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Particle Production From the Colour Field of Interacting Strings, Fragmentation of a Colour Rope

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Abstract

The fragmentation of a multi-string colour field with interaction (colour rope) has been studied based on a generalization of the well-known Lund String Fragmentation Model. The results from its Monte Carlo simulations are presented and discussed. The model reproduces the results of the fragmentation of independent strings when the interaction vanishes.

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1 Introduction

In relativistic heavy-ion collisions, for example, between S and Au at 200 AGeV/c, there are generally many "strings" formed through collisions. It is natural to ask if the fragmentation of the strings can be regarded as independent of each other when the density of strings is high. The question is, of course, closely related to the properties of the QCD vacuum.

In the FRITIOF Model [1], a hadron behaves like a massless relativistic string with a confined colour field similar to a vortex line (a thin quantized flux line) in a type II superconductor. In an electromagnetic superconductor a vortex line will interact with adjacent vortex lines.

In accordance with these considerations, a model of the fragmentation of such a multi-string field with interaction has been proposed [2] as a generalization of the fragmentation scheme of a single string in the Lund String Model [3] [4]. There is besides the "individual" string energy density (the string tension at rest $\kappa \simeq 1$ GeV/fm), also some interaction energy in the overlapping region of the strings.

In this paper, we are going to present further results obtained from the Monte-Carlo simulation of the multi-string fragmentation scheme. In our calculation, we have taken the conservations of energy-momentum and other quantum numbers into account. Some significant results are obtained when the interaction among strings is included, e.g. high p_T enhancement and the ratio of K meson to π meson increasing faster with transverse momentum when compared with non-interaction case.

In Sect.2, we give an example how this model of the multi-string fragmentation works by studying the fragmentation of three interacting strings. in Sect.3 we present the results of Monte-Carlo simulation of the model and we end in Sect.4 with a few concluding remarks.

2 The Fragmentation Mechanism of a Colour Rope

The fragmentation picture of two interacting strings has been presented in [2]. Here we briefly recall the scheme by generalizing the picture to the fragmentation of three interacting strings. Once we know how to handle the fragmentation of three interacting strings, the generalization to n strings is straight-forward.

2.1 The Breakup Scheme for Interacting Strings

The fragmentation scheme of three interacting strings is shown in Fig. 1 for the production of the first two mesons. Here, we use notations $V_{1,II}$ for the first vertex and $V_{0,II}$ for the initial particle in the 2-field, P_1 and P_2 for the points where the first meson and the second meson are formed.

The main features in the three-string case are similar to the two-string case, but the kinematics in present case is a little more complicated. For instance, a quark stemming from the 3-field will be accelerated first by the force $\mathcal{K}_3(\equiv \kappa_3 - \kappa_2)$, then by the force $\mathcal{K}_2(\equiv \kappa_2 - \kappa)$ when going into the

2-field and finally by κ in the 1-field before a meson is formed.

To be more clear, the first breakup occurs at the $V_{1,II}$ in the 2-field, then an antiquark $\bar{q}$ (assuming that the initial quarks are aligned along the positive lightcone) from the breakup is accelerated along the positive lightcone by the force $\mathcal{K}_2$ and later accelerated by the force κ after passing by an initial quark q (out of the turning point $V_{0,II}$) at the point A. The first meson is formed at the point P_1 by the $\bar{q}$ and the initial quark q (out of the turning point $V_{0,I}$) accelerated along the negative lightcone by the 1-field. The second meson is formed at the point P_2 by the q (out of the turning point $V_{0,II}$) accelerated first by the 2-field along the negative lightcone and then accelerated by the 1-field after the point A and another $\bar{q}$ (out of the vertex $V_{1,III}$) accelerated along the positive lightcone first by the force $\mathcal{K}_3$, then by the force $\mathcal{K}_2$ after the point C and finally by κ in the 1-field after the point B. Based on the statements above, we can write down the light-cone energy-momentum components of the first meson

$$\begin{aligned} p_{1+} &= \mathcal{K}_2(V_{0+,II} - V_{1+,II}) + \kappa(V_{0+,I} - V_{0+,II}) \\ p_{1-} &= \kappa(V_{1-,II} - V_{0-,I}). \end{aligned} \tag{1}$$

For the second meson, we have

$$\begin{aligned} p_{2+} &= \mathcal{K}_3(V_{0+,III} - V_{1+,III}) + \mathcal{K}_2(V_{1+,II} - V_{0+,III}) + \kappa(V_{0+,II} - V_{1+,II}) \\ p_{2-} &= \mathcal{K}_2(V_{1-,II} - V_{0-,II}) + \kappa(V_{1-,III} - V_{1-,II}). \end{aligned} \tag{2}$$

