarrow (1+\alpha)^{-1/2}V^\mu_\perp\\
B^\mu_\perp &\rightarrow (1+\alpha)^{1/2}B^\mu_\perp\\
\chi &\rightarrow M\chi\\
\sigma &\rightarrow \delta^{-1/2}M\sigma
\end{aligned}
\tag{188}
$$

and afterwards we diagonalize the mass terms

$$
\begin{aligned}
\mathcal{L}_{mass} &= \frac{1}{2}\frac{M^2}{1+\alpha}\left(V^\mu_\perp V_{\perp\mu}+\left(1-\frac{(1+\alpha)^2}{\gamma}\right)B^\mu_\perp B_{\perp\mu}\right)\\
&-\frac{1}{2}M^2\left(\beta\sigma^2+\delta^{-1/2}\chi\sigma\right)
\end{aligned}
\tag{189}
$$

by means of an appropriate $Sp(2)$ symplectic rotation of the fields $V^\mu_\perp$, $B^\mu_\perp$ and χ, σ

$$
V^\mu_\perp \rightarrow V^\mu_\perp\cosh\theta_V+B^\mu_\perp\sinh\theta_V
$$

$$
\begin{aligned}
B^\mu_\perp &\to V^\mu_\perp \sinh\theta_V + B^\mu_\perp \cosh\theta_V \\
\chi &\to \chi\cosh\theta_S + \sigma\sinh\theta_S \\
\sigma &\to \chi\sinh\theta_S + \sigma\cosh\theta_S.
\end{aligned} \tag{190}
$$

This is possible for $(1+\alpha)^2 > 4\gamma$ and $\beta^2 > 4\delta$, when the off-diagonal elements of the mass matrix vanish for

$$
\begin{aligned}
\tanh\theta_V &= \frac{(1+\alpha)^2 - 2\gamma - (1+\alpha)\sqrt{(1+\alpha)^2 - 4\gamma}}{2\gamma} \\
\tanh\theta_S &= \frac{\sqrt{\beta^2 - 4\delta} - \beta}{2\delta^{1/2}}.
\end{aligned} \tag{191}
$$

We get finally for the generating functional

$$
Z[J] = \int \mathcal{D}V_\perp \mathcal{D}B_\perp \mathcal{D}\Lambda \mathcal{D}\chi \mathcal{D}\sigma \exp\left(i \int \mathrm{d}^4x \mathcal{L}(V_\perp, B_\perp, \Lambda, \chi, \sigma, J, \ldots) \right) \tag{192}
$$

where

$$
\begin{aligned}
\mathcal{L}(V_\perp, B_\perp, \Lambda, \chi, \sigma, J, \ldots) &= \frac{1}{2}V^\mu_\perp \Box V_{\perp\mu} + \frac{1}{2}M^2_{V+} V^\mu_\perp V_{\perp\mu} - \frac{1}{2}B^\mu_\perp \Box B^\mu_\perp + \frac{1}{2}M^2_{V-} B^\mu_\perp B_{\perp\mu} \\
&+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}M^2_{S+}\sigma^2 - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}M^2_{S-}\chi^2 + \frac{1}{2}M^2\partial_\mu\Lambda\partial^\mu\Lambda \\
&+ \mathcal{L}'_{int}(\overline{V}^{(\theta)}, J, \ldots).
\end{aligned} \tag{193}
$$

where now

$$
\overline{V}^{(\theta)} = \frac{\exp\theta_V}{(1+\alpha)^{1/2}}(V_\perp + B_\perp) - \partial\chi\cosh\theta_S - \partial\sigma\sinh\theta_S - \partial\Lambda \tag{194}
$$

and where $M^2_{V\pm}$, $M^2_{S\pm}$ are the mass eigenvalues (21) and (23). The theory is now formulated in terms of two spin one and two spin zero fields, whereas two of them, namely $B^\mu_\perp$ and χ, are negative norm ghosts. The field Λ do not correspond to any dynamical degree of freedom, its role is merely to cancel the spurious poles of the free propagators of the transverse fields $V_\perp$ and $B_\perp$ at $p^2 = 0$.

B The additional degrees of freedom in the path integral- the antisymmetric tensor case

We assume the interaction Lagrangian to be of the form

$$
\mathcal{L}_{int} = \mathcal{L}_{ct} + \mathcal{L}'_{int}, \tag{195}
$$

where $\mathcal{L}_{ct}$ is given by (57) and re-express it in the terms of the longitudinal and transverse components of the original field $R_{\mu\nu}$

$$
\mathcal{L}_{int}(R^{\mu\nu}_\parallel - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}, J, \ldots) = \mathcal{L}_{ct}(R^{\mu\nu}_\parallel - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}, J, \ldots) + \mathcal{L}'_{int}(R^{\mu\nu}_\parallel - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}, J, \ldots) \tag{196}
$$

where

$$\mathcal{L}_{ct}(R^{\mu\nu}-\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}J,\ldots) = \frac{\alpha}{4}R^{\mu\nu}_{\parallel}\Box R_{\parallel\,\mu\nu}+\frac{\gamma}{4M^2}(\Box R^{\mu\nu}_{\parallel})(\Box R_{\parallel\,\mu\nu}) +\frac{\beta}{2}(\Box\Lambda^{\mu}_{\perp})(\Box\Lambda_{\perp\mu})-\frac{\delta}{2M^2}(\partial^{\alpha}\Box\Lambda^{\mu}_{\perp})(\partial_{\alpha}\Box\Lambda_{\perp\mu}). \quad (197)$$

