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On a possible compensation of the QCD vacuum contribution to the Dark Energy

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Abstract

We suggest one of the possible ways to compensate the large negative quantum-topological QCD contribution to the vacuum energy density of the Universe by means of a positive constant contribution from a cosmological Yang-Mills field. An important role of the exact particular solution for the Yang-Mills field corresponding to the finite-time instantons is discussed. An interesting connection of the compensation mechanism to the color confinement in the framework of instanton models has been pointed out. Besides the $\Lambda_{\rm QCD}$ scale, this proposal relies on one yet free dimensionless normalisation constant which cannot be fixed by the perturbative QCD theory, and thus should be fine-tuned for the exact compensation to hold.

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I. INTRODUCTION

Current accelerated expansion of the Universe is commonly attributed to the existence of the so-called Dark Energy which is confirmed in many cosmological observations so far, e.g. in studies of the type Ia Supernovae [1], cosmic microwave background anisotropies [2], large scale structure [3] etc. The Standard Cosmological Model is based on the time-independent Dark Energy approximation called the cosmological constant, or Λ-term, approximation which agrees well with current observational data. However, the problem of theoretical interpretation and prediction of fundamental properties of the Dark Energy (or the Λ-term) remains one of the major unsolved problem of Theoretical Physics [4]. For a comprehensive overview of existing theoretical models and interpretations of the Dark Energy, see e.g. Refs. [5–9] and references therein.

One of the traditional interpretations of the Λ-term is by means of the vacuum energy density satisfying the equation of state $P_\Lambda = -\Lambda$ with vacuum pressure P_Λ and energy density Λ. However, individual vacuum condensates known from particle physics e.g. those which are responsible for the chiral and gauge symmetries breaking in the Standard Model, contribute to the vacuum energy of the Universe individually exceeding the observable value of the Λ-term density $\Lambda_{\rm exp} = (3.0 \pm 0.7) \times 10^{-35}$ MeV^4 [2] by many orders of magnitude in absolute value [10]. This situation, which sometimes referred to as the "Vacuum Catastrophe" in the literature, requires extra hypotheses about (partial or complete) compensation of vacuum condensates of different types to the net vacuum energy density of the Universe (see e.g. Ref. [11]). A dynamical mechanism for such gross cancellations and corresponding major fine-tuning of vacuum parameters is yet not known and is a subject of ongoing intensive studies (for a review on this topic, see e.g. Ref. [5] and references therein).

Within the general problem of vacuum condensates cancellation, the QCD vacuum contribution has a special status. Various existing cancellation mechanisms refer essentially to an unknown high-scale physics beyond the Standard Model e.g. to Supersymmetry [5]. However, they cannot be applied for a compensation of the specifically non-perturbative and low-energy QCD contribution. In this paper, we focus primarily on elimination of this most "difficult" part of the vacuum energy of the Universe.

In the framework of the popular instanton liquid models [12], the topological (or instanton) modes of the QCD vacuum (which sometimes referred to as the quark-gluon condensate) are given essentially by the strong non-perturbative fluctuations of the gluon and light sea quark fields which are induced in processes of quantum tunneling of the gluon vacuum between topologically different classical states. The topological instanton-type contribution $\varepsilon_{vac(top)}$ to the energy density of the QCD vacuum is one of its main characteristics [13] and can be written as follows (see also Ref. [14])

$$\begin{aligned} \varepsilon_{vac(top)} \; &= \; -\frac{9}{32}\langle 0| : \frac{\alpha_s}{\pi} F^a_{\mu\nu}(x) F^{\mu\nu}_a(x) : |0\rangle + \frac{1}{4}\Big[\langle 0| : m_u \bar{u} u : |0\rangle + \langle 0| : m_d \bar{d} d : |0\rangle \\ &+ \; \langle 0| : m_s \bar{s} s : |0\rangle\Big] \simeq -(5 \pm 1) \times 10^9 \ \mathrm{MeV}^4 \,, \end{aligned} \tag{1.1}$$

which is composed of gluon and light sea u, d, s quark contributions. Clearly, other contributions of a different physical nature should compensate the topological QCD contribution (1.1) to the vacuum energy of the Universe since its value by far is not compatible with the cosmological observations and data on the Λ-term value [2]. This issue triggers the search for possible cancellation mechanisms, and one such mechanism will be discussed further in this paper.

