hep-ph/9410414
LU TP 94-17
October 1994

Matrix-Element Corrections to Parton Shower Algorithms

Michael H. Seymour*
Department of Theoretical Physics, University of Lund,
Sölvegatan 14A, S-22362 Lund, Sweden

Abstract

We discuss two ways in which parton shower algorithms can be supplemented by matrix-element corrections to ensure the correct hard limit: by using complementary phase-space regions, or by modifying the shower itself. In the former case, existing algorithms are self-consistent only if the total correction is small. In the latter case, existing algorithms are never self-consistent, a problem that is particularly severe for angular-ordered parton shower algorithms. We show how to construct self-consistent algorithms in both cases.

LU TP 94-17
October 1994

*Address after 1st January 1995: CERN TH Division, CH-1211 Geneva 23, Switzerland.

1 Introduction

Monte Carlo event generators[1–3] that combine the parton shower[4,5] or dipole cascade[6] approaches for perturbative jet evolution with local models of non-perturbative hadronisation[7,8] have been extremely successful in describing the hadronic final state of high energy reactions (see [9] for example). One feature that is essential for an accurate description of final state effects is the coherence of radiation from different partons in the event. In the parton shower approach, this can be implemented as an ordering in opening angles during the evolution away from the hard process. In the virtuality-ordered algorithm of [5], this is done by vetoing non-ordered emission, while in the algorithm of [4], it is more directly incorporated by using opening angle as the evolution variable.

However, because parton showers are based on expansions around the soft and collinear limits that dominate the total emission cross-section, there is no guarantee that they will perform well away from those limits. Many experimental observables are specifically sensitive to hard emission, such as event shapes in the 3-jet region of e^+e^- annihilation, or the E_T flow in DIS. To give a reliable prediction for such quantities, it is necessary to supplement the parton shower algorithm with the exact first-order matrix-element cross-section. This is particularly important for angular-ordered algorithms.

There are two approaches in current practice: either to split the phase-space into two parts, using the matrix-element cross-section in one region and the parton shower in the other[10]; or to modify the algorithm so that it faithfully reproduces the matrix-element cross-section in the hard limit[5]. It is worth noting that the dipole cascade model[6] automatically reproduces this hard limit by construction, so no such matrix-element corrections are required.

In this paper, we discuss both approaches and show that the former can be inconsistent if the total correction is large. In the latter, particular care must be taken to ensure the theoretical consistency. We show how self-consistency can be achieved in both cases, with minor modifications to existing algorithms.

2 Basics

We begin by recalling some of the features of parton shower evolution. We consider an arbitrary hard process with cross-section σ_0, and study the cross-section for additional radiation,

$$d\mathcal{P}^{\text{m.e.}}_{\text{incl}} = \frac{1}{\sigma_0}d\sigma. \qquad (1)$$

The subscript refers to the fact that the matrix-element cross-section describes the *inclusive* emission rate. After integrating over the whole of phase-space, this gives the average number of emissions,

$$N_{\mathrm{incl}} = \int \mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}. \tag{2}$$

Unless an infrared cutoff is applied, N_{incl} is a divergent quantity.

Parton shower algorithms are based on a 'sequential evolution' picture in which multiple emission occurs in a definite (but algorithm-specific) order, for example from largest to smallest angle in [4] or largest to smallest virtuality in [5]. This is defined by some *ordering variable* q^2†, with emissions with larger q^2 being treated earlier in the evolution than those with smaller q^2. An important quantity for the construction of the algorithm is the probability that there was no emission before some scale q^2 (which is generally referred to as a form factor, although it is formally the ratio of two form factors),

$$\Delta(q^2) = 1 - \int_{q^2} \mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}} + \mathcal{O}\left(\frac{1}{2!}\int_{q^2} \mathrm{d}\mathcal{P}^{\mathrm{m.e.}2}_{\mathrm{incl}}\right) - \ldots \approx \exp\left\{-\int_{q^2} \mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}\right\}, \tag{3}$$

where the approximation is valid when the cross-section is dominated by the soft and collinear regions, i.e. when q^2 is small. Note that we do not explicitly give the upper limit of the integral because it depends on the definition of q^2. One of the approximations used to construct a probabilistic parton shower algorithm is to replace the approximation by an equality,

$$\Delta(q^2) \equiv \exp\left\{-\int_{q^2} \mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}\right\}. \tag{4}$$