2.2 The Probability Distribution

We notice that the colour field from which mesons are produced has been divided into three parts when the interactions among the three strings are introduced. Each part has its own field intensity. The probability distribution for the fragmentation of a single string in the Lund String Model [4] consists of three parts: the first part is a phase space factor coming from the mass-shell condition; the second part is due to the requirement of the four-momentum conservation; the last part, an exponential area suppression law, describes "what is used up of possibilities" with a parameter b to be its weight. Apparently the probability distribution in the interacting string case should still contain these three parts with the difference coming from the fact that there are "localized" fields in the space-time of particle production when the interaction is introduced. The probability distribution is thus assumed as [2]

$$dP(p_1, ..., p_n) \simeq \prod_{j=1}^{n} N dp_j \delta(p_j^2 - m_j^2) \delta\left(\sum_{j=1}^{n} p_j - P_{tot}\right) \exp(-b_1 A_{,I} - b_2 A_{,II} - b_3 A_{,III}) \tag{3}$$

with the three parameters $b_1(\equiv b), b_2, b_3$ for each of the energy-momentum space areas of the three different fields which are related to the space-time picture as shown in Fig. 1 by

$$\begin{aligned} A_{,I} &= \kappa^2(F_{1,I} + F_{2,I} + ... + F_{n,I}) \\ A_{,II} &= \mathcal{K}_2^2(F_{1,II} + F_{2,II} + ... + F_{n,II}) \\ A_{,III} &= \mathcal{K}_3^2(F_{1,III} + F_{2,III} + ... + F_{n,III}). \end{aligned} \tag{4}$$

Similar to the two-string case, if $b_2\mathcal{K}_2^2 = 2b_1\kappa^2$ and $b_3\mathcal{K}_3^2 = 3b_1\kappa^2$, the exponential suppression factor can be written out as the product of three corresponding independent string factors. Furthermore, if $\mathcal{K}_3 = \mathcal{K}_2 = \kappa$ (no interaction among strings), we then get $b_3 = 3b_1$ and $b_2 = 2b_1$.

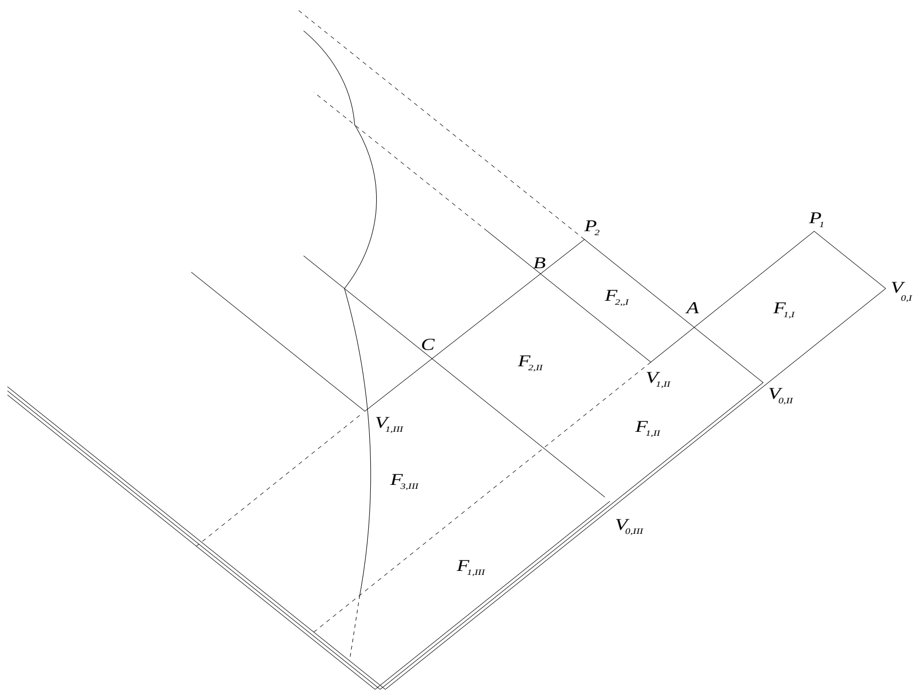

Figure 1: The breakup of a rope of three-string for the production of the first two mesons, to indicate the areas entering the fragmentation function. The curves correspond to three hyperbolas in the three fields along which the vertex $V_{1,III}$ is chosen.

