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Soft Isolated Photon Production as a Probe of the Parton Shower Mechanism

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Abstract

We discuss the rate of low-energy well-isolated photons in e^+e^- annihilation into hadrons, which is highly sensitive to the differences in scale-ordering used in parton shower models. We show that in the data already taken by each LEP experiment, differences of more than 5 standard deviations could be seen between the main models 'on the market', if the experimental errors are dominated by statistics. Requiring in addition a hadronic jet to be resolved by a $k_\perp$ jet algorithm increases this to almost 8 standard deviations. No other test has yet been proposed that is capable of seeing differences between the *perturbative* components of the models with such a high level of significance.

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1. Introduction

The hadronic final states of high-energy collisions are highly complex. Although we believe they are describable by QCD, we currently have no way to calculate their detailed structure. Nevertheless, due to the factorisation of QCD and its asymptotic freedom, many features of the final states can be attributed to perturbative effects, with only small corrections coming from the non-perturbative confining forces. Over the past decade, a great deal of progress has been made in the theoretical understanding of this perturbative component. Most of the important effects have been encoded in probabilistic Monte Carlo models, which also contain phenomenological models of the less well-understood confinement component. This paper is concerned with a comparison of three of the most commonly-used models, HERWIG[1], JETSET[2] and ARIADNE[3], using multihadron events in which a soft photon is observed at a large angle to the rest of the hadronic event.

Photon production in multihadron events has been widely studied at LEP, with all four experiments publishing measurements[4–8]. This is of interest partly to study the electroweak interactions of quarks[9], but mainly as a probe of the hadronic final state[4]. This is because photon emission is so similar to gluon emission, except that the photons do not suffer confinement effects, and so can be identified in phase-space regions where gluon emission cannot. An important consequence of the similarity between photon emission and gluon emission is that a test of a prediction for photons constitutes a test of the whole model, since there is no extra freedom or adjustable parameters in the inclusion of photons. To make a critical test of the model however, it is important to be able to make a direct connection between the prediction being tested and a specific feature of the underlying model.

The rate of low-energy large-angle photons was first discussed as a probe of the 'scale-ordering' used in different models in [10], and specific predictions were given in [11]. However, as we show in this paper, in order to make a solid connection between the scale-ordering and this rate, it is essential that the models not only correctly reproduce the soft limit, but also its first correction. In the case of HERWIG, this has had to await the introduction of a matrix-element correction to the parton shower prescription[12].

This paper is set out as follows: In section 2, we review the parton shower prescriptions used in each model, and describe the effect the scale-ordering has on the rate of large-angle soft photon emission. We also derive a rough estimate of the expected size of the effect. In section 3, we propose an event definition and give the predictions of each model. In section 4, we discuss the features of the hadronic part of the event, and show that requiring it to contain a high-p_t jet enhances the effect. Finally in section 5, we summarise the paper.

The specific versions of the models we use are ARIADNE4.04, JETSET7.3 and an as-yet unreleased version of HERWIG, provisionally called 6.0. We leave all parameters at their default values, except for switching on photon radiation, by setting `MSTA(20)=1`, `MSTJ(41)=2` and `VPCUT=0.4` respectively.

2. Parton Shower Prescriptions

The emission of a parton, whether from another parton or from a dipole, can quite generally be described by two variables—the scale Q^2, which has dimensions of energy-squared, and a dimensionless variable z, which describes the kinematics of the resulting state. When there is more than one emission, their relative azimuth ϕ is also required, but we neglect that here. The differential emission probability can then be written in the form

$$d\mathcal{P} = \frac{dQ^2}{Q^2} P(z) dz. \tag{1}$$

The parton shower is built around the Sudakov form factor, $\Delta(Q^2)$, which is defined as the probability that a parton formed at some scale Q produces no radiation that is resolvable above some infrared cutoff, Q_0. This can be found from the probabilities that n partons are produced, summed over all n, which exponentiates to give

$$\Delta(Q^2) = \exp\left\{ -\int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \int_{z_{\min}(k^2)}^{z_{\max}(k^2)} P(z) dz \right\}. \tag{2}$$

Thus the differential probability for an emission at scale Q, that is not accompanied by any resolvable radiation at scales above Q, is given by

$$\begin{aligned} d\mathcal{P} &= \frac{dQ^2}{Q^2} P(z) dz \exp\left\{ -\int_{Q^2}^{Q^2_{\max}} \frac{dk^2}{k^2} \int_{z_{\min}(k^2)}^{z_{\max}(k^2)} P(z) dz \right\} && (3) \\ &= \frac{dQ^2}{Q^2} P(z) dz \frac{\Delta(Q^2_{\max})}{\Delta(Q^2)} && (4) \end{aligned}$$

This can be interpreted in a very simple way as describing an ordered process, where Eq. (3) represents the probability distribution for the 'first' emission to occur at scale Q^2.

The subsequently produced system can go on to radiate further partons. In the strongly-ordered region $Q_1^2 \gg Q_2^2 \gg Q_3^2 \ldots$, which gives the leading logarithmic contribution, the corresponding probability distributions factorise into independent emission probabilities of the form given in Eq. (4). This, together with the scale-ordering, means that the process can be represented as a Markov chain, where a parton system evolves by a series of emissions, each at a smaller scale than, and only dependent on, the last. This is extremely suitable for a Monte Carlo implementation, where a direct correspondence can be made between phase-space points and generated events.