The cross-section for the *first* emission is then simply the product of the inclusive cross-section with the probability that there was no earlier emission,

$$\mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{1st}}(q^2) = \mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}(q^2)\exp\left\{-\int_{q^2} \mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}\right\}, \tag{5}$$

which is always finite. One then obtains the average number of first emissions,

$$N_{\mathrm{1st}} = \int \mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{1st}} \leq 1, \tag{6}$$

where the equality applies when there is no infrared cutoff. The other approximation used is to replace the exact cross-section by

$$\begin{aligned} \mathrm{d}\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{incl}} &= \frac{1}{\sigma_0}\mathrm{d}\sigma^{\mathrm{p.s.}}, && (7)\\ \mathrm{d}\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{1st}}(q^2) &= \mathrm{d}\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{incl}}(q^2)\exp\left\{-\int_{q^2} \mathrm{d}\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{incl}}\right\}, && (8) \end{aligned}$$

which is again valid in the soft and collinear limits. In general for multiple emission, only this approximation can be calculated, and not the exact matrix element. This paper is about how the exact cross-section can be used to improve the parton shower algorithm in cases in which it is known. Specifically, we consider the first-order matrix element, which dominates the cross-section when there is one emission that is much harder than all others.

† This is an arbitrary function of the emission kinematics, not necessarily the pair virtuality.

3 Complementary phase-spaces

Since the first-order matrix-element cross-section is reliable in the hard limit, and parton shower algorithms are reliable in the soft and collinear limits, the simplest approach is to split the phase-space into two parts, and use each in their reliable regions. If, as one would hope, there is reasonable overlap between the two regions of reliability, then the final result should not depend strongly on where the border between the two regions is drawn. To prevent double-counting of emissions, the border must be used consistently in both the matrix-element and parton shower phase spaces, i.e. the two regions must be exactly complementary.

In the model of [10], this is implemented for DIS by using an adjustable cutoff in invariant mass. On the other hand, using the angular-ordered parton shower algorithm of [4] one finds that the shower phase space has a natural border, from the requirement that all emissions be into the forward hemisphere in the particular Lorentz frame used. This was exploited in [11] for e^+e^- and [12] for DIS, to provide a similar correction, but without an adjustable cutoff.

Since the first-order matrix element describes the *inclusive* emission cross-section, it only reliably predicts the *single-emission* cross-section if the region in which it is used contributes a small fraction of the total cross-section. If this fraction approaches unity, then multiple emission is bound to play a rôle and the matrix-element cross-section will become unreliable. In the e^+e^- algorithm of [11], this fraction is a very safe 1 in 40, while for DIS[12] it is typically 1 in 10 so still reasonably safe. On the other hand, the cutoff in [10] is phenomenologically required to be rather small, leading to a fraction that is typically around 50%. They also find a residual dependence on the cutoff value. Thus we should consider how the way in which the matrix-element is used might be modified to improve the agreement.

Although we have said that the matrix-element cross-section only generates a single emission in its phase-space region, multiple emission is in fact generated by the algorithm. This is because after generating an emission within that region, the resulting hard process is parton showered with upper limits for emission controlled by the dynamics of the hard emission. In particular, subsequent emission from this system cannot be harder than the hard emission, but can be harder than the cutoff between the two regions. Thus, to the same accuracy as the usual parton shower, multiple emission into the matrix-element region is generated.