In order to determine the position of a vertex, we need two variables. But the constraint that a meson should be produced on mass shell i.e.

$$p_+p_- = m_\perp^2 \tag{5}$$

reduces the free parameters to one, which one can parameterize in coordinate space as the fraction z of the maximal field extension along the lightcone, which has been "gobbled up" by the meson.

As long as one stays within a field region, the relation Eq.(5) corresponds to choosing a point along a hyperbola with respect to the previous vertex. However, after one crosses the border between two field regions, he has to choose the production vertex along a new hyperbola. In the three regions, the phase space factors are, respectively

$$dp\delta(p^2-m^2)=\frac{dp_+}{p_+}=\begin{cases}\dfrac{dz}{z} & z<z_{b1},\ \text{1-field}\\[2mm] \dfrac{\mathcal{K}_2dz}{\mathcal{K}_2z-z_{b1}(\mathcal{K}_2-\kappa)} & z_{b1}<z<z_{b2},\ \text{2-field}\\[2mm] \dfrac{\mathcal{K}_3dz}{\mathcal{K}_3z-z_{b2}(\mathcal{K}_3-\mathcal{K}_2)-z_{b1}(\mathcal{K}_2-\kappa)} & z_{b2}<z<1,\ \text{3-field}\end{cases} \tag{6}$$

where z_{b1}, z_{b2} are the z value corresponding to the boundaries and are expressed by definition, together with z as (taking the production of the second meson as an example, see Fig. 1.)

$$\begin{aligned} z &= \frac{V_{0,II}-V_{1,III}}{V_{0,II}} \\ z_{b1} &= \frac{V_{0,II}-V_{1,II}}{V_{0,II}} \\ z_{b2} &= \frac{V_{0,II}-V_{0,III}}{V_{0,II}} \end{aligned} \tag{7}$$

Following the same derivation for the inclusive distribution in [2], we obtain the inclusive distributions of the string fragmentation in the three-string case,

$$f_I(z) = N\frac{(1-z)^{a_1}}{z}\exp\left(-\frac{b_I m_\perp^2}{z}\right) \quad \text{1-field} \tag{8}$$

$$f_{II}(z') = NC_2^{a_2}(1-z_{b1})^{a_1-a_2}\frac{(1-z')^{a_2}}{z'}\exp\left(-\frac{b_{II}m_\perp^2}{z'}\right) \quad \text{2-field} \tag{9}$$

$$\begin{aligned} f_{III}(z'') &= NC_3^{a_3}(1-z_{b2})^{a_2-a_3}(1-z_{b1})^{a_1-a_2}\frac{(1-z'')^{a_3}}{z''} \\ &\quad \cdot\exp\left(-\frac{b_{III}m_\perp^2}{z''}\right) \quad \text{3-field.} \end{aligned} \tag{10}$$

where

$$\begin{aligned} b_I &= b_1z_{b1}+b_2(z_{b2}-z_{b1})\frac{\mathcal{K}_2^2}{\kappa^2}+b_3(1-z_{b2})\frac{\mathcal{K}_3^2}{\kappa^2} \\ b_{II} &= b_I\frac{\kappa}{\mathcal{K}_2C_2} \\ b_{III} &= b_I\frac{\kappa}{\mathcal{K}_3C_3} \\ C_2 &= 1-z_{b1}\frac{\mathcal{K}_2-\kappa}{\mathcal{K}_2} \\ C_3 &= 1-z_{b2}\frac{\mathcal{K}_3-\mathcal{K}_2}{\mathcal{K}_3}-z_{b1}\frac{\mathcal{K}_2-\kappa}{\mathcal{K}_3} \\ z' &= \frac{z-z_{b1}\dfrac{\mathcal{K}_2-\kappa}{\mathcal{K}_2}}{C_2} \end{aligned} \tag{11}$$

$$z'' = \frac{z - z_{b2}\frac{\mathcal{K}_3 - \mathcal{K}_2}{\mathcal{K}_3} - z_{b1}\frac{\mathcal{K}_2 - \kappa}{\mathcal{K}_3}}{C_3}$$

The $a_1(\equiv a), a_2, a_3$ are parameters corresponding to the 1-field, the 2-field and the 3-field, respectively. N is the normalization constant determined by

$$\int f_I(z)dz + \int f_{II}(z)dz + \int f_{III}(z)dz = 1. \tag{12}$$

3 Monte Carlo Simulation

We see from the discussion of section 2 that the mesons are produced from the string fragmentation one by one along the positive light-cone and the fragmentation is ended on the negative light-cone. In fact, we could start from the negative light-cone as well and end on the positive light-cone. Therefore, in a Monte Carlo simulation we decide at each step on which side next meson is formed until the remaining energy in system is smaller than a given value, then the last six mesons (or four mesons in the two-string case) are formed such that the four-momentum and other quantum numbers are conserved. Practically, the stopping point is decided by the requirement that the rapidity distribution of particles produced has a central plateau.

