# Collectivity without plasma in hadronic collisions 

Christian Bierlich, Gösta Gustafson, and Leif Lönnblad<br>Lund University*

(Dated: October 27, 2017)


#### Abstract

We present a microscopic model for collective effects in high multiplicity proton-proton collisions, where multiple partonic subcollisions give rise to a dense system of strings. From lattice calculations we know that QCD strings are transversely extended, and we argue that this should result in a transverse pressure and expansion, similar to the flow in a deconfined plasma. The model is implemented in the PYTHIA8 Monte Carlo event generator, and we find that it can qualitatively reproduce the long range azimuthal correlations forming a near-side ridge in high multiplicity proton-proton events at LHC energies


Introduction - The general features of proton-proton collisions, such as jets, multiplicity distributions, and (approximate) particle ratios, can be described by dynamical models based on string [1] or cluster [2] hadronisation, e.g. Pythia8 [3, 4] and Herwig7 [5], in a very satisfactory way. In contrast, heavy ion collisions show collective features interpreted as flow in a deconfined, thermalized plasma [6, 7]. Nucleus collisions also show higher rates for strangeness [8]. These features have been interpreted as indicating fundamentally different dynamics in the two systems.

There are, however, also many similarities between collisions in small and in large systems. Many features in nucleus collisions, such as multiplicity distributions and particle distributions in rapidity and $p_{\perp}$, could fairly well be described by early models based on non-interacting strings (e.g. DPM [9] and Fritiof [10]). On the other hand recent precise measurements at the LHC show flow-like effects also in pp and $\mathrm{p} A$ collisions 11 13]. They also show increasing strangeness and baryon rates in pp events with high multiplicity [14]. This has raised the question if a QGP is formed also in small collision systems. Conversely, one could instead ask if collective effects in nucleus collisions could possibly originate from non-thermal interactions between string-like colour fields. This would entail a picture where collective phenomena does not arise from the formation of a deconfined QGP state, but rather as an emergent behaviour of dense configurations of confined QCD flux tubes. A third possibility is if the two pictures coexist, with a dense thermalised central "core" and an outer "corona". Such a picture is implemented in the quite successful EPOS model [15, 16].

It was early suggested that the many strings in an $A A$ collision may interact coherently as "ropes" 17]. Rope formation or "percolation" have subsequently been studied by several authors, see e.g. refs. [18-28]. Generally these analyses have predicted higher rates for strangeness and baryons, and larger transverse momenta, due to the stronger field in the rope.

The high energy density in overlapping strings also ought to give a transverse pressure, resulting in a transverse expansion seen as a transverse flow. This should give not only enhanced transverse momenta, in particular for high mass particles, but would also give rise to angular correlations. Such correlations were early considered by Abramovsky et al. [29], and a Monte Carlo "toy model" studying this effect in PbPb collisions was presented in [30].

A high string density can also be reached in pp and $\mathrm{p} A$ collisions at high enough energies,
allowing for rope formation also in these smaller systems. In ref. [31] we presented a proof of principle for a flow-like transverse expansion, due to overlapping strings in high energy pp collisions. In this letter we want use the concept of overlapping strings to study this effect more thoroughly, and compare the results with experimental data. To that effect we have implemented the resulting model in the PYTHIA8 event generator, and find good agreement between the model and the pp "ridge" first observed by the CMS collaboration [32].

The Lund string - The confining force field between coloured partons is in the Lund string model [1] described by a "massless relativistic string", which represents an idealised picture of a flux tube with no transverse extension. Like a linear electric field, the string is invariant under longitudinal boosts, and has no momentum in the direction of the string (apart from the force on the endpoints). Gluons are treated as point-like transverse excitations on the string [33], and for a set of colour connected partons, moving apart from a common origin, the string is stretched from a quark via the colour-ordered gluons to an antiquark.

The most simple situation is a quark and an antiquark produced in an $e^{+} e^{-}$annihilation, and moving apart. A string is stretched from the common production point, and breaks into pieces, hadrons, by repeated production of $q \bar{q}$ pairs, as sketched in fig. [1] The boost invariant string is reflected in a boost invariant distribution of the hadrons. The spacetime separations between the production points are space-like, and in each frame the hadrons with the smallest energies in this specific frame, are produced first in time.