Since the parton shower emission cannot be harder than the emission generated using the matrix element, the matrix-element emission must be the hardest in the event. However, this is in contradiction with the fact that the matrix element describes the inclusive emission cross-section. To avoid this contradiction, a form factor should be included to incorporate the probability that there was not an emission at a higher scale. That is, instead of generating events in the matrix-element region according to the inclusive cross-section, $d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}$, they should be generated according to $d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{1st}}$, which correctly accounts for the probability that

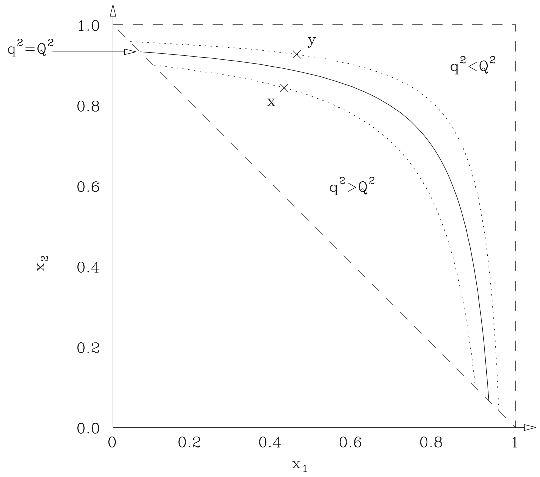

Figure 1: The phase-space for e^+e^- divided according to an ordering variable q^2 into a matrix-element region, $q^2 > Q^2$, and a parton shower region, $q^2 < Q^2$. Recall that the matrix element is divergent at $x_{1,2} = 1$.

there was no emission at a higher scale than the one generated.

In figure 1 we show an illustration for the specific case of e^+e^- annihilation. The probability distribution for the first emission to be at y is

$$\mathrm{d}\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{1st}}(q_y^2) = \mathrm{d}\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{incl}}(q_y^2)\exp\left\{-\int_{q_y^2}^{Q^2}\mathrm{d}\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{incl}}\right\}\exp\left\{-\int_{Q^2}\mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}\right\}. \tag{9}$$

The second integration arises because the first emission can only come from the parton shower if there was no matrix-element emission. If events at x in the matrix-element region are generated according to the exact first-order matrix element, as in the standard algorithm, they are distributed according to $\mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}(q_x^2)$. In the limit $q_{x,y}^2 \to Q^2$, where the points correspond to identical physical configurations, the two probability distributions are different, even if $\mathrm{d}\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{incl}}$ is a perfect approximation to $\mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}$, leading to a residual dependence on the cutoff between the two regions, Q^2. If

$$\int_{Q^2}\mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}} \tag{10}$$

is large, this dependence is strong. On the other hand, if we generate the matrix-element events according to

$$\mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{1st}}(q_x^2) = \mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}(q_x^2)\exp\left\{-\int_{q_x^2}\mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}\right\}, \tag{11}$$

as we propose, then in the $q_{x,y}^2 \to Q^2$ limit we obtain

$$\mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{1st}}(Q^2) = \mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}(Q^2)\exp\left\{-\int_{Q^2}\mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}\right\}, \tag{12}$$

$$\mathrm{d}\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{1st}}(Q^2) = \mathrm{d}\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{incl}}(Q^2)\exp\left\{-\int_{Q^2}\mathrm{d}\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{incl}}\right\}. \tag{13}$$

If the parton shower cross-section exactly reproduced the first-order matrix element, the two would then be perfectly matched, with no dependence on Q^2.

4 Correcting the algorithm

In [5], a method was described to correct the first emission of a virtuality-ordered algorithm to reproduce the first-order matrix-element cross-section. Although we shall show that this is not self-consistent, it is similar to the method we propose, so it is worth describing the details.

If one stops a parton shower after one emission then the final state is exactly that described by the first-order matrix element: three partons in the e^+e^- case. It is straightforward to then work through the kinematic reconstruction used by the algorithm, to relate the variables generated in the parton shower branching to those used to describe the matrix-element cross-section, and calculate the differential cross-section produced by the algorithm, $d\mathcal{P}_{\rm incl}^{\rm p.s.}$. In the algorithm of [5], this was found to be everywhere larger than the matrix-element cross-section, $d\mathcal{P}_{\rm incl}^{\rm m.e.}$, so the veto algorithm was used to correct the distribution of first branchings[‡].

However, we recall that the time-ordered language used to describe parton shower evolution is not fundamental to the theory, so when we use a concept like the 'first' emission we must be extremely careful that we have retained theoretical consistency. Indeed, the important point has been made above, that the first-order matrix-element cross-section describes the inclusive distribution of all emissions from the original current, and not just the first. Thus *all* hard emission should be corrected to the first-order matrix-element cross-section, and not just the first.