This semi-classical evolution approach is capable of describing the logarithmic terms arising from collinear singularities, but once quantum interference is taken into account the simple probabilistic picture is no longer valid. This is particularly important for soft emission which is also enhanced by large logarithms, and where many different branching configurations can lead to the same final state, so interference effects can be large. In fact, these are destructive outside angular-ordered regions of phase-space, due to the coherence of the radiation from each emitter[13]. Thus a Markov approach can still be used, provided that each branch is generated with ordered opening angle as well as scale[14]. All the Monte Carlo models discussed here incorporate this effect, although by different methods in each case. The interference also gives rise to azimuthal correlations between different emissions, which are included in HERWIG and ARIADNE but not JETSET, but which are not important for the predictions discussed here.

Although the definitions of all the variables appearing in Eq. (4) are model-dependent, the three models are actually all equivalent to leading logarithmic order (in the case of ARIADNE only up to corrections of order $1/N_c^2$, where $N_c = 3$ is the number of colours). They also agree with analytical calculations to that order. However, when constructing real events one must ensure that energy-momentum is conserved, and although the first corrections this introduces are of next-to-next-to-leading order, they can be numerically important[15]. It is these effects that produce dependence upon the definition of emission scale used, which we discuss in more detail below.

Many of the next-to-leading corrections can be included by appropriate definitions of variables. The most important of these is the scale that is used for the running coupling, which in the soft and collinear limits should be the transverse momentum of the emitted parton. The three models use different approximations to the transverse momentum, which all become equivalent in the appropriate limits.

Although the models describe the logarithmic regions well, many experimental observables emphasise phase-space regions well away from these. All three models match the hardest emission to the leading order matrix element to improve the treatment of such regions.

It is in principle straightforward to include photon emission within a parton shower model, and all the three models discussed here have done so. It is important to note that this does not introduce any new adjustable parameters, so studies of prompt photons are capable of constraining the underlying models, and not just the photon emission parts of them. Since photons escape from the partonic system without undergoing hadronisation, many regions of phase-space can be selected that are experimentally inaccessible for gluon emission. As we show in this paper, at least one such region—low-energy large-angle emission, is much more sensitive to the model-dependence of the scale definition than are the bulk properties of events that have hitherto been measured.

2.1 The HERWIG model

In HERWIG, the angular ordering of emissions is incorporated into the scale ordering, by defining the scale as

$$
\begin{aligned}
Q^2 &= E^2\xi \approx E^2(1-\cos\theta), && (5)\\
\xi &= p_i\cdot p_j\,/E_iE_j \approx 1-\cos\theta, && (6)\\
E &= E_i + E_j, && (7)
\end{aligned}
$$

where $p_{i,j}$, $E_{i,j}$ are the 4-momentum and energy of the resulting partons, and θ is their opening angle. The approximation for ξ holds when the partons are massless. Colour-connected pairs of partons with invariant-mass M are treated in separate frames where they have initial energy $E = M/\sqrt{2}$ and upper limit $\xi < 1$, and are boosted to their overall rest-frame after showering. This has the consequence of excluding very hard emission, which is corrected by using the lowest-order matrix-element in the excluded region, and the parton shower algorithm elsewhere. Thus it should be remembered that this region, while important for the properties of hadronic events containing very hard photons, does not test the fundamental properties of the underlying model.

The other variable, z, is defined as the energy fraction carried away by the gluon or photon. The distribution functions $P(z)$ are then just the standard DGLAP ones. The definition of transverse momentum used for the running coupling is defined by

$$p_t^2 = z^2(1-z)^2Q^2. \tag{8}$$

The two-loop running coupling is used, leading to the incorporation of some important next-to-leading order terms. The infrared cutoff is described by a parameter Q_0 that can be separately adjusted for quarks and gluons, which sets the minimum transverse momentum of emission that can be resolved. This leads to a lower bound of $z > Q_0/Q$. After all partons have been evolved down to the infrared cutoff scale, the partonic system is hadronised using the cluster fragmentation model.

Matching to the leading order matrix element is performed for every emission that is harder than all previous emissions, which correctly ensures that the correction enters into the form factor[12]. This is a new feature since the most recent public version, 5.7.

2.2 The JETSET model

In JETSET, a more traditional definition of scale is used; the virtuality of the emitting parton,

$$Q^2 = (p_i + p_j)^2 \approx E_i E_j (1 - \cos\theta), \tag{9}$$

where the definitions are as for HERWIG. Parton showers are generated in their overall centre-of-mass system, where the upper limit is $Q^2 < M^2$. This leads to the phase-space for the first emission being covered twice—once from each jet, and the sum of the two emission probabilities is larger everywhere than the matrix-element probability, which is then used to correct the distribution of first emissions.