We can introduce the auxiliary (longitudinal) antisymmetric tensor field $B^{\mu\nu}_{\parallel}$ and (transverse) vector fields $\chi^{\mu}_{\perp}$, $\rho^{\mu}_{\perp}$, $\sigma^{\mu}_{\perp}$ and $\pi^{\mu}_{\perp}$ in order to avoid the higher derivative terms and write in complete analogy with the Proca field case

$$Z[J]=\int\mathcal{D}R_{\parallel}\mathcal{D}B_{\parallel}\mathcal{D}\Lambda_{\perp}\mathcal{D}\chi_{\perp}\mathcal{D}\rho_{\perp}\mathcal{D}\sigma_{\perp}\mathcal{D}\pi_{\perp}\exp\left(\mathrm{i}\int\mathrm{d}^4x\mathcal{L}(R_{\parallel},B_{\parallel},\Lambda_{\perp},\chi_{\perp},\rho_{\perp},\sigma_{\perp},\pi_{\perp},J,\ldots)\right) \quad (198)$$

where the measures and fields are

$$\mathcal{D}B_{\parallel} = \mathcal{D}B\delta(\partial_{\alpha}B_{\mu\nu}+\partial_{\nu}B_{\alpha\mu}+\partial_{\mu}B_{\nu\alpha}) \quad (199)$$

$$B^{\mu\nu}_{\parallel} = -\frac{1}{2\Box}(\partial^{\mu}g^{\nu\alpha}\partial^{\beta}+\partial^{\nu}g^{\mu\beta}\partial^{\alpha}-(\mu\leftrightarrow\nu))B_{\alpha\beta} \quad (200)$$

and for $\phi^{\mu}=\chi^{\mu}$, ρ^{μ}, σ^{μ} and π^{μ}

$$\mathcal{D}\phi_{\perp} = \mathcal{D}\phi\delta(\partial_{\mu}\phi^{\mu}) \quad (201)$$

$$\phi^{\mu}_{\perp} = \left(g^{\mu\nu}-\frac{\partial^{\mu}\partial^{\nu}}{\Box}\right)\phi_{\perp\nu}. \quad (202)$$

The Lagrangian is then

$$\begin{aligned}\mathcal{L} = {} & \frac{1+\alpha}{4}R^{\mu\nu}_{\parallel}\Box R_{\parallel\,\mu\nu}+\frac{1}{4}M^2R^{\mu\nu}_{\parallel}R_{\parallel\,\mu\nu}\\ & -\frac{1}{\gamma}M^2B^{\mu\nu}_{\parallel}B_{\parallel\,\mu\nu}+B^{\mu\nu}_{\parallel}\Box R_{\parallel\,\mu\nu}\\ & +\frac{1}{2}M^2\Lambda^{\mu}_{\perp}\Box\Lambda_{\perp\mu}-\frac{1}{2\beta}\chi^{\mu}_{\perp}\chi_{\perp\mu}-\chi^{\mu}_{\perp}\Box\Lambda_{\perp\mu}\\ & +\frac{1}{2\delta}M^2\partial^{\alpha}\rho^{\mu}_{\perp}\partial_{\alpha}\rho_{\perp\mu}-\partial^{\alpha}\rho^{\mu}_{\perp}\partial_{\alpha}\sigma_{\perp\mu}-\partial^{\alpha}\Lambda^{\mu}_{\perp}\partial_{\alpha}\pi_{\perp\mu}-\pi^{\mu}_{\perp}\sigma_{\perp\mu}\\ & +\mathcal{L}_{int}\left(R^{\mu\nu}-\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta},J,\ldots\right).\end{aligned} \quad (203)$$

Note that, the fields χ^{μ}, ρ^{μ}, σ^{μ} and π^{μ} mix with Λ^{μ} and are therefore pseudovectors. The Lagrangian (203) is completely analogical to (182) up to the more Lorentz indices, so will be brief in the next steps. First we identify the redundant degrees of freedom diagonalizing the kinetic terms by means of the following sequence of shifts (cf. (183))

$$\begin{aligned}R^{\mu\nu}_{\parallel} &\to R^{\mu\nu}_{\parallel}-2(1+\alpha)^{-1}B^{\mu\nu}_{\parallel}\\ \Lambda^{\mu}_{\perp} &\to \Lambda^{\mu}_{\perp}+\frac{1}{M^2}\chi^{\mu}_{\perp}-\frac{1}{M^2}\pi^{\mu}_{\perp}\\ \rho^{\mu}_{\perp} &\to \rho^{\mu}_{\perp}+\frac{\delta}{M^2}\sigma^{\mu}_{\perp}\end{aligned}$$

$$\chi^{\mu}_{\perp} \;\rightarrow\; \chi^{\mu}_{\perp} + \pi^{\mu}_{\perp}. \tag{204}$$

As a result we get the Lagrangian in the form (cf. (184))

$$\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{4}(1+\alpha)R^{\mu\nu}_{\parallel}\Box R_{\parallel\,\mu\nu} + \frac{1}{4}M^2 R^{\mu\nu}_{\parallel} R_{\parallel\,\mu\nu} \\
& -(1+\alpha)^{-1}B^{\mu\nu}_{\parallel}\Box B_{\parallel\,\mu\nu} + (1+\alpha)^{-2}M^2 B^{\mu\nu}_{\parallel} B_{\parallel\,\mu\nu} - \frac{1}{\gamma}M^2 B^{\mu\nu}_{\parallel} B_{\parallel\,\mu\nu} \\
& -(1+\alpha)^{-1}M^2 R^{\mu\nu}_{\parallel} B_{\parallel\,\mu\nu} \\
& +\frac{1}{2}M^2\Lambda^{\mu}_{\perp}\Box\Lambda_{\perp\mu} - \frac{1}{2M^2}\chi^{\mu}_{\perp}\Box\chi_{\perp\mu} - \frac{1}{2\beta}(\chi^{\mu}_{\perp}+\pi^{\mu}_{\perp})(\chi_{\perp\mu}+\pi_{\perp\mu}) \\
& +\frac{1}{2\delta}M^2\partial^{\alpha}\rho^{\mu}_{\perp}\partial_{\alpha}\rho_{\perp\mu} - \frac{\delta}{2M^2}\partial^{\alpha}\sigma^{\mu}_{\perp}\partial_{\alpha}\sigma_{\perp\mu} - \pi^{\mu}_{\perp}\sigma_{\perp\mu} \\
& +\mathcal{L}_{int}\left(\overline{R}, J, \ldots\right),
\end{aligned} \tag{205}$$

where

$$\overline{R}^{\mu\nu} = R^{\mu\nu}_{\parallel} - 2(1+\alpha)^{-1}B^{\mu\nu}_{\parallel} - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}(\widehat{\Lambda}_{\alpha\beta} + \frac{1}{M^2}\widehat{\chi}_{\perp\alpha\beta}). \tag{206}$$