At this point it becomes necessary to define more specifically what we mean by 'hard' emission. The most suitable definition is in terms of transverse momentum, since this avoids in a single variable both the soft and collinear singularities. By contrast, the virtuality and opening angle, which are used as ordering variables in the algorithms of [5] and [4] respectively, do not prevent emission at low transverse momenta where the running coupling becomes large. To prevent them from entering the non-perturbative region, the infrared cutoff is active throughout the evolution. Furthermore, the QCD matrix elements for multiple emission factorise in the limit of strongly-ordered transverse momenta, so using it as the variable to measure hardness allows simple construction of algorithms.

In terms of the transverse momentum, the recoil of parton a from the emission of parton b is extremely simple—parton a's direction is perturbed by an amount proportional to the transverse momentum. Thus we can simply analyse the effect of a later emission from parton a in the two strongly-ordered regions, in which the second emission is much harder or much softer than the first. In the first case, as far as the second emission is concerned, the recoil from the first emission is insignificant, so the emitting current is identical to the original current, and the emission should be described the first-order matrix element. Furthermore,

[‡] See the appendix for a brief discussion of the veto algorithm.

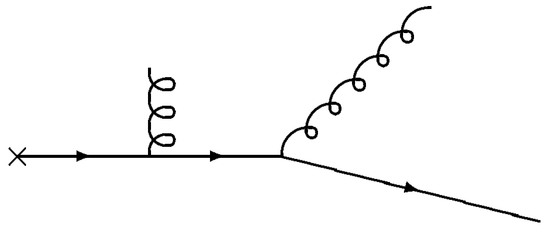

Figure 2: Emission of two gluons in which the first (according to the ordering variable of the algorithm) is softer, and at a larger angle, than the second. Both should be described by the first-order matrix element: the second because the recoil from the first is negligible, so it is effectively emitted by the original current; and the first because it represents the coherent sum of emissions from the external lines which, after azimuthal averaging, is equivalent to a single emission from the internal line pretending that the later emission did not happen, i.e. it is also effectively emitted by the original current.

since the first emission can be considered infinitely soft by comparison, the second emission can be related to the matrix-element cross-section by pretending that the first emission never occurred, i.e. exactly as in the method described above. On the other hand if the second emission is much softer than the first, it is effectively emitted by a completely different current and should not be corrected to the first-order matrix-element cross-section. Instead, the second-order matrix element factorises in this limit, and the parton shower algorithm is reliable without correction.

Thus, we see that the parton shower algorithm can be corrected to the first-order matrix-element cross-section by applying the method given above to *every emission that is the hardest so far,* instead of just the first.

One might suppose that having found a later emission that was harder than the first, the first should be considered as coming from a modified current, so should not have been corrected. This is not the case, owing to the coherence of large-angle radiation from different partons in the event. Since the parton shower is generated with ordered opening angles, the first gluon must be at a larger angle than the later one. The coherence of soft large-angle radiation from the two emitters then allows them to be described as a single emission from the internal line imagined to be on shell, ie. pretending that the later emission did not happen. Thus the earlier gluon is effectively emitted by the lowest-order current, and both emissions should be corrected. This is illustrated in figure 2.

It is worth considering what goes wrong with an algorithm that corrects the first emission, rather than the hardest. For events in which the hardest emission is first there is clearly no difference, but if a soft gluon is emitted beforehand, then no correction would be applied to the hard gluon. However, the probability that this occurs is strongly (logarithmically) dependent on the infrared cutoff, leading to the result that the hard emission cross-section of the algorithm is unphysically dependent on its soft infrared cutoff. That is,

$$\begin{aligned} d\mathcal{P}_{\mathrm{hard}} &= d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{hard}} \exp\left\{-\int_\epsilon d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{soft}}\right\} + d\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{hard}}\left(1-\exp\left\{-\int_\epsilon d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{soft}}\right\}\right) && (14) \\ &\sim d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{hard}} \exp(\alpha_s \log\epsilon) + d\mathcal{P}^{\mathrm{p.s.}}_{\mathrm{hard}}\left(1-\exp(\alpha_s\log\epsilon)\right). && (15) \end{aligned}$$