The other variable is also the energy fraction of the gluon or photon as in HERWIG. The transverse momentum used for the running coupling is defined by

$$p_t^2 = z(1-z)Q^2 \tag{10}$$

The infrared cutoff is set by the minimum virtuality that can be reached, Q_0, and except for photon emission, by requiring $p_t > Q_0/2$. These lead to rather complex limits for z, which we will not reproduce here. Angular-ordering is imposed by vetoing emissions that are generated with non-ordered angles, and continuing the evolution from the chosen value of Q^2. After parton showering, the partonic system is hadronised using the string fragmentation model.

2.3 The ARIADNE model

In contrast to HERWIG and JETSET, which treat gluon emission as arising from quasi-independent partons, ARIADNE uses the colour dipole model, where gluons are supposed to be emitted by quasi-independent expanding dipoles between colour-connected pairs of partons. The scale used is the invariant transverse momentum of the emitted parton,

$$Q^2 = k_t^2 = s_{12}s_{23}/s_{123}, \tag{11}$$

where s denotes the invariant-mass-squared, and parton 2 is emitted between partons 1 and 3. The whole procedure is boost-invariant, so can be performed in any frame—in fact each emission is generated in the rest-frame of its parent dipole.

The other variable used is the rapidity of the emitted parton relative to the emitting dipole,

$$y = \tfrac{1}{2}\log\frac{s_{12}}{s_{23}}. \tag{12}$$

The infrared cutoff is simply a cutoff in k_t. The scale used for the running coupling is

$$p_t^2 = Q^2. \tag{13}$$

Unlike JETSET and HERWIG, the kinematic variable y is never bounded by the infrared cutoff, but only by $\cosh y < \sqrt{s_{123}}/2k_t$. The emission distributions $P(y)$, are given by the first-order matrix-elements for the $13 \to 123$ processes, which agree with the DGLAP splitting functions in the soft and collinear limits, up to corrections of order $1/N_c^2$. Angular ordering and azimuthal correlations are automatically included by the prescription. The orientation of the two-dipole system after emission relative to the original dipole, is decided by minimising the disturbance to neighbouring dipoles. The partonic system is hadronised by the string model, as implemented in JETSET.

2.4 The effect of scale ordering

The central point of this proposal is that the differences in definition of scale are most apparent in low-energy large-angle emission. However, in the case of gluon emission, these gluons are never detectable due to hadronisation effects. On the contrary, photons can be detected with energies down to a few GeV, especially if they are well isolated, as we require in this analysis.

If we consider emitting a photon of energy E_γ at $\theta_\gamma \sim 90^\circ$ from a quark of energy $E(\gg E_\gamma)$, the scale at which this occurs is entirely different in the three models: in HERWIG, it is $\sim (E_\gamma + E)^2(1-\cos\theta_\gamma) \sim Q^2_{\rm max}$; in JETSET, it is $\sim 2EE_\gamma(1-\cos\theta_\gamma) \sim Q_{\rm max}Q_{\rm min}$, and in ARIADNE, it is $\sim E_\gamma^2 \sin^2\theta_\gamma \sim Q^2_{\rm min}$. This means that the energy of the emitting quark will be lower in ARIADNE than in JETSET than in HERWIG, so although the three models would predict the same rate of such photons in the absence of gluon radiation, their rate will be lowest in ARIADNE and highest in HERWIG. This is known as a competition for phase-space. As we shall see shortly when we make these qualitative statements more quantitative, it is formally a next-to-next-to-leading order effect, and as such is neglected in most analytical calculations. Until a complete calculation of these corrections can be made, it is a matter for experiment to decide in tests such as the one discussed here, which scale is the most natural to think about parton showers being ordered in.

It is straightforward to find the probability distributions for soft photons in terms of x, the energy of the photon scaled by its maximum allowed energy ($\sqrt{s}/2$ for the first emission), and θ_γ, the angle between the photon and the charged thrust axis. For small x we obtain the same expression for each model,

$$d\mathcal{P} = 4\frac{e_q^2\alpha}{2\pi}\frac{dx}{x}\frac{d\cos\theta_\gamma}{\sin^2\theta_\gamma}\left(1 - x + \mathcal{O}(x^2)\right), \qquad (14)$$

where e_q is the charge of the quark in units of the electron charge. To this order in x, this is identical to the leading order matrix element. In the case of HERWIG, this agreement is only true because of the recently-implemented matrix-element matching.

Note that the leading term is energy-independent, $dx/x = dE_\gamma/E_\gamma$, while the next-to-leading term is energy-dependent, $dx = dE_\gamma/E$, and negative.

The essence of this proposal is straightforward: The later the emission of the photon in the shower, the lower the energy of the emitting quark, and so the larger the suppression due to the next-to-leading term.

It might appear that this contradicts Low's Theorem, which states that the rate of soft photons depends only on the momenta of external particles, and not on their production mechanism. However, this only applies to the leading term in the soft limit, which we have seen is correct in all three cases, and not to its correction terms.