II. CLASSICAL EVOLUTION OF THE COSMOLOGICAL YANG-MILLS FIELDS

Consider one of the possible ways to eliminate the *microscopic* QCD vacuum contribution (1.1) to the vacuum energy density of the Universe introducing the hypothesis about the existence of the cosmological *macroscopic* Yang-Mills fields in early Universe.

Cosmological solutions for classical Yang-Mills fields have a long history referring back to the late seventies, when there was an active search for solutions to the Einstein-Yang-Mills field equations [15]. Later, the role of non-Abelian gauge fields in the early Universe evolution has been intensively studied in many different aspects, in particular, in the context of the Dark Energy [16] and non-Abelian fields driven inflation without a presence of a scalar field ("gauge-flation") [17]. Practically, there are no any physical arguments which could forbid the existence of the homogeneous non-Abelian gauge field with unbroken $SU(N)$ symmetry at cosmological scales with an isotropic energy-momentum tensor [18], possibly originating from the inflationary stage of the Universe evolution [17].

Let us now assume that a chromodynamical (gluon) field with unbroken color $SU(3)_c$ symmetry exists as a real physical object filling up the early Universe, and the subject of our further discussion concerns possible physical states of this field and their real-time dynamics. For simplicity, we work in the flat Friedmann Universe with conformal metric $g_{\mu\nu} = a^2(\eta)g_{\mu\nu(\mathrm{M})}$, where $g_{\mu\nu(\mathrm{M})}$ is the Minkowski metric. The Einstein equations with energy-momentum tensor of classical Yang-Mills fields are [15]

$$\begin{aligned}
&\frac{1}{\varkappa}\left(R^{\nu}_{\mu}-\frac{1}{2}\delta^{\nu}_{\mu}R\right)=\frac{1}{g^2_{\mathrm{YM}}}\frac{1}{\sqrt{-g}}\left(-F^a_{\mu\lambda}F^{\nu\lambda}_a+\frac{1}{4}\delta^{\nu}_{\mu}F^a_{\sigma\lambda}F^{\sigma\lambda}_a\right)\,, \qquad \sqrt{-g}=a^4(\eta)\,,\\
&\left(\frac{\delta^{ab}}{\sqrt{-g}}\partial_\nu\sqrt{-g}-f^{abc}A^c_\nu\right)\frac{F^{\mu\nu}_b}{\sqrt{-g}}=0\,, \quad F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+f^{abc}A^b_\mu A^c_\nu\,.
\end{aligned} \tag{2.1}$$

This system is written in the most trivial form without taking into account interactions of the macroscopic Yang-Mills field with the physical vacuum (no vacuum polarisation effects are included here) and other forms of matter (i.e. $\varepsilon = 0$). Here and below, raising and lowering Lorenz indices are done by the Minkowski metric $g_{\mu\nu(\mathrm{M})}$ as usual.

Since initial conditions in the early Universe are quite arbitrary, it is meaningful to start with the study of spatially-homogeneous and isotropic modes of the gluon field [18]. A specific feature of such modes concerns their distinct topological structure where the isotopic and spatial indices are mixed up. In the case of Hamiltonian gauge $A^a_0 = 0$ and homogeneous and isotropic 3-space we have the following simple structure of these modes:

$$A^a_i=\begin{cases}\ \delta^a_i A(\eta), & i,a=1,2,3\\ 0, & i=1,2,3;\ a>3\,,\end{cases} \tag{2.2}$$

with a single non-trivial time-dependent degree of freedom $A(\eta)$ to be studied in what follows. In this case, the classical Yang-Mills equations (2.1) read

$$\frac{3}{\varkappa}\frac{a'^2}{a^4}\;=\;\frac{3}{2g^2_{\mathrm{YM}}a^4}\left(A'^2+A^4\right)\,, \qquad A''+2A^3=0\,, \tag{2.3}$$

and thus completely determine the conformal time evolution of the spatially-homogeneous and isotropic Yang-Mills field. The second equation in Eq. (2.3) can be exactly integrated, and its general solution implicitly corresponds to non-linear oscillations, i.e.