Expanding in powers of α_s, this dependence might seem sub-leading,

$$d\mathcal{P}_{\mathrm{hard}} = d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{hard}} + \mathcal{O}(\alpha_s^2), \qquad (16)$$

so irrelevant, but in the realm of applicability of parton showers, logarithms of the ratio of the hard scale to the infrared cutoff are large enough to overcome the smallness of α_s, so the dependence is formally leading order. That is, when counting powers of α_s, $-\alpha_s \log\epsilon$ should be considered $\mathcal{O}(1)$. Correcting only the first emission is therefore formally inconsistent. In our solution, both the softer earlier emission and the later harder one would be corrected to the matrix element, so

$$\begin{aligned} d\mathcal{P}_{\mathrm{hard}} &= d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{hard}} \exp\left\{-\int_\epsilon d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{soft}}\right\} + d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{hard}}\left(1-\exp\left\{-\int_\epsilon d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{soft}}\right\}\right) && (17) \\ &= d\mathcal{P}^{\mathrm{m.e.}}_{\mathrm{hard}}, && (18) \end{aligned}$$

as claimed. This makes a particularly large difference in angular-ordered algorithms because it is common for several soft gluons to be emitted at large angles before reaching the hardest emission.

We finally note that the parton shower cross-section is not in general guaranteed to be larger than the matrix-element one, so we need to be able to enhance emission, as well as reduce it. As we show in the appendix, it is straightforward to uniformly enhance the emission probability by any integer factor. The veto algorithm can then be used to reduce this down to the appropriate level.

5 Summary

We have discussed the two ways in which first-order matrix elements can be used to improve parton shower algorithms, and showed that both must be carefully defined to ensure self-consistency. For the complementary phase-space method, this requires either that the fraction of emissions that go in to the matrix-element region be small, or that a form factor be included to generate the hardest emission in the region exclusively, rather than all inclusive emissions. For the correction to the algorithm, it requires that the correction should be applied to every emission that is the hardest so far, rather than just to the first emission, as was done in previous algorithms.

There is no conceptual difficulty with combining both types of correction within the same algorithm. Indeed for parton shower algorithms that are capable of covering the whole of phase-space with their first emission, the two corrections are identical for the first emission. The only difference is whether the phase-space points are generated directly according to the matrix element or first according to the parton shower algorithm, and then corrected to the matrix element. By then going on to correct any subsequent emissions that are harder than the first, we ensure that the correction is self-consistently applied to the whole shower, and not just to the first emission.

Having implemented both types of correction[11,12], we have found that while the complementary phase-space method is phenomenologically important, the correction to the algorithm has little effect, at least within the algorithm of [4], in which the parton shower is not able to cover the whole of phase-space, but is a good approximation in the regions it does cover.

We finally note that although such corrections have only been applied to e^+e^- or DIS so far, our arguments apply equally well to any other hard process. In particular, it would be straightforward to use the simple prescription of [13] to provide a matrix-element correction to the Drell-Yan process that successfully unified the high- and low-p_t regions. This is in progress.

Acknowledgements

I am grateful to Pino Marchesini and Bryan Webber for many discussions of these and related subjects. The comments of Gösta Gustafson, Gunnar Ingelman and Leif Lönnblad are also gratefully acknowledged. This work is supported in part by the EEC Programme "Human Capital and Mobility", Network "Physics at High Energy Colliders", contract CHRX-CT93-0357 (DG 12 COMA).

Appendix: The Veto Algorithm

In this appendix we briefly recall three applications of the veto algorithm.