We can easily make a rough estimate of the size of this effect. The rate predicted by a

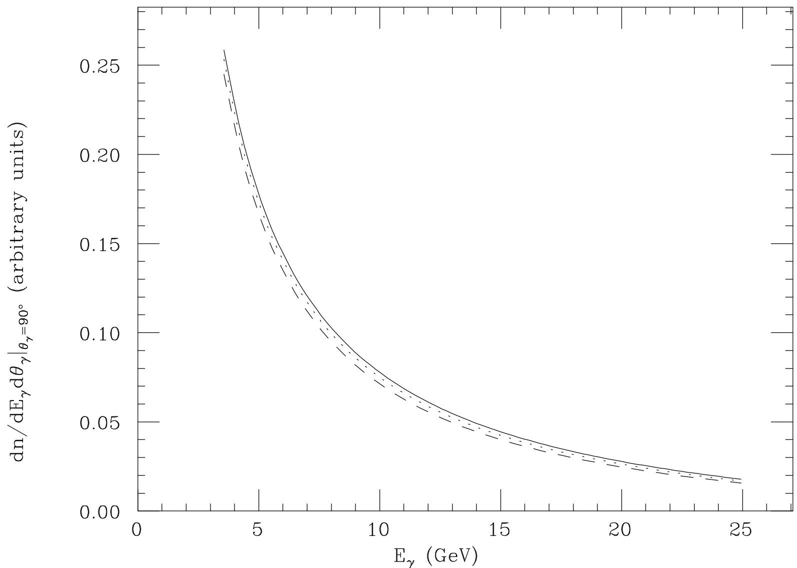

Figure 1: Rough expectations for the rate of large angle photons for models in which they are treated early (solid), late (dashed), and in-between (dotted).

model in which the emission comes at some scale Q_γ is

$$\frac{dn}{dE_\gamma\, d\theta_\gamma} \sim 4\frac{e_q^2\alpha}{2\pi}\left(\frac{1}{E_\gamma} - \left\langle\frac{1}{E}\right\rangle\right), \tag{15}$$

where E is the energy of the quark after the treatment of emission at scales between $\sqrt{s}$ and Q_γ. One might estimate this from

$$\left\langle\frac{1}{E}\right\rangle^{-1} \approx \langle E\rangle \approx \frac{\sqrt{s}}{2}\left(1 - \frac{C_F}{2\pi}\int_{Q_\gamma^2}^{s}\frac{dp_t^2}{p_t^2}\alpha_s(p_t^2)\int_0^1 dz(1+(1-z)^2)\right), \tag{16}$$

but empirically we find that $\left\langle\frac{1}{E}\right\rangle^{-1}$ is much smaller than $\langle E\rangle$ for this distribution. A fit to results from ARIADNE gives

$$\left\langle\frac{1}{E}\right\rangle^{-1} \approx \frac{\sqrt{s}}{2}\left(1 - 0.15\log\frac{\sqrt{s}/2}{Q_\gamma}\right), \tag{17}$$

which is accurate to a few percent over the whole range of Q_γ from 1 GeV to $\sqrt{s}/2$, for $\sqrt{s} \sim 90$ GeV. One can obtain estimates of the expected rate from models in which such emission is treated early (HERWIG), late (ARIADNE), and in-between (JETSET), by setting

$$Q_{e,\mathrm{early}} = \sqrt{s}/2, \tag{18}$$
$$Q_{e,\mathrm{late}} = Q_\gamma, \tag{19}$$
$$Q_{e,\mathrm{mid}} = \sqrt{Q_\gamma\sqrt{s}/2}, \tag{20}$$

respectively. In figure 1 we plot the differential distribution according to each of these scale choices. Since we only consider the average behaviour and not fluctuations away from this average, this should only be considered a rough estimate of the size of the effect. It varies between 5% when E_γ is a few GeV, and 10% when E_γ is about ten GeV. There is a competition between two effects that prevents this ratio from varying much with E_γ:

As E_γ gets smaller, the difference between the scales increases, so the absolute difference increases, while the total rate also increases, decreasing the relative difference. In this simple approximation, the decrease marginally wins out, while as we shall see in the next section, the increase marginally wins in the actual models.

We conclude that a difference of around 10% should be observable due to the difference in scale order, if indeed the programs are accurate implementations of the simple models considered above.

3. Predictions for Soft Isolated Photons

To make quantitative predictions, we must use a specific event definition. In particular, since the inclusive rate of photons is not infrared safe, we must define an isolation criterion. We must also define the axis from which the photon angle is measured in an infrared safe way.

Most studies of photons in e^+e^- annihilation to hadrons have used a jet-like event definition, first clustering the hadronic part of the event into jets, and then requiring the photon to be isolated from these jets. However, to keep the interpretation of the soft photon rate simple, we should try to make an event definition that is as similar as possible to an inclusive photon, while remaining infrared safe. This means that we should make as few requirements on the hadronic event as possible. To this end, we make a simple cone-isolation cut, by requiring there to be less than 1 GeV of total hadronic energy in a cone of 15° half-angle centred on the photon. The general reliability of such a procedure has been questioned[16], but since in our case we also require the photon to be at a large angle to the event axis it does not suffer the problems due to collinear emission discussed there. In events in which more than one photon passes these criteria, all are used in the analysis, although this is so rare that it is unimportant to the result.

We define the event axis as the thrust axis, but calculated using only charged particles, which is more reliable experimentally than using all hadrons. Since it excludes the photon, it also means that the event axis is independent of the photon energy when the photon is at a large angle to it.

Having defined an isolated photon and an event axis, we would ideally be interested in the double-differential distribution over the angle from the axis, θ_γ, and the photon energy, E_γ. However, since the event rate is reasonably small, we consider distributions that have been integrated over some range of one or other variable.