$$A'^2+A^4=C^4, \quad \int_{A_0}^{A}\frac{dA}{\sqrt{C^4-A^4}}=\eta, \tag{2.4}$$

with C, A_0 being integration constants. Numerical solution of Eq. (2.3) for the gluon field potential with initial condition $A'(0) = 0$ and an arbitrary amplitude $A_0 = C$ to a good accuracy can be approximated by

$$A(\eta) \simeq A_0 \cos\left(\frac{6}{5}\, A_0 \eta\right). \tag{2.5}$$

An essentially non-linear character of oscillations of the classical YM field is thus emerged in explicit dependence of their amplitude on frequency. According to Eqs. (2.3) and (2.4), the spatially-homogeneous classical YM field in the isotropic Universe behaves as an ultra-relativistic medium with energy density $\varepsilon_{\rm YM} \sim 1/a^4$ and equation of state $p_{\rm YM} = \varepsilon_{\rm YM}/3$ [18].

III. ROLE OF THE VACUUM POLARISATION

Can a classical Yang-Mills field be a component of the cosmological medium in the radiation-dominated Universe? A simple analysis have shown that the classical spatially-homogeneous Yang-Mills field cannot exist in the early Universe since the classical Yang-Mills equations (2.1) are not form-invariant and unstable with respect to radiative corrections. Such an instability emerges due to the fact that there is no any threshold for vacuum polarization of a massless non-linear gauge field, i.e. any infinitesimally small external field is capable of reconstruction of the classical Yang-Mills vacuum [19]. Due to non-linearity of initial operator Yang-Mills equations the vacuum polarization of the massless quantum gluon field by its classical component leads to a modification of classical equations. In practice, we deal with the Savvidy equations for the Savvidy vacuum fluctuations [19] and look for their spatially-homogeneous modes. As was also stressed in Ref. [20], similar quantum effects such as the gluon condensation and the vacuum polarization effects can be important for generation of an effective cosmological constant with a negative equation of state in the system of coupled Born-Infeld and gravitational fields in early Universe.

Let us analyze the Yang-Mills equations of motion incorporating the vacuum polarisation effects. The Lagrangian of the gluon field taking into account the vacuum polarisation in the one-loop approximation has the following form [19]:

$$L_{\rm YM} = -\frac{11}{128\pi^2}\frac{F^a_{\mu\nu}F^{\mu\nu}_a}{\sqrt{-g}} \ln\left(\frac{J}{\Lambda^4_{\rm QCD}}\right), \qquad J = \frac{1}{\xi^4}\frac{|F^a_{\alpha\beta}F^{\alpha\beta}_a|}{\sqrt{-g}}\,.$$

Here, the numerical parameter ξ is not fixed and reflects an ambiguity in normalisation of the corresponding gauge/Lorentz invariant J. Such a Lagrangian leads to a modified system of equations for gravitational and Yang-Mills fields in the isotropic Universe with vacuum polarisation effects incorporated, namely,

$$\begin{aligned} \frac{1}{\varkappa}\left(R^\nu_\mu - \frac{1}{2}\delta^\nu_\mu R\right) = T^{\nu,\,{\rm mat}}_\mu + \bar\Lambda\delta^\nu_\mu + \frac{11}{32\pi^2}\frac{1}{\sqrt{-g}}\Bigg[\Bigg(-F^a_{\mu\lambda}F^{\nu\lambda}_a \\ + \frac{1}{4}\delta^\nu_\mu F^a_{\sigma\lambda}F^{\sigma\lambda}_a\Bigg)\ln\frac{e|F^a_{\alpha\beta}F^{\alpha\beta}_a|}{\sqrt{-g}\,(\xi\Lambda_{\rm QCD})^4} - \frac{1}{4}\delta^\nu_\mu F^a_{\sigma\lambda}F^{\sigma\lambda}_a\Bigg]\,, \end{aligned} \tag{3.1}$$