The probability for a specific final state is given by [34]

$$
\begin{equation*}
d \mathcal{P} \propto \exp (-b A) \times d \Omega \tag{1}
\end{equation*}
$$

Here $b A$ is (the imaginary part of) the action for the relativistic string, with $A$ equal to the space-time area (in units of the string tension $\kappa$ ) covered by the string before its breakup into hadrons (see fig. [1). For a single hadron species with mass $m$, the $n$ particle phase space $\Omega$ (in $1+1$ dimensions) is given by $d \Omega=\prod_{i=1}^{n}\left[N d^{2} p_{i} \delta\left(p_{i}^{2}-m^{2}\right)\right] \delta^{(2)}\left(\sum p_{i}-P_{\text {tot }}\right)$. The parameter $N$ here specifies the relation between the phase space for $n+1$ and $n$ particles.

For a straight string the area $A$ is easily expressed in terms of the hadron momenta, and the result can be generated by successively peeling off the hadrons from the quark or antiquark ends. Here each hadron is given a fraction $z$ of the remaining light-cone


FIG. 1. Breakup of a string between a quark and an antiquark in a $x-t$ diagram. New $q \bar{q}$ pairs are produced around a hyperbola, and combine to the outgoing hadrons. The original $q$ and $\bar{q}$ move along light-like trajectories. The area enclosed by the quark lines is the coherence area $A$ in eq. (11), in units of the string tension $\kappa$.
momentum, determined by the distribution (in case of a single hadron species with mass $m$ )

$$
\begin{equation*}
f(z)=N z^{a} \exp \left(-b m^{2} / z\right) \tag{2}
\end{equation*}
$$

The three parameters $N, a$, and $b$ are related by normalisation, leaving two parameters to be determined by experiments.

The breakup points for the string are located around a hyperbola in spacetime, with a typical proper time given by

$$
\begin{equation*}
\left\langle\tau^{2}\right\rangle=\frac{1+a}{b \kappa^{2}} \tag{3}
\end{equation*}
$$

where $\kappa$ is the tension of the string. With values $a=0.68, b=0.98 \mathrm{GeV}^{-2}$ (the default values in Pythia8), and $\kappa=0.9-1 \mathrm{GeV} / \mathrm{fm}$, we obtain the typical breaking time around 2 fm . Thus the string breaks typically when the original quark and antiquark are about 4 fm apart. The string then breaks in two pieces, which move apart keeping their size, with the new antiquark trailing after the initial quark increasing its energy due to the pull from the string, as illustrated in fig. [1. Eventually the string breaks again, and in the successive breaks the string pieces become smaller and smaller.

Interactions between strings - We note that just after the production of the initial $q \bar{q}$ pair, the colour field is necessarily compressed, not only longitudinally but also transversely. Also in a high energy pp collision the strings are stretched between charges emerging from a very limited region within two Lorentz contracted pancakes. As illustrated in fig. 2, the flux tube expands both longitudinally and transversely with the speed of light. Two neighbouring flux tubes will then start to overlap and interact close to $z=0$ (where $z$ is the longitudinal


FIG. 2. (a) The forcefield between a quark and an antiquark separating from a common origin expands both longitudinally and transversely, until the transverse extension saturates. (b) The expansion is boost invariant. Therefore, in any frame two parallel flux tubes begin to overlap and interact in the centre in the specific frame chosen.
coordinate) in the specific frame used in the analysis, as this is where the flux tube expands most rapidly. As seen in fig. 1 this is also the region where those particles are produced, which are slow in this particular frame.

The repulsion gives the flux tubes a transverse velocity, a process which we will refer to as shoving. In a pp collision the density of strings is not too high, and the time until breakup ( $\tau \sim 2 \mathrm{fm}$ ) is large compared to both the width of the flux tubes and the radius of the proton (both $<1 \mathrm{fm}$ ). We therefore expect that the force field has (almost) reached its equilibrium transverse width, before it breaks up into hadrons. These equilibrium flux tubes may be a single triplet string, or a rope stretched between higher colour multiplets, as discussed in ref. [28].