In figure 2 we show the angular distribution of the isolated photons, integrated over photon energies between 3 and 10 GeV. The distribution is plotted in such a way that without the isolation cut it would be flat. We see that for θ_γ larger than about 45° it is indeed flat. To ensure that we stay well away from the region where the isolation cut becomes important, and in the region where the scales are most different in the different models, we restrict ourselves to $\theta_\gamma > 60°$.

In figure 3 we show the energy distribution, integrated over the angular region $\theta_\gamma > 60°$. We see that HERWIG is indeed about 10% above ARIADNE. The first bin is dominated by effects of the infrared cut, and should be ignored.

JETSET lies about 30% below our theoretical expectation, which is consistent with the deficit observed in experimental analyses. Since this phase-space region does not contribute a great deal to the total cross-section observed in other analyses, this is an interesting cross-check showing that the deficit is rather constant over phase-space. However, since

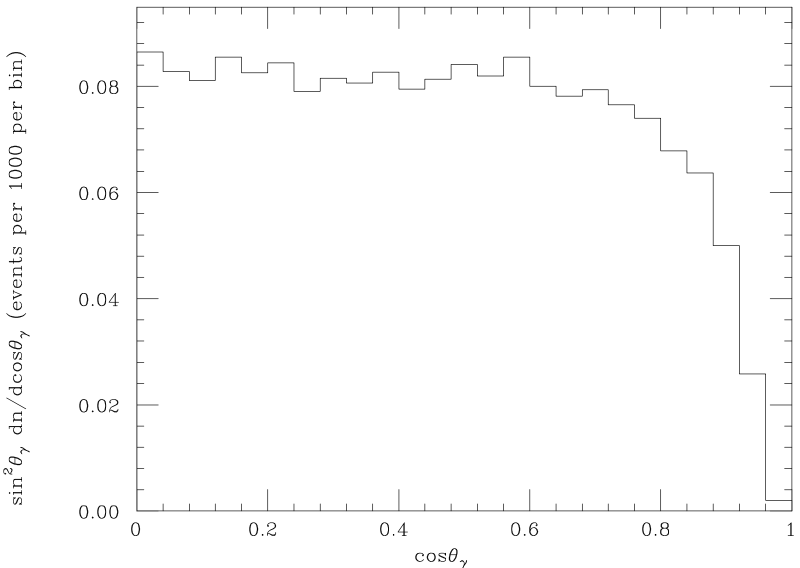

Figure 2: The angular distribution of isolated photons, with energy between 3 and 10 GeV, from ARIADNE. Pure QED, without gluon radiation or an isolation cut, would predict a flat distribution on these axes.

it is not yet understood, it makes the results impossible to interpret purely in terms of scale-ordering, and we disregard JETSET for the remainder of this paper.

Although the difference between HERWIG and ARIADNE is in accord with our expectations from the previous section, it would be useful to check that the difference really is due to scale-ordering, and not some coincidental effect of the implementation. One way in which we can do this is to run ARIADNE, but with the ordering altered so that all photon emission is treated before any gluon emission (the 'photon-first' version discussed in [17]). If the difference is solely due to the scale-ordering, then the results from 'photon-first' ARIADNE should be identical to those from HERWIG. As seen in figure 4, although they are not identical they become so at low energy, showing that the differences for soft photons are dominated by scale-ordering. As the energy gets larger, recoil effects from subsequent

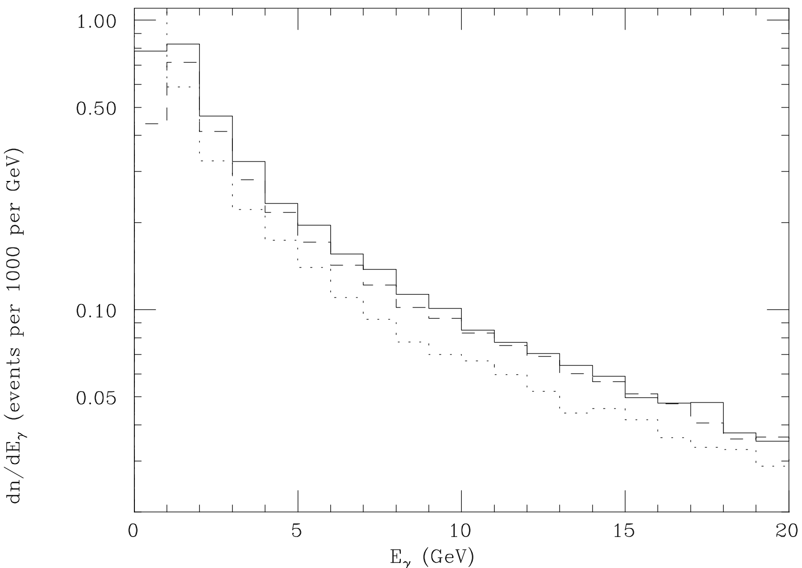

Figure 3: Energy spectrum of isolated photons that are more than 60° from the thrust axis, from HERWIG (solid), ARIADNE (dashed) and JETSET (dotted).