$$\left(\frac{\delta^{ab}}{\sqrt{-g}}\partial_\nu\sqrt{-g} - f^{abc}A^c_\nu\right)\left(\frac{F^{\mu\nu}_b}{\sqrt{-g}}\ln\frac{e|F^a_{\alpha\beta}F^{\alpha\beta}_a|}{\sqrt{-g}\,(\xi\Lambda_{\rm QCD})^4}\right) = 0\,,$$

where $e \simeq 2.71$ is the base of the natural logarithm; $\Lambda_{\rm QCD}$ is the QCD energy scale; $T_\mu^{\nu,\,\rm mat} = (\varepsilon + p)u_\mu u^\nu - \delta_\mu^\nu p$ is the energy-momentum tensor of all components of the cosmological medium except for the macroscopic Yang-Mills field; $\bar\Lambda = \Lambda_{\rm inst} + \Lambda_{\rm cosm} + \ldots$ is the total contribution to the vacuum energy density which consists of the *non-perturbative* spatially-inhomogeneous (topological) quantum fluctuations of the gluon and quark fields (quark-gluon condensate) of an instanton nature (1.1), $\Lambda_{\rm inst} \equiv \varepsilon_{vac(top)} \simeq -(5\pm 1)\times 10^9\,{\rm MeV}^4$, an uncompensated contribution from the observable cosmological Λ-term, $\Lambda_{\rm cosm}$, and dots represent all other *perturbative* vacua contributions. The Λ-term value, $\Lambda_{\rm cosm}$, could have a different nature, other than topological non-perturbative one in QCD or perturbative ones in high-energy particle physics, so we explicitly separated it from the rest. Form now on, we implicitly assume that perturbative components of the net vacuum energy density from all other microscopic vacuum condensates in particle physics are compensated elsewhere at high energy scales and do not enter the vacuum energy density of the Universe, so $\bar\Lambda = \Lambda_{\rm inst} + \Lambda_{\rm cosm}$.

The system of equations (3.1) is written in the most general form including all forms of matter, as well as the uncompensated quark-gluon condensate contribution $\Lambda_{\rm inst}$ and the observable cosmological Λ-term $\Lambda_{\rm cosm}$. The components of the energy-momentum tensor for the homogeneous and isotropic modes specified in Eq. (2.2) have the following generic form:

$$
\begin{aligned}
T_0^{0,\,\rm tot} &= T_0^{0,\,\rm mat} + \bar\Lambda + \frac{33}{64\pi^2}\frac{1}{a^4}\left[(A'^2 + A^4)\ln\frac{6e|A'^2 - A^4|}{a^4(\xi\Lambda_{\rm QCD})^4} + A'^2 - A^4\right], \quad T_0^{\beta,\,\rm tot} = T_0^{\beta,\,\rm mat}, \\
T_\alpha^{\beta,\,\rm tot} &= T_\alpha^{\beta,\,\rm mat} + \bar\Lambda\delta_\alpha^\beta + \frac{11}{32\pi^2}\frac{1}{a^4}\delta_\alpha^\beta\left[-\frac{1}{2}(A'^2 + A^4)\ln\frac{6e|A'^2 - A^4|}{a^4(\xi\Lambda_{\rm QCD})^4} + \frac{3}{2}(A'^2 - A^4)\right] \qquad (3.2)
\end{aligned}
$$

In flat and isotropic Universe, trace of the Einstein equations and the equation of motion of the macroscopic gluon field read, respectively,

$$
\frac{6}{\varkappa}\frac{a''}{a^3} = \varepsilon - 3p + 4\bar\Lambda + T_\mu^{\mu,\,\rm YM}, \quad T_\mu^{\mu,\,\rm YM} = \frac{33}{16\pi^2}\frac{1}{a^4}\left(A'^2 - A^4\right), \qquad (3.3)
$$

$$
\frac{\partial}{\partial\eta}\left(A'\ln\frac{6e|A'^2 - A^4|}{a^4(\xi\Lambda_{\rm QCD})^4}\right) + 2A^3\ln\frac{6e|A'^2 - A^4|}{a^4(\xi\Lambda_{\rm QCD})^4} = 0. \qquad (3.4)
$$