Integrating out the superfluous fields $\rho_{\perp\mu}$ and $\pi_{\perp\mu}$ which are decoupled from the interaction we get

$$Z[J] = \int \mathcal{D}R_{\parallel}\mathcal{D}B_{\parallel}\mathcal{D}\Lambda_{\perp}\mathcal{D}\chi_{\perp}\mathcal{D}\rho_{\perp}\mathcal{D}\sigma_{\perp}\mathcal{D}\pi_{\perp}\exp\left(\mathrm{i}\int \mathrm{d}^4x\mathcal{L}(R_{\parallel}, B_{\parallel}, \Lambda_{\perp}, \chi_{\perp}, \rho_{\perp}, \sigma_{\perp}, \pi_{\perp}, J, \ldots)\right) \tag{207}$$

with (cf. (187))

$$\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{4}(1+\alpha)R^{\mu\nu}_{\parallel}\Box R_{\parallel\,\mu\nu} + \frac{1}{4}M^2 R^{\mu\nu}_{\parallel} R_{\parallel\,\mu\nu} \\
& -(1+\alpha)^{-1}B^{\mu\nu}_{\parallel}\Box B_{\parallel\,\mu\nu} + (1+\alpha)^{-2}M^2 B^{\mu\nu}_{\parallel} B_{\parallel\,\mu\nu} - \frac{1}{\gamma}M^2 B^{\mu\nu}_{\parallel} B_{\parallel\,\mu\nu} \\
& -(1+\alpha)^{-1}M^2 R^{\mu\nu}_{\parallel} B_{\parallel\,\mu\nu} \\
& +\frac{1}{2}M^2\Lambda^{\mu}_{\perp}\Box\Lambda_{\perp\mu} \\
& -\frac{1}{2M^2}\chi^{\mu}_{\perp}\Box\chi_{\perp\mu} + \frac{\delta}{2M^2}\sigma^{\mu}_{\perp}\Box\sigma_{\perp\mu} + \frac{1}{2}\beta\sigma^{\mu}_{\perp}\sigma_{\perp\mu} + \chi^{\mu}_{\perp}\sigma_{\perp\mu} \\
& +\mathcal{L}_{int}(S, J, \ldots)
\end{aligned} \tag{208}$$

Again, assuming $\alpha > -1$ and $\delta > 0$ we have two pairs of fields with opposite signs of the kinetic terms, namely $(R^{\mu\nu}_{\parallel}, B^{\mu\nu}_{\parallel})$ and $(\chi^{\mu}_{\perp}, \sigma^{\mu}_{\perp})$ respectively.The fields within both of these these pairs mix. After re-scaling

$$\begin{aligned}
R^{\mu\nu}_{\parallel} \;&\rightarrow\; (1+\alpha)^{-1/2}R^{\mu\nu}_{\parallel} \\
B^{\mu\nu}_{\parallel} \;&\rightarrow\; \frac{1}{2}(1+\alpha)^{1/2}B^{\mu\nu}_{\parallel} \\
\chi^{\mu}_{\perp} \;&\rightarrow\; M\chi^{\mu}_{\perp} \\
\sigma^{\mu}_{\perp} \;&\rightarrow\; \frac{M}{\sqrt{\delta}}\sigma^{\mu}_{\perp}
\end{aligned} \tag{209}$$

the form of the mass matrix becomes identical to that of (189) (with obvious identifications) and we can therefore perform the same symplectic rotations as in the Proca field case and under the same assumptions to get diagonal mass terms corresponding to the eigenvalues (21, 23). As a result we have found four spin-one states, two of them being negative norm ghosts, namely $B^{\mu\nu}_{\parallel}$ and $\sigma^{\mu}_{\perp}$ and two of them with opposite parity, namely $\chi^{\mu}_{\perp}$ and $\sigma^{\mu}_{\perp}$. As in the Proca field case, the field $\Lambda^{\mu}_{\perp}$ effectively compensates for the spurious $p^2 = 0$ poles in the $R^{\mu\nu}_{\parallel}$ and $B^{\mu\nu}_{\parallel}$ propagators within Feynman graphs.

C Path integral formulation of the first order formalism

Within the first order formalism, the path integral formulation is merely a generalization of the previous two cases, so we will be as brief as possible in what follows. Note that, now the kinetic term is invariant with respect to the both transformations (26) and (60), therefore the manifestation of the degrees of freedom within the the path integral formalism can be done in analogy with the previous two cases. Using triple Faddeev-Popov trick in the path integral

$$Z[J] = \int \mathcal{D}R \exp\left(\mathrm{i}\int \mathrm{d}^4x \left(MV_\nu \partial_\mu R^{\mu\nu} + \frac{1}{2}M^2 V_\mu V^\mu + \frac{1}{4}M^2 R_{\mu\nu}R^{\mu\nu} + \mathcal{L}_{int}(V^\alpha, R^{\mu\nu}, J, \ldots)\right)\right) \tag{210}$$