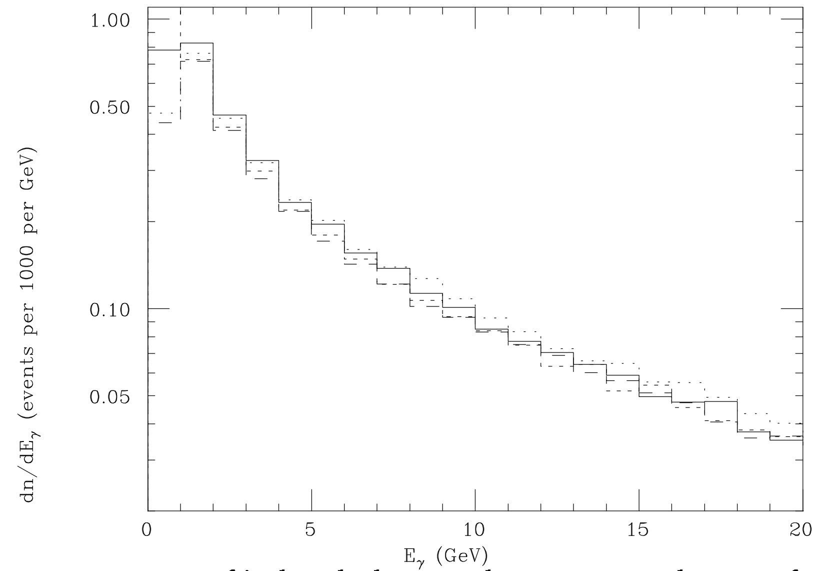

Figure 4: Energy spectrum of isolated photons that are more than 60° from the thrust axis, from HERWIG (solid), ARIADNE (dashed), 'photon-first' ARIADNE (dotted), and 'photon-first' JETSET (short-dashed).

emissions (which affect the neglected terms in (14)) begin to become important, and one would not expect the models to agree. It is interesting to note that 'photon-first' JETSET agrees with HERWIG and 'photon-first' ARIADNE.

To estimate the statistical significance of the difference, we integrate the rate over the region

$$\theta_{\min} < \theta_\gamma, \tag{21}$$
$$E_{\min} < E_\gamma < E_{\max}, \tag{22}$$

giving a number of events $N_{\rm AR}$ and $N_{\rm HW}$ from each model. We then calculate the significance of the difference,

$$\sigma = \frac{N_{\rm HW} - N_{\rm AR}}{\sqrt{N_{\rm HW}}}. \tag{23}$$

Studying σ as a function of $\theta_{\min}$, $E_{\min}$ and $E_{\max}$, we find that it is strongly dependent on $E_{\min}$. The value used will depend on the experimental procedure, and in particular how well soft pions can be rejected (see below). We choose a representative value of $E_{\min} = 3$ GeV. The significance is maximised for $\theta_{\min} = 60°$, but is almost as large for $\theta_{\min} = 70°$, the value used in [11]. Nevertheless we use 60° here. The statistical significance only varies slowly with $E_{\max}$, peaking for $E_{\max} \sim 10$ GeV, but still almost as large for $E_{\max} \sim 20$ GeV. If the experimental errors are dominated by the systematic uncertainty however, it is an advantage to have as large a ratio between the two model predictions as possible. This favours smaller values of $E_{\max}$, so we set $E_{\max} = 10$ GeV.

Using these cuts, and assuming a total of two million hadronic Z decays, we obtain 2520 events from HERWIG and 2260 from ARIADNE, with a statistical significance of 5.2 standard deviations. Increasing $E_{\min}$ reduces this significance: For $E_{\min} = 4$ or 5 GeV, it becomes 4.1 or 3.9 standard deviations respectively.

We conclude that, at least in the cases of HERWIG and ARIADNE, the Monte Carlo implementations are good representations of the underlying models, and that the model-

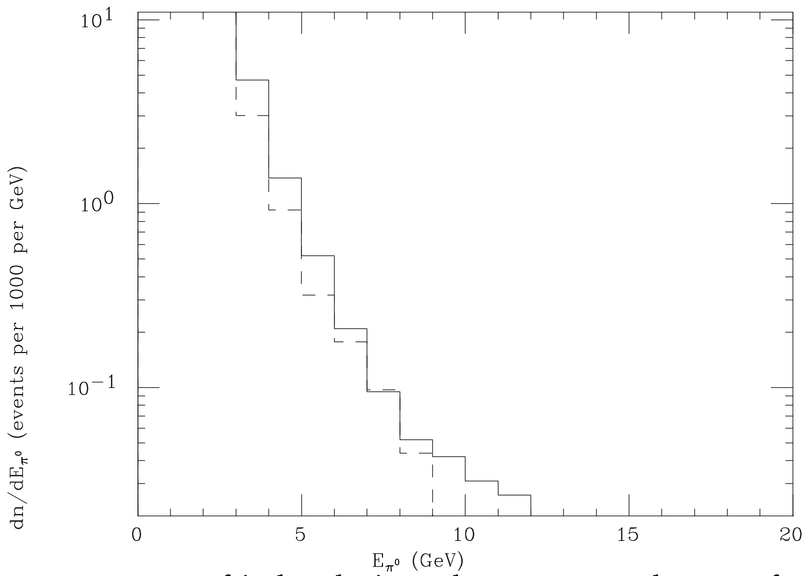

Figure 5: Energy spectrum of isolated pions that are more than 60° from the thrust axis, from HERWIG (solid) and JETSET (dashed).

dependence of the predictions for soft isolated photons is largely determined by the scale-ordering. The difference is sufficiently large to have a high statistical significance in the data already taken by each LEP experiment. Thus, if the systematic errors due to backgrounds can be kept down to a reasonable level, an experimental measurement of the rate of soft isolated photons would give important evidence for which (if any) of the ordering variables provides a good description of the physical showering process.