It is straightforward to show that the $(0,0)$ Einstein equation

$$
\frac{3}{\varkappa}\frac{a'^2}{a^4} = \varepsilon + \bar\Lambda + \frac{33}{64\pi^2}\frac{1}{a^4}\left[\left(A'^2 + A^4\right)\ln\frac{6e|A'^2 - A^4|}{a^4(\xi\Lambda_{\rm QCD})^4} + A'^2 - A^4\right] \qquad (3.5)
$$

is the exact first integral of the system of equations (3.3) and (3.4), while the exact first integral of second equation (3.4) is

$$
\frac{6e(A'^2 - A^4)}{a^4(\xi\Lambda_{\rm QCD})^4} = 1\,. \qquad (3.6)
$$

The latter leads to a considerable simplification of the energy-momentum tensor, namely,

$$
\begin{aligned}
T_0^{0,\,\rm tot} &= T_0^{0,\,\rm mat} + \bar\Lambda + \frac{33}{64\pi^2}\frac{(\xi\Lambda_{\rm QCD})^4}{6e}\,, \\
T_\alpha^{\beta,\,\rm tot} &= T_\alpha^{\beta,\,\rm mat} + \left(\bar\Lambda + \frac{33}{64\pi^2}\frac{(\xi\Lambda_{\rm QCD})^4}{6e}\right)\delta_\alpha^\beta\,. \qquad (3.7)
\end{aligned}
$$

Now we can observe an interesting possibility to eliminate the microscopic *negative QCD contribution* to the vacuum energy, $\Lambda_{\rm inst}$, by means of the *constant positive contribution* from the spatially-homogeneous mode of macroscopic gluon field. A small non-compensated remnant – the observable Λ-term, $\Lambda_{\rm cosm}$ – can, in principle, have a different nature which will be discussed in our forthcoming publication. The corresponding condition for the $\Lambda_{\rm inst} \simeq -265^4\,{\rm MeV}^4$ compensation

$$\frac{33}{64\pi^2}\frac{(\xi\Lambda_{\rm QCD})^4}{6e} + \Lambda_{\rm inst} = 0, \quad \Lambda_{\rm QCD} \simeq 280\,{\rm MeV}\,, \tag{3.8}$$

however, is not fully automatic; it is satisfied for a certain value of the normalisation parameter only, $\xi \simeq 4$, which should be constrained in a complete theory of the QCD vacuum. Therefore, in principle, one succeeds to eliminate the huge negative contribution from spatially-inhomogeneous non-perturbative quantum fluctuations of the gluon field by means of a positive contribution from fluctuations of spatially-homogeneous macroscopic gluon field. This is achieved by fixing the remaining freedom in normalization of the Yang-Mills invariant J in the Lagrangian (3.1). As we will see below, both mutually compensating contributions to the vacuum energy density of the Universe have a common instanton nature.

IV. COSMOLOGICAL EVOLUTION OF FINITE-TIME INSTANTONS

Together with the compensation condition (3.8) and the first integrals (3.5) and (3.6), the resulting system of equations (3.3) and (3.4) is dramatically reduced to the following simple form:

$$\frac{3}{\varkappa}\frac{a'^2}{a^4} = \varepsilon + \Lambda_{\rm cosm}, \tag{4.1}$$

$$A'^2 - A^4 = a^4\frac{(\xi\Lambda_{\rm QCD})^4}{6e}\,. \tag{4.2}$$

Notice that under the exact cancellation condition (3.8) the cosmological (macroscopic) evolution of the Friedmann Universe given by the scale factor $a = a(\eta)$ is now completely decoupled from the microscopic evolution of the gluon field $A = A(\eta)$. The physical time scale for the cosmological evolution is of the order of the Universe age $t_{\rm cosm} \sim 1/H$ (in terms of the Hubble parameter H), while the typical time scale for the Yang-Mills field evolution is of the order of the hadronisation time $t_{\rm hadr} \sim 1/\Lambda_{\rm QCD}$. So at present epoch the right hand side of Eq. (4.2) can be taken to be constant in time to a good accuracy, or more precisely, given by a classical solution of the Friedmann equation (4.1). In practice, this means that the dynamical cancellation under the condition (3.8) and, hence, the decoupling of the QCD vacuum fluctuations from the hot cosmological plasma have effectively happened at the end of the hadronisation epoch in the early Universe evolution.