we get

$$Z[J] = \int \mathcal{D}R_{\parallel}\mathcal{D}\Lambda_{\perp}\mathcal{D}V_{\perp}\mathcal{D}\Lambda \exp\left(\mathrm{i}\int \mathrm{d}^4x \mathcal{L}(R^{\mu\nu}_{\parallel}, \Lambda^{\rho}_{\perp}, V^{\alpha}_{\perp} \ldots, \Lambda, J, \ldots)\right) \tag{211}$$

where

$$\begin{aligned}
\mathcal{L}(R^{\mu\nu}_{\parallel}, \Lambda^{\rho}_{\perp}, V^{\alpha}_{\perp} \ldots, \Lambda, J, \ldots) &= MV_{\perp\nu}\partial_\mu R^{\mu\nu}_{\parallel} + \frac{1}{2}M^2 V_{\perp\mu}V^{\mu}_{\perp} + \frac{1}{4}M^2 R_{\parallel\mu\nu}R^{\mu\nu}_{\parallel} \\
&+\frac{1}{2}M^2\Lambda^{\mu}_{\perp}\Box\Lambda_{\perp\mu} + \frac{1}{2}M^2\partial_\mu\Lambda\partial^\mu\Lambda \\
&+\mathcal{L}_{int}(R^{\mu\nu}_{\parallel} - \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta}, V^{\alpha}_{\perp} - \partial^\alpha\Lambda, J,)
\end{aligned} \tag{212}$$

and, as in the previous subsections

$$\begin{aligned}
\mathcal{D}R_{\parallel} &= \mathcal{D}R\delta(\partial_\alpha R_{\mu\nu} + \partial_\nu R_{\alpha\mu} + \partial_\mu R_{\nu\alpha}) \\
\mathcal{D}\Lambda_{\perp} &= \mathcal{D}\Lambda\delta(\partial_\mu\Lambda^\mu) \\
\mathcal{D}V_{\perp} &= \mathcal{D}V\delta(\partial_\mu\Lambda^\mu) \\
R^{\mu\nu}_{\parallel} &= -\frac{1}{2\Box}(\partial^\mu g^{\nu\alpha}\partial^\beta + \partial^\nu g^{\mu\beta}\partial^\alpha - (\mu \leftrightarrow \nu))R_{\alpha\beta} \\
\Lambda^{\mu}_{\perp} &= \left(g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{\Box}\right)\Lambda_\nu \\
V^{\mu}_{\perp} &= \left(g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{\Box}\right)V_\nu.
\end{aligned} \tag{213}$$

In order to diagonalize the kinetic terms we perform a shift

$$V^{\mu}_{\perp} \to V^{\mu}_{\perp} - \frac{1}{M}\partial_\nu R^{\nu\mu}_{\parallel} \tag{214}$$

and get

$$\begin{aligned}\mathcal{L}(R_{\parallel}^{\mu\nu},\Lambda_{\perp}^{\rho},V_{\perp}^{\alpha}\ldots,\Lambda,J,\ldots) &= \frac{1}{4}R_{\parallel}^{\mu\nu}\Box R_{\parallel\,\mu\nu}+\frac{1}{4}M^2R_{\parallel\mu\nu}R_{\parallel}^{\mu\nu}+\frac{1}{2}M^2V_{\perp\mu}V_{\perp}^{\mu}\\ &+\frac{1}{2}M^2\Lambda_{\perp}^{\mu}\Box\Lambda_{\perp\mu}+\frac{1}{2}M^2\partial_{\mu}\Lambda\partial^{\mu}\Lambda\\ &+\mathcal{L}_{int}(R_{\parallel}^{\mu\nu}-\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta},V_{\perp}^{\alpha}-\frac{1}{M}\partial_{\nu}R_{\parallel}^{\nu\mu}-\partial^{\alpha}\Lambda,J,).\end{aligned} \tag{215}$$

The discussion of the role of the field $R_{\parallel}^{\mu\nu}$ and the $\Lambda_{\perp}^{\mu}$ is the same as in the antisymmetric tensor case. The extra fields $V_{\perp}^{\mu}$ and Λ do not correspond to the original degree of freedom, their free propagators are

$$\Delta_{V\perp}^{\mu\nu}(p) = \frac{P^{T\,\mu\nu}}{M^2} \tag{216}$$

$$\Delta_{\Lambda}(p) = \frac{1}{M^2}\frac{1}{p^2} \tag{217}$$

with spurious poles at $p^2=0$. According to the form of the interaction, only the combination with spurious poles cancelled, namely

$$\Delta_{V\perp}^{\mu\nu}(p)+p^{\mu}p^{\nu}\Delta_{\Lambda}(p)+\frac{1}{M^2}p_{\alpha}p_{\beta}\Delta_{\parallel}^{\alpha\mu\beta\nu}(p)=-\frac{P^{T\,\mu\nu}}{p^2-M^2}+\frac{P^{L\,\mu\nu}}{M^2} \tag{218}$$

enters the Feynman graphs.

Alternatively, we could make in (212) the following shift

$$R_{\parallel}^{\mu\nu}\to R_{\parallel}^{\mu\nu}+\frac{1}{M}\left(\partial^{\mu}V_{\perp}^{\nu}-\partial^{\nu}V_{\perp}^{\mu}\right) \tag{219}$$

leading to

$$\begin{aligned}\mathcal{L}(R_{\parallel}^{\mu\nu},\Lambda_{\perp}^{\rho},V_{\perp}^{\alpha}\ldots,\Lambda,J,\ldots) &= \frac{1}{2}V_{\perp\mu}\Box V_{\perp}^{\mu}+\frac{1}{2}M^2V_{\perp\mu}V_{\perp}^{\mu}+\frac{1}{4}M^2R_{\parallel\mu\nu}R_{\parallel}^{\mu\nu}\\ &+\frac{1}{2}M^2\Lambda_{\perp}^{\mu}\Box\Lambda_{\perp\mu}+\frac{1}{2}M^2\partial_{\mu}\Lambda\partial^{\mu}\Lambda\\ &+\mathcal{L}_{int}(R_{\parallel}^{\mu\nu}+\frac{1}{M}\left(\partial^{\mu}V_{\perp}^{\nu}-\partial^{\nu}V_{\perp}^{\mu}\right)-\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\alpha\beta},V_{\perp}^{\alpha}-\partial^{\alpha}\Lambda,J,\ldots).\end{aligned} \tag{220}$$

In this formulation, the role of the fields $V_{\perp}^{\mu}$ and the field Λ is the same as in the Proca field case. $R_{\parallel}^{\mu\nu}$ does not correspond to the original degree of freedom and, as in the previous formulation, it serves together with $\Lambda_{\perp\mu}$ to cancel the spurious $p^2=0$ poles.