The principal background comes from isolated neutral meson production, mainly π^0s, where the meson decays into two photons. Unfortunately this is a prediction for which the difference between the different Monte Carlo models is known to be large. An experimental analysis would therefore need to either reduce the background rate well below the signal or measure it from the data, for example by measuring it in the region where the two photons are separately measurable and extrapolating into the region where they are not. In figure 5 we show the energy spectrum of π^0s that pass the isolation and angular cut. We see that to reduce the background rate to the level of the scale-dependence in the models, a rejection factor of at least 100 would be needed at $E_\gamma = 3$ GeV. The pion spectrum falls much more quickly than the photon one, so the situation improves with increasing E_γ, for example by 5 GeV, a rejection factor of 20 would be sufficient. It is also likely that a more stringent isolation cut would improve the situation, although we have not investigated this.

4. The Hadronic Event

In terms of the time-ordered picture described earlier, the suppression of soft photons in ARIADNE is because the evolving quark has been 'offered the chance' to emit gluons before the photon in question. However, this chance is not always taken, and so in some fraction of events the emitting quark still has its full energy. Therefore it seems that if events are selected containing a jet with p_t greater than the photon's, the difference between the models should be increased. We should beware of relying on events in which the jet p_t is a significant fraction of the available energy, since the results would then depend on the model-dependent hard limits. But since all the models reduce to DGLAP evolution at small p_t, as long as we work at reasonably small p_t the differences will still be dominated by scale-ordering.

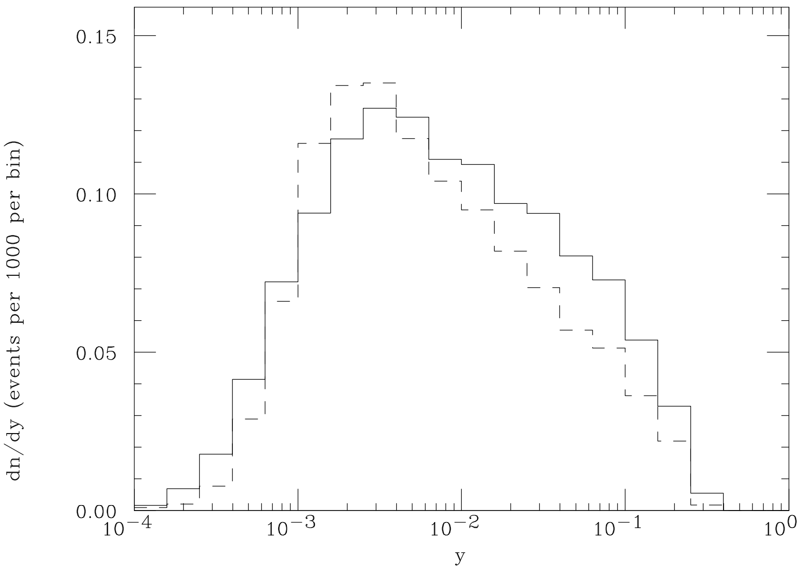

Figure 6: Distribution of jet $p_t(\approx\sqrt{ys})$ for events that pass the cuts of the previous section. Solid is from HERWIG, dashed from ARIADNE.

To resolve the jet in an infrared safe way, we use the $k_\perp$ jet algorithm for e^+e^- annihilation[18] (sometimes known as the Durham algorithm), continuing to cluster jets until exactly three remain. We exclude the photon from the clustering procedure, and set the denominator of the $k_\perp$ measure to the total visible energy excluding the photon. The y value at which the three jets get clustered to two gives the p_t of the emission scaled by the available energy,

$$p_t = \sqrt{y}\,(\sqrt{s} - E_\gamma)\,. \tag{24}$$

In figure 6 we show the y distribution of events that pass the previous section's cuts. It can be seen that the extra HERWIG events relative to ARIADNE are predominantly accompanied by higher p_t jets. Applying a cut of

$$y > y_{\rm cut}, \tag{25}$$

and optimising for statistical significance we obtain $y_{\rm cut} = 0.01$, corresponding to a p_t of about 8.5 GeV. This gives 1090 events from HERWIG and 830 from ARIADNE, with a significance of 7.9 standard deviations. It is worth noting that in hadronic events without the photon selection, HERWIG gives 5% more events with $y > 0.01$ than ARIADNE so that most, but not all, of the 20% gained by this cut can be attributed to the scale-ordering.