The different time scales imply that the process has three separate phases: i) An initial phase where the individual flux tubes expand but still do not interact. ii) A second phase where the flux tubes interact, repel each other, and reach equilibrium triplet strings or ropes specified by definite $\mathrm{SU}(3)$ multiplets. iii) A final phase in which the flux tubes break into hadrons. Naturally the boundaries between the phases are not sharp, but we will in this paper assume that they can be treated separately.

Flux tube repulsion - Colour flux tubes are similar to vortex lines in a superconductor. In a type I superconductor there is a homogenous field within the tube (similar to the confined field in the bag model), while in a type II superconductor the field falls off exponentially
outside a thin core. Lattice calculations indicate that the properties of a colour flux tube lie in between type I and II superconductors, with field strength close to a Gaussian [35].

If the energy in the flux tube would be dominated by a longitudinal colour-electric field, the interaction between them would be given by their overlap. For Gaussian distributions this would give a force per unit string length

$$
\begin{equation*}
\frac{\delta F\left(d_{\perp}\right)}{\delta z} \equiv f\left(d_{\perp}\right)=\frac{g \kappa d_{\perp}}{R^{2}} \exp \left(-\frac{d_{\perp}^{2}(t)}{4 R^{2}}\right) \tag{4}
\end{equation*}
$$

Here $d_{\perp}$ is the transverse separation between the flux tubes, $R$ is the (time dependent) width of the energy distribution in a single flux tube, and $\kappa$ the tension in a single string. In eq. (4) we have included a tunable parameter $g$, and the situation with a dominating colour-electric field would correspond to $g=1$. We note, however, that in the bag model (similar to a type I superconductor) half of the string energy corresponds to destroying the condensate inside the flux tube, and in a type II superconductor the larger part of the energy lies in the current keeping the flux together. Due to these uncertainties we expect that $g$ might deviate from 1, possibly within an order of magnitude. We motivated the Gaussian form in eq. (4) by lattice results, but the end result is more sensitive to the value of $g$, than the shape of the field.

For a boost invariant system it is convenient to introduce hyperbolic coordinates

$$
\begin{equation*}
\tilde{\tau}=\sqrt{t^{2}-z^{2}}, \quad \eta=\ln ((t+z) / \tilde{\tau}) \tag{5}
\end{equation*}
$$

(Note that $\tilde{\tau}$ is boost invariant, but not equal to the eigentime, given by $\tau=\sqrt{t^{2}-z^{2}-\mathbf{x}_{\perp}^{2}}$.) Near $z=0$ we get $\delta z=t \delta \eta$, and the force in eq. (4) gives $d p_{\perp} / d t \delta z=f\left(d_{\perp}\right)$. (For nonrelativistic velocities this gives the acceleration $d v_{\perp} / d t=\delta z f\left(d_{\perp}\right) /(\delta z \kappa)=f\left(d_{\perp}\right) / \kappa$.) Boost invariance then gives the two equations

$$
\begin{equation*}
\frac{d p_{\perp}}{\tilde{\tau} d \tilde{\tau} d \eta}=f\left(d_{\perp}\right), \quad \frac{d^{2} d_{\perp}}{d \tilde{\tau}^{2}}=\frac{f\left(d_{\perp}\right)}{\kappa} . \tag{6}
\end{equation*}
$$

When the flux tubes have separated, the boost invariant total $p_{\perp}$ per unit $\eta$ is given by integration of eqs. (6).

Hadronisation - The dynamics of the relativistic string is well described in ref. [36]. The motion of a string stretched such that $d p_{\perp} / d \eta$ is constant and along the $x$ direction, is described by a smoothly bent curve in space-time (for $t \geq 0$ )

$$
\begin{equation*}
x_{\mu}=(\tilde{\tau} \cosh (\eta) ; \alpha \operatorname{arcsh}(\tilde{\tau} / \alpha), 0, \tilde{\tau} \sinh (\eta)) \tag{7}
\end{equation*}
$$

The transverse momentum is here specified by the parameter $\alpha=\left(d p_{\perp} / d \eta\right) / \kappa$. We note that the transverse coordinate $x$ is independent of $\eta$, and the result is explicitly boost invariant. The expression in eq. (7) does not explicitly depend on the total energy W , which only shows up in the kinematical boundaries $0<|\sigma|<W / 2$ and $t<W / 2$, after which time the string ends are pulled back again.