Let us end up this subsection with the path integral treatment of the toy quadratic interaction Lagrangian (105). Using the same transformations as before we get

$$\mathcal{L}(R_{\parallel}^{\mu\nu},\Lambda_{\perp}^{\rho},V_{\perp}^{\alpha}\ldots,\Lambda,J,\ldots) = MV_{\perp\nu}\partial_{\mu}R_{\parallel}^{\mu\nu}+\frac{1}{2}M^2V_{\perp\mu}V_{\perp}^{\mu}+\frac{1}{4}M^2R_{\parallel\mu\nu}R_{\parallel}^{\mu\nu}$$

$$
\begin{aligned}
&+\frac{1}{2}M^2\Lambda_\perp^\mu\Box\Lambda_{\perp\mu}+\frac{1}{2}M^2\partial_\mu\Lambda\partial^\mu\Lambda \\
&+\frac{\alpha_V}{2}V_{\perp\mu}\Box V_\perp^\mu-\frac{\beta_V}{2}(\Box\Lambda)^2+\frac{\alpha_R}{4}R_{\parallel\mu\nu}\Box R_\parallel^{\mu\nu}+\frac{\beta_R}{4}\Box\Lambda_\perp^\mu\Box\Lambda_{\perp\mu} \\
&+\mathcal{L}_{int}^{'}(R_\parallel^{\mu\nu}-\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\perp\alpha\beta},V_\perp^\alpha-\partial^\alpha\Lambda,J,\ldots) \qquad (221)
\end{aligned}
$$

Introducing the auxiliary fields analogous to the previous two examples, we have

$$
\begin{aligned}
\mathcal{L}(R_\parallel^{\mu\nu},\Lambda_\perp^\mu,\chi,\chi_\perp^\mu,\sigma_\perp^\mu,\pi_\perp^\mu,J,\ldots) &= \frac{\alpha_V}{2}V_{\perp\mu}\Box V_\perp^\mu+\frac{1}{2}M^2V_{\perp\mu}V_\perp^\mu \\
&+\frac{\alpha_R}{4}R_{\parallel\mu\nu}\Box R_\parallel^{\mu\nu}+\frac{1}{4}M^2R_{\parallel\mu\nu}R_\parallel^{\mu\nu} \\
&+MV_{\perp\nu}\partial_\mu R_\parallel^{\mu\nu} \\
&+\frac{1}{2}M^2\Lambda_\perp^\mu\Box\Lambda_{\perp\mu}-\frac{1}{2}M^2\Lambda\Box\Lambda \\
&+\frac{1}{2\beta_V}\chi^2+\chi\Box\Lambda-\frac{1}{2\beta_R}\chi_\perp^\mu\chi_{\perp\mu}-\chi_\perp^\mu\Box\Lambda_{\perp\mu} \\
&+\mathcal{L}_{int}^{'}(R_\parallel^{\mu\nu}-\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\perp\alpha\beta},V_\perp^\alpha-\partial^\alpha\Lambda,J,\ldots) \qquad (222)
\end{aligned}
$$

The kinetic terms can be diagonalized now by means of the shifts

$$
\begin{aligned}
\Lambda_\perp^\mu &\rightarrow \Lambda_\perp^\mu+\frac{1}{M^2}\chi_\perp^\mu \qquad (223) \\
\Lambda &\rightarrow \Lambda+\frac{1}{M^2}\chi \qquad (224)
\end{aligned}
$$

to the form

$$
\begin{aligned}
\mathcal{L}(R_\parallel^{\mu\nu},\Lambda_\perp^\mu,\chi,\chi_\perp^\mu,J,\ldots) &= \frac{\alpha_V}{2}V_{\perp\mu}\Box V_\perp^\mu+\frac{1}{2}M^2V_{\perp\mu}V_\perp^\mu \\
&+\frac{\alpha_R}{4}R_{\parallel\mu\nu}\Box R_\parallel^{\mu\nu}+\frac{1}{4}M^2R_{\parallel\mu\nu}R_\parallel^{\mu\nu} \\
&+MV_{\perp\nu}\partial_\mu R_\parallel^{\mu\nu} \\
&+\frac{1}{2}M^2\Lambda_\perp^\mu\Box\Lambda_{\perp\mu}-\frac{1}{2}M^2\Lambda\Box\Lambda \\
&-\frac{1}{2M^2}\chi_\perp^\mu\Box\chi_{\perp\mu}-\frac{1}{2\beta_R}\chi_\perp^\mu\chi_{\perp\mu} \\
&+\frac{1}{2M^2}\chi\Box\chi+\frac{1}{2\beta_V}\chi^2 \\
&+\mathcal{L}_{int}^{'}(S,W,J,\ldots), \qquad (225)
\end{aligned}
$$

where

$$
\overline{R}^{\mu\nu} = R_\parallel^{\mu\nu}-\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\Lambda}_{\perp\alpha\beta}-\frac{1}{2M^2}\varepsilon^{\mu\nu\alpha\beta}\widehat{\chi}_{\perp\alpha\beta}
$$

$$\overline{V}^{\mu} = V_{\perp}^{\alpha} - \partial^{\alpha}\Lambda - \frac{1}{M^2}\partial^{\alpha}\chi.$$

In the formula (225) the scalar and axial-vector ghost field as well as two propagating dynamically mixed spin-1 degrees of freedom are explicit.