A.1 The Veto Algorithm

Imagine that we want to generate a probability distribution

$$F(x) = f(x)\exp\left\{-\int_x^{x_{\max}} f(x)dx\right\} \quad \text{with } x < x_{\max}, \tag{19}$$

but only know how to generate some other distribution

$$G(x) = g(x)\exp\left\{-\int_x^{x_{\max}} g(x)dx\right\} \quad \text{with } x < x_{\max}, \tag{20}$$

with

$$g(x) \geq f(x). \tag{21}$$

The veto algorithm consists of the following steps to generate $F(x)$:

1. Generate a value of x according to $G(x)$, with $x < x_{\max}$.

2. With probability $f(x)/g(x)$, keep the generated x value.

3. Otherwise, set $x_{\max} = x$ and go to step 1.

The probability distribution produced by this procedure satisfies the integral equation

$$\begin{aligned} P(x) &= \frac{f(x)}{g(x)}\Bigg(g(x)\exp\Big\{-\int_x^{x_{\max}} g(x)dx\Big\} \\ &\quad+\int_x^{x_{\max}} dx' P(x')\frac{g(x')-f(x')}{f(x')}g(x)\exp\Big\{-\int_x^{x'} g(x)dx\Big\}\Bigg). \end{aligned} \tag{22}$$

The term outside the bracket is the probability that the generated value was kept, the first term in brackets is the probability that this value was generated at the first attempt. The second term is the integral over all higher values, of the probability that they were generated, but rejected, with the next value being generated at x. It is straightforward to show that this is satisfied by

$$P(x) = F(x) \tag{23}$$

as claimed.

A.2 Competing Processes

Imagine that we want to generate a probability distribution $F(x)$, but only know how to generate $G(x)$ and $H(x)$, with

$$f(x) = g(x) + h(x). \tag{24}$$

$F(x)$ is then generated by choosing one x value according to each of $g(x)$ and $h(x)$, and using the larger. In this case the resulting probability distribution satisfies

$$P(x) = G(x)\exp\Big\{-\int_x^{x_{\max}} h(x)dx\Big\} + H(x)\exp\Big\{-\int_x^{x_{\max}} g(x)dx\Big\}, \tag{25}$$

where the first term corresponds to cases where the value chosen according to $G(x)$ is the larger, reduced by the probability that the other value is not larger, and vice versa. Clearly this is satisfied by

$$P(x) = F(x) \tag{26}$$

as claimed. In the case that $g(x)$ and $h(x)$ correspond to different physical processes, the process from which x was generated is the one that happened.

A.3 Enhancing Emission

Finally, imagine that we want to generate $F(x)$, but only know how to generate $G(x)$, with

$$f(x) = ng(x), \tag{27}$$

with n an integer. This can be done as a special case of A.2, by considering $F(x)$ to be the sum of n identical competing processes, $G(x)$. We then choose n values of x according to $G(x)$ and use the largest of them.

References

1. G. Marchesini, B.R. Webber, G. Abbiendi, I.G. Knowles, M.H. Seymour and L. Stanco, Computer Phys. Comm. 67 (1992) 465.
2. T. Sjöstrand and M. Bengtsson, Computer Phys. Comm. 43 (1987) 367.
3. L. Lönnblad, Computer Phys. Comm. 71 (1992) 15.
4. G. Marchesini and B.R. Webber, Nucl. Phys. B238 (1984) 1.
5. M. Bengtsson and T. Sjöstrand, Phys. Lett. 185B (1987) 435; Nucl. Phys. B289 (1987) 810.
6. G. Gustafson and U. Pettersson, Nucl. Phys. B306 (1988) 746.
7. B.R. Webber, Nucl. Phys. B238 (1984) 492.
8. B. Andersson, G. Gustafson, G. Ingelman and T. Sjöstrand, Phys. Rep. 97 (1983) 31.
9. L3 Collaboration, B. Adeva et al., Zeit. Phys. C55 (1992) 39.
   OPAL Collaboration, R. Akers et al., Zeit. Phys. C63 (1994) 181.
10. G. Ingelman, in *Physics at HERA*, vol. 3, p. 1366.
11. M.H. Seymour, Zeit. Phys. C56 (1992) 161.
12. M.H. Seymour, Lund preprint LU TP 94-12, submitted to the International Conference on High Energy Physics, Glasgow, U.K., 20–27 July 1994.
13. M.H. Seymour, Lund preprint LU TP 94-13, submitted to Nucl. Phys. B.