For convenience, let us rewrite the Yang-Mills equation (4.2) in terms of dimensionless time and gauge field as follows

$$\left(\frac{d\tilde{A}}{d\tilde{\eta}}\right)^2 - \tilde{A}^4 = 1\,, \qquad \tilde{A} = A\,\frac{(6e)^{1/4}}{\xi\Lambda_{\rm QCD}} \simeq \frac{A}{2\Lambda_{\rm QCD}}\,, \qquad \tilde{\eta} = \eta\,\frac{\xi\Lambda_{\rm QCD}}{(6e)^{1/4}} \simeq 2\Lambda_{\rm QCD}\,\eta\,, \tag{4.3}$$

for $\xi \simeq 4$. For simplicity, we have chosen the initial values of the Yang-Mills field and the scale factor as follows:

$$\tilde{A}(\tilde{\eta}_0 = 0) \equiv \tilde{A}_0 = 0, \qquad a(\tilde{\eta}_0 = 0) \equiv a_0 = 1\,. \tag{4.4}$$

We can see now that, indeed, the time scale of the Yang-Mills field fluctuations is essentially microscopic and corresponds to the $\Lambda_{\rm QCD}$ energy scale. The equation (4.3) can then be easily integrated, and its general solution can be written in the following implicit form:

$$\int_{\tilde{A}_0}^{\tilde{A}} \frac{d\tilde{A}}{\sqrt{1+\tilde{A}^4}} = \tilde{\eta}\,, \qquad (4.5)$$

where $\tilde{A}_0$ is an integration constant. Notably, the analytical solution (4.5) taking into account the QCD vacuum polarisation, in fact, differs from the classical Yang-Mills solution (2.4) by sign in front of $\tilde{A}^4$ under the square root only, having though a significant effect on its time dependence. Moreover, since the solution (4.5) was obtained under the exact cancellation condition (3.8), it corresponds to the minimal energy of the QCD system in the ground state of the Universe and, hence, is physically preferable.

For the initial conditions given by Eq. (4.4) (independently on a particular $\dot{a}_0$ value), the solution for $\tilde{A}(\tilde{\eta})$ (4.5) obeys the following properties:

- Symmetry: $\tilde{A}(-\tilde{\eta}) = -\tilde{A}(\tilde{\eta})$.
- Periodicity: $\tilde{A}(\tilde{\eta} \pm T) = \tilde{A}(\tilde{\eta})$.
- Continuous intervals and singularities: $\tilde{A}(\tilde{\eta} \to \pm T/4) = \pm\infty$.

Most importantly, it is continuous only at a finite microscopically-small time interval $T \sim 1/\Lambda_{\rm QCD}$ and corresponds to spatially-homogeneous pulses of the gluon field potential with a constant energy-density. An analogous effect in a modified Maxwell-$F(R)$ gravity was observed in Ref. [7] where the large-scale magnetic fields are generated due to the breaking of the conformal invariance of the electromagnetic field through its non-minimal gravitational coupling.

We stress also that once the compensation condition (3.8) has been satisfied at a particular moment in time, it holds true for any later times, and the YM fields and large negative $\Lambda_{\rm inst}$ disappear from the resulting equations and do not participate in the Universe evolution any longer. We, therefore, arrive at quasistationary regime when the Universe evolution is completely determined by usual matter and uncompensated cosmological Λ-term only.