We note that the string state discussed here is very different from a straight string boosted transversely with some velocity $v_{\perp}$. The directions of the initial quark and antiquark would then be rotated an angle $\theta=\operatorname{arcsine} v_{\perp}$, and the produced hadrons would be limited to the rapidity range between plus and minus $\log (\cot (\theta / 2))$. The distribution would thus not be boost invariant, but depend on the Lorentz frame used for the transverse boost.

When calculating the breakup into hadrons, of a string described in eq. (7), we realise that when the string is not straight, the relation between the space-time area $A$ in eq. (1) and the final hadron momenta in the splitting function in eq. (2) will be modified. We plan to discuss how to solve this problem in a coming paper (inspired by earlier work in ref. [37]), and we will in the implementation used in this letter apply an approximate method, described in the next paragraph.

Monte Carlo Implementation - We have used the PyTHiA8 event generator [4] to study the effect of string repulsion, and we simulate the shoving mechanism by adding soft gluons to the strings. As the string interaction depends on their positions in impact parameter space, and since the models for multi-parton interaction (MPI) and parton showers in Pythia8 make no attempt to calculate parton positions in transverse space, we have devised and implemented a simple model for this. Here we assume a 2D Gaussian mass distribution of the protons, and pick the transverse coordinates for each MPI according to the convolution of the mass distributions at the impact parameter of the collision [38].

In a real pp collision many strings are stretched between partons forming minijets, or occasional hard jets. The string pieces between these partons, which we call (colour) dipoles, will not be fully aligned with the beam axis. In our current implementation the system is separated in segments along the beam axis, which are presently 0.1 rapidity units wide. One string may here overlap with several other strings, and within each segment the shoving is implemented as the sum of many small kicks between all pairs of overlapping string pieces. This is done in several time steps. After each time step, the string configurations are updated, and the process continues until hadronisation time is reached ( $\tau \sim 2 \mathrm{fm}$ after
the initial collision). We allow the partons to move a finite time ( $\sim 1 \mathrm{fm}$ ) before they begin to interact with each other. This also accounts for the fact that it takes some time for the strings to extend transversely, before they feel the transverse repulsion.

For technical reasons PYTHIA8 cannot handle very soft gluons properly. Thus, if the state contains a group of colour-connected gluons with invariant mass below a cut ( $\sim 0.2 \mathrm{GeV}$ ), they will in the program be replaced by a single, harder, gluon. The result of this substitution is that the extra transverse momentum per unit rapidity, due to the shoving, is correctly reproduced. However, the hadron multiplicity will be artificially somewhat increased, which implies that the effect on $p_{\perp}$ per particle will be underestimated. Such an increase in multiplicity would also give an artificial widening of a jet and to avoid this we also abstain from adding excitations to string segments which already before the shoving has a high $p_{\perp}>2 \mathrm{GeV}$.

In fig. 3 we show the extra transverse momentum per unit rapidity due to shoving for particles within the region $|y|<2.5$ (matching the experimental acceptance in ref. [32]). The result is presented in four final-state particle multiplicity bins, and for three values of the parameter $g$ determining the strength of the repulsion between strings in eq. (4). As discussed there, we expect that a reasonable value for $g$ would be 1 within an order of magnitude. In fig. 3 we show results for $g=4$, and $g=10$. For illustration we also show results for $g=40$, which we regard as unreasonably large. We note that as expected the extra $d p_{\perp} / d y$ increases both with the strength parameter $g$ and with multiplicity. We also note that even for the lowest value, $g=4$, the added $p_{\perp}$ per unit rapidity can reach quite large values.

Flow-like effects - We mentioned above that the problem with hadronising soft gluons implies that the multiplicity is somewhat overestimated in events with low multiplicity. We do, however, not expect a noticeable effect from this problem for two-particle correlations. This expectation has been checked by comparing two-particle correlations for default Pythia8 and for shoving with $g=0.01$. We here note that although the multiplicity increases when adding even very small excitations, these excitations do not affect the twoparticle correlations. Thus there is no appearance of a ridge for this small $g$-value. In fig. 3 we also see that the extra $p_{\perp}$ grows as expected, with increasing event multiplicity.