D The parameters α_i and β_i in terms of LECs

In this appendix we present the expressions for the renormalization scale independent polynomial parameters entering the self-energies (cf. Section 4).

D.1 The Proca field case

$$\begin{aligned}
\alpha_0 &= \left(\frac{4\pi F}{M}\right)^2 Z_M^r(\mu) \\
\alpha_1 &= \left(\frac{4\pi F}{M}\right)^2 Z_V^r(\mu) - \frac{40}{3}\sigma_V^2\left(\ln\frac{M^2}{\mu^2} + \frac{1}{3}\right) \\
\alpha_2 &= \left(\frac{4\pi F}{M}\right)^2 M^2 X_V^r(\mu) + \frac{40}{9}\sigma_V^2\left(\ln\frac{M^2}{\mu^2} + \frac{1}{3}\right) \\
\alpha_3 &= \left(\frac{4\pi F}{M}\right)^2 M^4 U_V^r(\mu) + g_V^2\left(\frac{M}{F}\right)^2\left(\ln\frac{M^2}{\mu^2} - \frac{2}{3}\right) \\
\beta_0 &= \left(\frac{4\pi F}{M}\right)^2 Z_M^r(\mu) = \alpha_0 \\
\beta_1 &= \left(\frac{4\pi F}{M}\right)^2 Y_V^r(\mu) \\
\beta_2 &= \left(\frac{4\pi F}{M}\right)^2 M^2 X_V^{\prime r}(\mu) \\
\beta_3 &= \left(\frac{4\pi F}{M}\right)^2 M^4 V_V^r(\mu).
\end{aligned}$$

Here U_V and V_V are certain linear combinations of the couplings of $\mathcal{L}_V^{ct(6)}$ renormalized as

$$\begin{aligned}
U_V &= U_V^r(\mu) - 2g_V^2\left(\frac{M}{F}\right)^4\frac{1}{M^4}\lambda_\infty \\
V_V &= V_V^r(\mu)
\end{aligned}$$

D.2 The antisymmetric tensor case

$$\alpha_0 = \left(\frac{4\pi F}{M}\right)^2 Z_M^r(\mu) - \frac{40}{3}d_1^2\ln\frac{M^2}{\mu^2} - \frac{20}{9}(3d_1^2 - d_3^2) - 5\left(\frac{\lambda^{VVV}}{M}\right)^2\left(\frac{F}{M}\right)^2\left(7 - 6\ln\frac{M^2}{\mu^2}\right)$$

$$
\begin{aligned}
\alpha_1 &= \left(\frac{4\pi F}{M}\right)^2 (Z_R^r(\mu)+Y_R^r(\mu)) - \frac{40}{9}(3d_1^2+2d_3^2)\ln\frac{M^2}{\mu^2} - \frac{20}{3}\left(d_1^2+\frac{1}{9}d_2^2\right) \\
&\quad +\frac{10}{3}\left(\frac{\lambda^{VVV}}{M}\right)^2\left(\frac{F}{M}\right)^2\left(7-6\ln\frac{M^2}{\mu^2}\right) \\
\alpha_2 &= \left(\frac{4\pi F}{M}\right)^2 M^2(X_R^r(\mu)+W_R^r(\mu)) - \frac{40}{9}d_3^2\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right) + \frac{1}{2}\left(\frac{G_V}{F}\right)^2\left(\ln\frac{M^2}{\mu^2}-\frac{2}{3}\right) \\
&\quad -\frac{5}{3}\left(\frac{\lambda^{VVV}}{M}\right)^2\left(\frac{F}{M}\right)^2\left(2-3\ln\frac{M^2}{\mu^2}\right) \\
\alpha_3 &= \left(\frac{4\pi F}{M}\right)^2 M^4 U_R^r(\mu) + \frac{40}{9}d_3^2\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right) \\
\beta_0 &= \left(\frac{4\pi F}{M}\right)^2 Z_M^r(\mu) - \frac{40}{3}d_1^2\ln\frac{M^2}{\mu^2} - \frac{20}{9}(3d_1^2+d_3^2) - \frac{5}{3}\left(\frac{\lambda^{VVV}}{M}\right)^2\left(\frac{F}{M}\right)^2\left(11-6\ln\frac{M^2}{\mu^2}\right) \\
\beta_1 &= \left(\frac{4\pi F}{M}\right)^2 Y_R^r(\mu) - \frac{20}{9}(6d_1^2-12d_1(d_3+d_4)+5d_3^2+9d_4^2-6d_3d_4)\ln\frac{M^2}{\mu^2} \\
&\quad -\frac{20}{27}(9d_1^2-18d_1(d_3+d_4)-7d_3^2-12d_4^2+18d_3d_4) \\
&\quad \frac{20}{3}\left(\frac{\lambda^{VVV}}{M}\right)^2\left(\frac{F}{M}\right)^2\left(7+3\ln\frac{M^2}{\mu^2}\right) \\
\beta_2 &= \left(\frac{4\pi F}{M}\right)^2 M^2 W_R^r(\mu) - \frac{20}{9}(d_3^2+6d_3d_4-5d_4^2)\ln\frac{M^2}{\mu^2} - \frac{80}{27}(d_3^2+4d_4^2) \\
&\quad -\frac{5}{3}\left(\frac{\lambda^{VVV}}{M}\right)^2\left(\frac{F}{M}\right)^2\left(4-3\ln\frac{M^2}{\mu^2}\right) \\
\beta_3 &= \left(\frac{4\pi F}{M}\right)^2 M^4 V_R^r(\mu) - \frac{40}{9}d_4^2\left(\ln\frac{M^2}{\mu^2}-\frac{2}{3}\right).
\end{aligned}
$$