A $q\bar{q}$ pair in LUND MODEL obtains its transverse momenta by the tunneling mechanism with the probability

$$\exp(-\pi m^2/\kappa_{eff})\exp(-\pi p_T^2/\kappa_{eff}) \tag{13}$$

Here κ_{eff} is the coupling constant of the field region from which the $q\bar{q}$ is tunneled out with quark (antiquark) mass m and transverse momentum p_T $(-p_T)$. Eq.(13) shows that the possibility of heavier quark production would be enhanced in a field region with larger κ_{eff}. Assuming the average transverse momentum and strangeness suppression for the single string fragmentation are σ_1 and μ_1, respectively, then the same quantities in, say, the two-field region are

$$\sigma_2 = \sigma_1(\mathcal{K}_2/\kappa)^{1/2} \tag{14}$$

$$\mu_2 = \mu_1^{\kappa/\mathcal{K}_2}. \tag{15}$$

We have interfaced our Monte-Carlo program for the fragmentation of a colour rope with FRITIOF, which is used to produce the initial strings for p-Cu collisions at 200AGeV/c. The strings fragment assuming there is interaction among them. We introduce a parameter δ to indicate the mean strength of the interaction between two strings, so we could write $\kappa_2 = 2\kappa + \delta$ and $\kappa_3 = 3\kappa + 3\delta$. In present calculations, the gluon emission is not included and the primordial transverse momenta of the quarks in a nucleon have been neglected. Therefore, we do not intend to compare our simulation results with experimental data in this paper. Finally, for simplicity, we only fragment two strings at a time, i.e. a colour rope consists of only two interacting strings.

In the following calculations we use all the parameter values defaulted in JETSET for the single string fragmentation and take $b_2 = 2b_1$ and $a_2 = a_1$. Fig. 2a and Fig.2b show the transverse momentum distributions and K/π ratios with respect to p_T calculated for the fragmentation of colour rope and for the fragmentation of independent string. The calculations show a high p_T enhancement and kaon enhancement at large transverse momenta when there is interaction among strings.

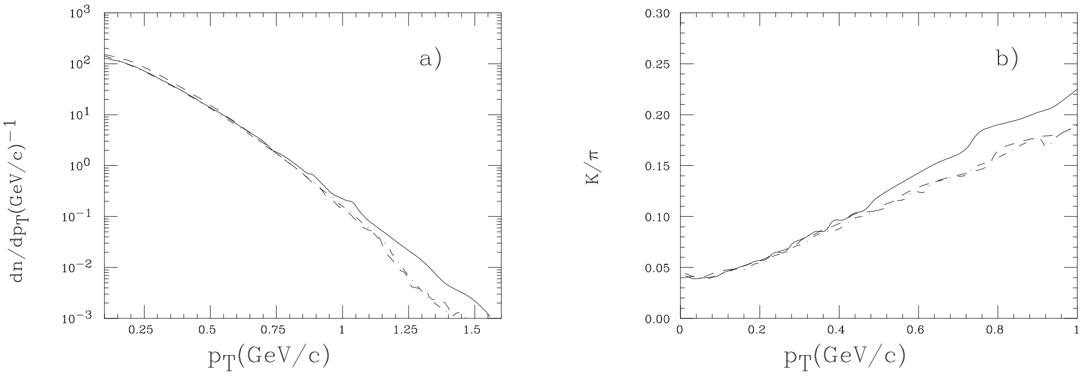

Figure 2: The transverse momentum distributions (a) and the K/π ratios with respect to p_T (b) of the particles produced from the fragmentation of string in 200AGeV/c p-Cu collisions for three cases: δ=0.2 GeV^2 (solid line), δ=0.0 GeV^2 (dashed line) and the independent fragmentation (dash-dotted line).