To calculate the "ridge effect" we employ an analysis similar to the one chosen by experiments [32], where a signal distribution $S(\Delta \phi, \Delta \eta)$ is divided by a random background


FIG. 3. The extra transverse momentum per unit rapidity, $d p_{\perp} / d y$ (measured in the interval $|y|<2.5$ ), for four different final state particle multiplicities, and three different values for the shoving parameter $g$.
distribution, $B(\Delta \phi, \Delta \eta)$, constructed by combining particles from two different events in the same centrality class. In fig. 4 we show results for correlations in the range $2<|\Delta \eta|<4.8$ for two values of the shoving parameter $g$, and compare with default PYTHIA8, where no ridge is expected. Data from CMS [32] is added to the figure, but the comparison was carried out by ourselves, and is not guaranteed to include all experimental corrections. It would be beneficial for further model development to have the analysis available in Rivet [39]. We note that the ridge appearing at high event multiplicity is qualitatively reproduced with a value for the interaction strength $g=4$.

Conclusion - We have shown, that long-range two-particle correlations in pp collisions can be accounted for in a model, where the strings interact as overlapping flux tubes. The assumed interaction potential is inspired by lattice calculations. The model is implemented in Pythia8 and contains a single free parameter, which determines the strength of the interaction. We show that for a physically reasonable value of this parameter, the model qualitatively reproduces the "ridge" structure in high multiplicity pp collisions, as observed by several experiments.

We wish to emphasise that the model does not rely on the assumption of a deconfined plasma, normally employed by thermodynamical models of collective effects. On the contrary the collective behaviour is a consequence of the confining fields, resulting in an interaction between the strings that is without diffusion or loss of energy. Thus, although the string


FIG. 4. Di-hadron correlation functions for pp collisions at 7 TeV , in four centrality intervals, for two values of the shoving parameter $g$, compared to default Pythia8. For $g=4$, adding shoving produces a ridge similar to the data from CMS [32].
system is not deconfined nor thermalised, the transverse expansion has important similarities with the expansion of a boost-invariant perfect (non-viscous) liquid.

In a coming publication we want to improve the approximations in the implementation of the "shoving model" presented here, and combine it with the rope hadronisation model in ref. [28]. Our plan is then to include these effects in our model for collisions with nuclei [40], to see if they can adequately describe data showing collective effects in these larger systems. Would such a comparison turn out successful, this would challenge the current paradigm in heavy ion physics. It would then be necessary to find observables sensitive to dynamical differences between the traditional approach assuming a thermalised plasma, and the nonthermalised dynamics described here.