Here U_R and V_R are certain linear combinations of the couplings of $\mathcal{L}_R^{ct(6)}$ with the infinite parts fixed as

$$
\begin{aligned}
U_R &= U_R^r(\mu) - \frac{80}{9}\left(\frac{M}{F}\right)^2\frac{1}{M^4}d_3^2\lambda_\infty \\
V_R &= V_R^r(\mu) + \frac{80}{9}\left(\frac{M}{F}\right)^2\frac{1}{M^4}d_4^2\lambda_\infty.
\end{aligned}
$$

D.3 The first order formalism

$$
\begin{aligned}
\alpha_0^{RV} &= \left(\frac{4\pi F}{M}\right)^2 Z_{RV}^r(\mu) + \frac{10}{9}(\sigma_{RV}+2\sigma_V)\left[(d_1-d_3)+3(2d_1-\sigma_{RV})\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right)\right] \\
\alpha_1^{RV} &= \left(\frac{4\pi F}{M}\right)^2 M^2 X_{RV}^r(\mu) + \frac{10}{9}(\sigma_{RV}+2\sigma_V)(4d_3+\sigma_{RV})\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right) \\
\alpha_2^{RV} &= \left(\frac{4\pi F}{M}\right)^2 M^4 Y_{RV}^r(\mu) - \frac{20}{9}(\sigma_{RV}+2\sigma_V)d_3\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{2}\frac{g_V G_V}{M}\left(\frac{M}{F}\right)^2\left(\ln\frac{M^2}{\mu^2}-\frac{2}{3}\right) \\
\alpha_0^{VV} &= \left(\frac{4\pi F}{M}\right)^2 Z_{MV}^r(\mu) \\
\alpha_1^{VV} &= \left(\frac{4\pi F}{M}\right)^{22} Z_V^r(\mu) - \frac{10}{3}\left(\sigma_{RV}(\sigma_{RV}+2\sigma_V)+4\sigma_V^2\right)\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right) \\
\alpha_2^{VV} &= \left(\frac{4\pi F}{M}\right)^2 M^2 X_V^r(\mu) + \frac{10}{9}\left(\sigma_{RV}(\sigma_{RV}+2\sigma_V)+4\sigma_V^2\right)\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right) \\
\alpha_3^{VV} &= \left(\frac{4\pi F}{M}\right)^2 M^4 U_V^r(\mu) + g_V^2\left(\frac{M}{F}\right)^2\left(\ln\frac{M^2}{\mu^2}-\frac{2}{3}\right) \\
\beta_0^{VV} &= \left(\frac{4\pi F}{M}\right)^2 Z_{MV}^r(\mu) = \alpha_0^{VV} \\
\beta_1^{VV} &= \left(\frac{4\pi F}{M}\right)^2 Y_V^r(\mu) \\
\beta_2^{VV} &= \left(\frac{4\pi F}{M}\right)^2 M^2 X_V^{\prime r}(\mu) \\
\beta_3^{VV} &= \left(\frac{4\pi F}{M}\right)^2 M^4 V_V^r(\mu) \\
\alpha_0^{RR} &= \left(\frac{4\pi F}{M}\right)^2 Z_{MR}^r(\mu) + \frac{10}{3}\left(\sigma_{RV}(2d_1-\sigma_{RV})-4d_1^2\right)\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right) \\
& -\frac{10}{9}\left(d_1-d_3\right)\left(2d_1+2d_3-\sigma_{RV}\right) \\
\alpha_1^{RR} &= \left(\frac{4\pi F}{M}\right)^2 \left(Z_R^r(\mu)+Y_R^r(\mu)\right) - \frac{40}{9}(3d_1^2+2d_3^2)\ln\frac{M^2}{\mu^2} - \frac{20}{3}\left(d_1^2+\frac{1}{9}d_2^2\right) \\
& +\frac{10}{9}\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right)\sigma_{RV}(4d_3+\sigma_{RV}) \\
\alpha_2^{RR} &= \left(\frac{4\pi F}{M}\right)^2 M^2\left(X_R^r(\mu)+W_R^r(\mu)\right) - \frac{20}{9}d_3(2d_3+\sigma_{RV})\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right) \\
& +\frac{1}{2}\left(\frac{G_V}{F}\right)^2\left(\ln\frac{M^2}{\mu^2}-\frac{2}{3}\right) \\
\alpha_3^{RR} &= \left(\frac{4\pi F}{M}\right)^2 M^4 U_R^r(\mu) + \frac{40}{9}d_3^2\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right) \\
\beta_0^{RR} &= \left(\frac{4\pi F}{M}\right)^2 Z_{MR}^r(\mu) + \frac{10}{3}(\sigma_{RV}(2d_1-\sigma_{RV})-4d_1^2)\left(\ln\frac{M^2}{\mu^2}+\frac{1}{3}\right) \\
& -\frac{20}{9}(d_1^2+d_3^2)+\frac{10}{9}\sigma_{RV}\left(d_1+d_3-\sigma_{RV}\right) \\
&= \alpha_0^{RR} - \frac{40}{9}d_3^2 + \frac{10}{9}\sigma_{RV}\left(2d_3-\sigma_{RV}\right)
\end{aligned}
$$