In figure 7 we show the ratio of y to the scaled energy of the photon, which, since we work at large angles, is equivalent to its scaled transverse momentum. We apply a cut of

$$y > r_{\rm min} E_\gamma^2/s. \tag{26}$$

Setting $r_{\rm min} = 1$ might seem the simplest situation, and gives 1630 events from HERWIG and 1350 from ARIADNE, with a significance of 6.9 standard deviations. Optimising again, we obtain a slightly higher value, $r_{\rm min} = 2.5$, corresponding to a p_t cut 60% above that of the photon. This gives 1170 events from HERWIG and 920 from ARIADNE, with a significance of 7.3 standard deviations. Thus $y_{\rm cut}$ is more effective than $r_{\rm min}$. We have not investigated the possibility of optimising the photon and jet cuts simultaneously, which could also improve the significance.

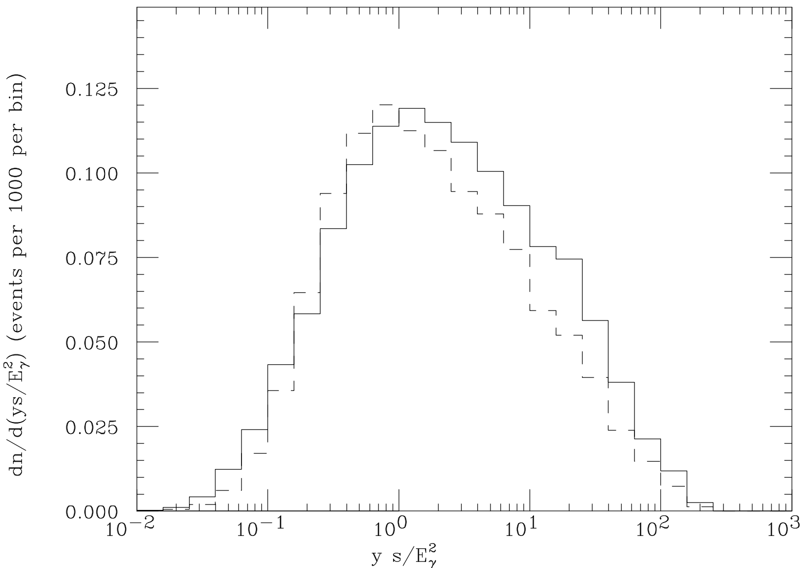

Figure 7: Distribution of jet p_t relative to the photon energy for events that pass the cuts of the previous section. Solid is from HERWIG, dashed from ARIADNE.

Since the cut on y is so successful in enhancing the ratio of the two models it raises another possibility, of measuring the *fraction* of photon events that contain such a jet. This fraction is predicted to be 43.3% by HERWIG and 36.7% by ARIADNE. With the numbers of events given above, this would have a statistical error of about 1%, so a significance of 6.6 standard deviations. Although this is not as statistically discriminating as the absolute rate of events that pass the cuts, one would expect many of the systematics from the background uncertainty to cancel in the ratio. It is therefore possible that the total error on the relative fraction is smaller than on the absolute number of events passing the cuts.

Once we have infrared safe definitions of the photon and jet system, we can also study correlations between them. In particular, the argument of [17] suggests an anticorrelation between the azimuths of the photon and the softest of the three jets. In the limit of high p_t, all three models give similar results[10], since large p_t tends also to mean large angles and large virtualities, so the scale-ordering has little effect. In the phase-space region we have selected however, we do have large differences in the scale-ordering, and so would expect more model-dependence. We expect HERWIG to be almost uniform in azimuth, with ARIADNE showing the anticorrelation, although it will not be large at our small p_t values. Using the $r_{\min} = 1$ cut we obtain $\langle \cos \Delta\phi \rangle_{\mathrm{AR}} = -0.041 \pm 0.019$, where the error is that expected for 2 million events. $\langle \cos \Delta\phi \rangle_{\mathrm{HW}}$ is consistent with zero, even in our sample of 10 million Monte Carlo events. Although the effect is barely significant, it does have the advantage that the background is unlikely to depend significantly on $\Delta\phi$, so the background systematics would again largely cancel.

We conclude that a p_t cut does enhance the significance of the model-dependence, while remaining largely determined by the scale-ordering.

5. Summary

The Monte Carlo models we have been discussing are important components of many experimental analyses of high-energy collisions. It is therefore essential that they be tested as thoroughly as possible, so that we know where they can be relied upon, and where they cannot. Although general comparisons between model predictions and data are useful for disproving models, critical tests require a direct connection between the quantity being measured and a specific feature of the model. A well-known example is the measurement of the 'string effect', which has helped show the importance of coherence effects in the description of multihadron final states.

In this paper we have discussed another such test, the rate of low-energy large-angle photons, which has a direct connection with the variable used to define the 'time order' of the parton showering process. By comparing with the 'photon-first' version of ARIADNE, we have shown that the model-dependence is indeed dominated by the scale-ordering. If the experimental errors are dominated by statistics, and our representative cuts are used, differences between the models should be observable with a significance of more than 5 standard deviations, in the data already taken by each LEP experiment. Also requiring a hadronic jet to be resolved in the $k_\perp$ algorithm with $y_{\rm cut} = 0.01$ increases this to almost 8 standard deviations, while the relative fraction of events passing this jet cut has a significance of over 6 standard deviations.

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