What is the physical interpretation of the result (4.5)? Obviously, such a solution with regular singularities does not have a quasiclassical interpretation (except for a vicinity of the midpoints of the continuous intervals where the gluon field potential is small and slowly changing). In practice, we deal with a sequence of quantum fluctuations of the YM field in time, or, in fact, with *the finite-time instantons*. The creation and annihilation of such finite-time instantons should have essentially quantum nature like the QCD instantons. In order to regularize singularities in the quasiclassical solution (4.5) one needs to turn into a complete quantum theory taking into account quantum corrections due to e.g. fermion-antifermion pair creation and annihilation processes in the early Universe.

V. CONCLUSION

From the group theory point of view, our proposal is analogical to the standard instantons theory in QCD [12]. In both these cases one deals with the mapping of 3-space onto $SU(2)$ subgroup elements, so the analogy between the resulting instanton solutions is rather close.

Indeed, based on the quasiclassical result (4.5) only, one can naively conjecture that *the cosmological evolution of the YM field emerges a sequence of quantum tunneling transitions through the time barriers* represented by the regular singularities in the solution (4.5). We have observed that the positive constant energy density of spatially-homogeneous finite-time instantons in the early Universe can be cancelled with the negative constant QCD contribution from spatially-inhomogeneous gluon field fluctuations induced by a similar quantum tunneling of the gluon field, but through spatial (not time!) topological barriers between different classical vacua. This exhibits a remarkable similarity and interplay between instantons of different types in the early Universe evolution. Besides the $\Lambda_{\rm QCD}$ scale parameter, a degree of such a cancellation at the moment relies on one yet free dimensionless normalisation constant ξ which cannot be fixed by known perturbative QCD theory, and thus should be fine-tuned for the exact compensation to hold. This freedom must be eventually fixed by non-perturbative QCD dynamics.

At the level of the Einstein equations, we have explicitly shown that an arbitrary $SU(2)$ configuration of the cosmological Yang-Mills field leads to a color neutral ("white") contribution to the energy-momentum tensor which is represented in the form of Lorentz-invariant Λ-term. Thus, there is no any danger that the cosmological Yang-Mills field leading to explicitly "white" observables upon averaging over all stochastic $SU(2)$ configurations would violate well-known symmetries of the QCD theory, and quark fields cannot affect this picture.

From the point of view of quantum tunneling, the chain of quantum fluctuations in a certain approximation can be considered as non-linear oscillations. In order to regularize the infinite end-points of the continuous time intervals within the quasiclassical approach, one could therefore consider a continuous smearing of the resulting fluctuations by means of a non-linear continuous parameterization such as

$$\tilde{A}_{appr}(\tilde{\eta}) = \frac{1}{a\,\sin(\omega\tilde{\eta}) + b\,\dfrac{\cos^2(\omega\tilde{\eta})}{\sin(\omega\tilde{\eta})}} \tag{5.1}$$

with adjustable parameters a, b and ω. This approximation has been qualitatively compared to the exact quasiclassical solution (4.5) and approaches it in the limit of small $a \to 0$, while the classical non-linear solution (2.4) is reached in the limit $a \to b$ (up to an arbitrary initial phase) with ω being dependent on the initial amplitude. Thus, the parameterization (5.1) represents a simple continuous interpolation between the classical and quasiclassical solutions, and can be used in practical calculations in the quasiclassical limit of the theory.

To summarize, the space-time dynamics of colored quarks and gluons has to be considered from the QCD confinement point of view. One should emphasize that a formal singularity in the gluon field potential (4.5) simply means that a quark (and a gluon) in the Universe cannot experience free motion in 3-space during the time periods larger than the typical time scale of confinement, $\sim \Lambda_{\rm QCD}^{-1}$. In this sense, our solution (4.5) reflects the "time" aspect of confinement. Besides the extremely small Λ-term problem, the exact compensation mechanism, proposed above, can be viewed as a manifestation of the QCD confinement since there are practically no non-zeroth gluon fields propagating at the length scales larger than the typical hadron scale $\sim$ 1 fm, and they certainly disappear at macroscopically large cosmological scales typical for modern Universe. The perturbative higher order QCD corrections which affect the QCD β-function are typically small and do not change the qualitative picture described above. A deeper theoretical investigation of these aspects is essential for both non-perturbative QCD and Cosmology, and is planned for further studies.

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