$$
\begin{aligned}
\beta_1^{RR} &= \left(\frac{4\pi F}{M}\right)^2 Y_R^r(\mu) - \frac{20}{9}(6d_1^2 - 12d_1(d_3+d_4) + 5d_3^2 + 9d_4^2 - 6d_3d_4)\ln\frac{M^2}{\mu^2} \\
&\quad -\frac{20}{27}(9d_1^2 - 18d_1(d_3+d_4) - 7d_3^2 - 12d_4^2 + 18d_3d_4) \\
&\quad -\frac{5}{27}\sigma_{RV}\left(32d_3 + 6(d_3+9d_4)\ln\frac{M^2}{\mu^2} - 3\sigma_{RV}\left(\ln\frac{M^2}{\mu^2} - \frac{2}{3}\right)\right) \\
\beta_2^{RR} &= \left(\frac{4\pi F}{M}\right)^2 M^2 W_R^r(\mu) - \frac{20}{9}(d_3^2 + 6d_3d_4 - 5d_4^2)\ln\frac{M^2}{\mu^2} - \frac{80}{27}(d_3^2 + 4d_4^2) \\
&\quad +\frac{5}{27}\sigma_{RV}\left(8d_3 + 6(d_3+3d_4)\ln\frac{M^2}{\mu^2} - 3\sigma_{RV}\left(\ln\frac{M^2}{\mu^2} - \frac{2}{3}\right)\right) \\
\beta_3^{RR} &= \left(\frac{4\pi F}{M}\right)^2 M^4 V_R^r(\mu) - \frac{40}{9}d_4^2\left(\ln\frac{M^2}{\mu^2} - \frac{2}{3}\right).
\end{aligned}
$$

Here U_V, V_V, U_R, V_R and Y_{RV} are certain linear combination of the couplings from $\mathcal{L}_{RV}^{ct(6)}$ with infinite parts fixed according to

$$
\begin{aligned}
U_V &= U_V^r(\mu) - 2g_V^2\left(\frac{M}{F}\right)^4\frac{1}{M^4}\lambda_\infty \\
V_V &= V_V^r(\mu) \\
U_R &= U_R^r(\mu) - \frac{80}{9}\left(\frac{M}{F}\right)^2\frac{1}{M^4}d_3^2\lambda_\infty \\
V_R &= V_R^r(\mu) + \frac{80}{9}\left(\frac{M}{F}\right)^2\frac{1}{M^4}d_4^2\lambda_\infty \\
Y_{RV} &= Y_{RV}^r(\mu) + \frac{40}{9}\left(\frac{M}{F}\right)^2\frac{1}{M^4}(\sigma_{RV}+2\sigma_V)d_3\lambda_\infty + \frac{g_VG_V}{M}\left(\frac{M}{F}\right)^2\frac{1}{M^4}\lambda_\infty
\end{aligned}
$$

E Proof of the positivity of the spectral functions

Here we prove the positivity of the spectral functios $\rho_{L,T}(\mu^2)$ defined as

$$
(2\pi)^{-3}\theta(p^0)\left[\rho_T(p^2)p^2\Pi^T_{\mu\nu\alpha\beta}(p) - \rho_L(p^2)p^2\Pi^L_{\mu\nu\alpha\beta}(p)\right] = \sum_N \delta^{(4)}(p-p_N)\langle 0|R_{\mu\nu}(0)|N\rangle\langle N|R_{\alpha\beta}(0)|0\rangle. \tag{226}
$$

Let us define for $p^2 > 0$

$$
\begin{aligned}
u^{(\lambda)}_{\mu\nu}(p) &= \frac{\mathrm{i}}{\sqrt{p^2}}\left(p_\mu\varepsilon^{(\lambda)}_\nu(p) - p_\nu\varepsilon^{(\lambda)}_\mu(p)\right) \\
w^{(\lambda)}_{\mu\nu}(p) &= \frac{1}{2}\varepsilon_{\mu\nu}{}^{\alpha\beta}u^{(\lambda)}_{\alpha\beta}(p)
\end{aligned}
$$

where $\varepsilon^{(\lambda)}_\mu(p)$ are the usual spin-one polarization vectors corresponding to the mass $\sqrt{p^2}$. Then for $p^2 > 0$ we get the following orthogonality relations

$$
u^{(\lambda)}_{\mu\nu}(p)u^{(\lambda')\mu\nu}(p)^* = -2\delta^{\lambda\lambda'}
$$

$$
\begin{aligned}
w^{(\lambda)}_{\mu\nu}(p) w^{(\lambda')\mu\nu}(p)^* &= 2\delta^{\lambda\lambda'} \\
u^{(\lambda)}_{\mu\nu}(p) w^{(\lambda')\mu\nu}(p)^* &= 0
\end{aligned}
$$

and the projectors can be written for $p^2 > 0$ in terms of the polarization sums as

$$
\begin{aligned}
\Pi^L_{\mu\nu\alpha\beta}(p) &= -\frac{1}{2}\sum_\lambda u^{(\lambda)}_{\mu\nu}(p) u^{(\lambda)}_{\alpha\beta}(p)^* \\
\Pi^T_{\mu\nu\alpha\beta}(p) &= \frac{1}{2}\sum_\lambda w^{(\lambda)}_{\mu\nu}(p) w^{(\lambda)}_{\alpha\beta}(p)^*.
\end{aligned}
$$

Multiplying (155) by $u^{(\lambda)}_{\mu\nu}(p)^* u^{(\lambda)}_{\alpha\beta}(p)$ and $w^{(\lambda)}_{\mu\nu}(p)^* w^{(\lambda)}_{\alpha\beta}(p)$ respectively we get the positivity constraints for the spectral functions

$$
\begin{aligned}
0 &\leq \sum_N \delta^{(4)}(p - p_N) |\langle 0|R_{\mu\nu}(0)|N\rangle u^{(\lambda')\mu\nu}(p)^*|^2 = 2(2\pi)^{-3}\theta(p^0)\rho_L(p^2)p^2 \\
0 &\leq \sum_N \delta^{(4)}(p - p_N) |\langle 0|R_{\mu\nu}(0)|N\rangle w^{(\lambda')\mu\nu}(p)^*|^2 = 2(2\pi)^{-3}\theta(p^0)\rho_T(p^2)p^2.
\end{aligned}
$$

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