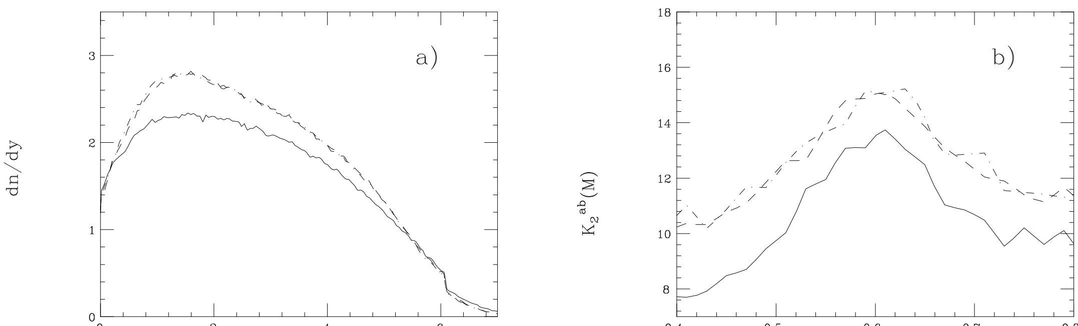

Figure 3: The rapidity distributions (a) and the correlation functions near ρ_0 mass (b) of the particles produced from the fragmentation of strings in 200AGeV/c p-Cu collisions for three cases: δ=0.2GeV^2 (solid line), δ=0.0GeV^2 (dashed line) and the independent fragmentation (dash-dotted line).

The rapidity distributions are shown in Fig. 3a. We have also calculated the two-particle correlation functions around ρ^0 mass as shown in Fig. 3b. The correlation function is defined to be

$$K_2^{ab}(M) = \frac{\rho_2^{ab}(M)}{\rho_1^a \otimes \rho_1^b(M)} - 1. \tag{16}$$

Where ρ_2^{ab} and $\rho_1^{a,b}$ are the two-particle inclusive density and single inclusive densities, respectively; M is the invariant-mass of the two particles.

All the results we obtain show that our simulation of the rope fragmentation reproduces the results of the independent string fragmentation when δ is zero.

4 Concluding Remarks

The idea for this work was motivated by the fact that some phenomena observed in relativistic heavy-ion collisions (say, the high p_T enhancement, strange particle production and so on) can not be well described by viewing nucleus-nucleus collisions as a simple superposition of independent nucleon-nucleon collisions. Before resorting to some new physics like the formation of a quark-gluon plasma state (QGP) in order to explain the experimental data we would like to see if they could be understood by conventional physics.

From the simulation results, we see that the particles produced from the interacting field will on the average obtain larger transverse momenta and more strange particles will be produced from the interacting region. The feature may help us to understand those phenomena observed in heavy-ion collisions and it encourages a further study of the model, e.g. to include the contribution of gluons.

In our earlier study [2] an interesting phenomenon was found that in the fragmentation of the interacting strings a few particles produced tend to acquire a large longitudinal momentum, which we called the "slingshot effect". In a more realistic simulation as we have done here with the strings of different initial masses and cms rapidities, and with the resonance decay the effect has been smeared out quite a lot. However, it can still be seen by the diminishing production of particles in the central rapidity region for a nonvanishing δ, see 3a.

It is natural to ask in the framework of this model if the strings could " melt up " into an object of the same character corresponding to the breakdown of a type II superconductor when the density of string (or the energy density of string per unit length) reaches a critical value, which is an intuitive picture of the phase transition from the hadronic state to the QGP state.

In a type II superconductor if several vortex lines are arranged on a close-packed hexagonal lattice the interaction energy per vortex line per unit length is given [5] as

$$\delta = 6\kappa \frac{\ln(\lambda/d)}{\ln(\lambda/\xi)}, \qquad \xi \leq d \ll \lambda. \tag{17}$$

with the area per vortex line $A = \sqrt{3}d^2/2$. λ and ξ are two material constants of a supercondutor. One of them, λ, is the size of the (exponential) fall-off of a magnetic field in the supercoducting state and the other, ξ, the size of the Cooper pairs, which is the transverse size of a vortex line for a type II superconductor. d is the distance of two vortex lines.

We could imagine that the breakdown of the vortex state occurs at $d = 2\xi$ where the cores of the two vortex lines start to overlap and the two vortex lines " melt up ". This leads to a critical energy density $\epsilon_c \approx \frac{7\kappa}{\sqrt{3}d^2/2} \simeq 2$ GeV/fm^3 if $\xi = 1$ fm, which is the typical transverse size of the string in our model.

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