* This work was funded in part by the Swedish Research Council, contracts number 201603291, 2016-05996 and 2017-0034, in part by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme, grant agreement No 668679, and in part by the MCnetITN3 H2020 Marie Curie Initial Training Network, contract 722104.
[1] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjöstrand, Phys. Rept. 97, 31 (1983).
[2] G. Marchesini and B. R. Webber, Nucl. Phys. B238, 1 (1984).
[3] T. Sjöstrand, S. Mrenna, and P. Z. Skands, JHEP 05, 026 (2006), arXiv:hep-ph/0603175 [hep-ph].
[4] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen, and P. Z. Skands, Comput. Phys. Commun. 191, 159 (2015), arXiv:1410.3012 [hep-ph].
[5] J. Bellm et al., Eur. Phys. J. C76, 196 (2016), arXiv:1512.01178 [hep-ph].
[6] J.-Y. Ollitrault, Phys. Rev. D46, 229 (1992).
[7] K. H. Ackermann et al. (STAR), Phys. Rev. Lett. 86, 402 (2001), arXiv:nucl-ex/0009011 [nucl-ex].
[8] J. Baechler et al. (NA35), Nucl. Phys. A525, 221C (1991).
[9] A. Capella, C. Pajares, and A. V. Ramallo, Nucl. Phys. B241, 75 (1984).
[10] B. Andersson, G. Gustafson, and B. Nilsson-Almqvist, Nucl. Phys. B281, 289 (1987).
[11] M. Aaboud et al. (ATLAS), arXiv:1609.06213 (2016), arXiv:1609.06213 [nucl-ex].
[12] V. Khachatryan et al. (CMS), arXiv:1606.06198 (2016), arXiv:1606.06198 [nucl-ex].
[13] B. Abelev et al. (ALICE), Phys. Lett. B719, 29 (2013), arXiv:1212.2001 [nucl-ex],
[14] J. Adam et al. (ALICE), Nature Phys. 13, 535 (2017), arXiv:1606.07424 [nucl-ex].
[15] K. Werner, Phys. Rev. Lett. 98, 152301 (2007), arXiv:0704.1270 [nucl-th]
[16] T. Pierog, I. Karpenko, J. M. Katzy, E. Yatsenko, and K. Werner, Phys. Rev. C92, 034906 (2015), arXiv:1306.0121 [hep-ph].
[17] T. S. Biro, H. B. Nielsen, and J. Knoll, Nucl. Phys. B245, 449 (1984).
[18] A. Białas and W. Czyż, Phys. Rev. D31, 198 (1985).
[19] M. Gyulassy and A. Iwazaki, Phys. Lett. 165B, 157 (1985).
[20] C. Merino, C. Pajares, and J. Ranft, Phys. Lett. B276, 168 (1992).
[21] H. Sorge, M. Berenguer, H. Stoecker, and W. Greiner, Phys. Lett. B289, 6 (1992).
[22] N. S. Amelin, M. A. Braun, and C. Pajares, Z. Phys. C63, 507 (1994).
[23] M. A. Braun, F. Del Moral, and C. Pajares, Phys. Rev. C65, 024907 (2002), arXiv:hep-ph/0105263 [hep-ph].
[24] S. Soff, J. Randrup, H. Stoecker, and N. Xu, Phys. Lett. B551, 115 (2003), arXiv:nucl-th/0209093 [nucl-th].
[25] A. Capella and E. G. Ferreiro, Phys. Rev. C75, 024905 (2007),
arXiv:hep-ph/0602196 [hep-ph].
[26] M. A. Braun, C. Pajares, and V. V. Vechernin, Nucl. Phys. A906, 14 (2013), arXiv:1204.5829 [hep-ph].
[27] M. A. Braun, C. Pajares, and V. V. Vechernin, Eur. Phys. J. A51, 44 (2015), arXiv:1407.4590 [hep-ph].
[28] C. Bierlich, G. Gustafson, L. Lönnblad, and A. Tarasov, JHEP 03, 148 (2015), arXiv:1412.6259 [hep-ph].
[29] V. A. Abramovsky, E. V. Gedalin, E. G. Gurvich, and O. V. Kancheli, JETP Lett. 47, 337 (1988), [Pisma Zh. Eksp. Teor. Fiz.47,281(1988)].
[30] I. Altsybeev, AIP Conf. Proc. 1701, 100002 (2016), arXiv:1502.03608 [hep-ph].
[31] C. Bierlich, G. Gustafson, and L. Lönnblad, (2016), arXiv:1612.05132 [hep-ph].
[32] V. Khachatryan et al. (CMS), JHEP 09, 091 (2010), arXiv:1009.4122 [hep-ex].
[33] B. Andersson and G. Gustafson, Z. Phys. C3, 223 (1980).
[34] B. Andersson, G. Gustafson, and B. Söderberg, Z. Phys. C20, 317 (1983).
[35] P. Cea, L. Cosmai, F. Cuteri, and A. Papa, Phys. Rev. D89, 094505 (2014), arXiv:1404.1172 [hep-lat].
[36] X. Artru, Phys. Rept. 97, 147 (1983).
[37] B. Andersson, G. Gustafson, J. Hakkinen, M. Ringner, and P. Sutton, JHEP 09, 014 (1998), arXiv:hep-ph/9807541 [hep-ph].
[38] The shoving model has been implemented in PYTHIA8, which can be obtained at http://home.thep.lu.se/Pythia. The implementation allows for the user to supply their own model of MPI distribution in impact parameter space.
[39] A. Buckley, J. Butterworth, L. Lönnblad, D. Grellscheid, H. Hoeth, J. Monk, H. Schulz, and F. Siegert, Comput. Phys. Commun. 184, 2803 (2013), arXiv:1003.0694 [hep-ph].
[40] C. Bierlich, G. Gustafson, and L. Lönnblad, JHEP 10, 139 (2016), arXiv:1607.04434 [hep-ph].

