BOSE-EINSTEIN CORRELATIONS IN THE LUND MODEL

Talk given at the XXV Multiparticle Symposium

Work Done in Lund Together With My Student Markus Ringnér

E-mail:markus@thep.lu.se

Bo Andersson

Department of Theoretical Physics, University of Lund, Sölvegatan 14A, 22362 Lund, Sweden

E-mail: bo@thep.lu.se

ABSTRACT

I will present a generalisation to identical multi-boson states of an earlier model [1], for the Bose-Einstein correlations, which are inherent in a quantum-mechanical interpretation of the Lund Fragmentation Model. The resulting formulas are for simplicity applied to a toy model, only containing three (charged) boson species ("the pions") and one non-bosonic hadron. (There is now a computer program in C++, available on request from Markus Ringnér, which may be used to obtain the general results from the Lund Model as implemented by JETSET). The resulting predictions imply a difference for the π^0 correlations (two π^0's can be produced as neighbours in rank) compared to the charged $\pi^{\pm}$, which cannot be neighbours in rank. The model also exhibits irregularities in small rapidity bins,of the kind which has been termed "intermittency".

1. Introduction

In an earlier paper [1], a model for Bose-Einstein correlations has been proposed, based upon the properties of the Lund Fragmentation Model. The predictions of the model does fit very well to the data which were available at the time of the PETRA-PEP accelerators for e^+e^--annihilation events. In that reasonably low multiplicity regime it is sufficient to consider the two-particle correlations alone.

For larger energies more and more particles are produced and it is the intention of this note to exhibit the necessary corrections to the model in case there are many identical bosons available [2] .We will in forthcoming publications extend the model both to the Bose-Einstein properties of the hadrons inside gluon jets (which, with our dynamical considerations will be different from the ones we present in here), to Deep Inelastic Scattering events and also to hadronic interactions generally.

The main problem is to be able to handle states with $n \gg 1$ identical bosons. It is then necessary to introduce $(n)!$ terms in the symmetrisation process for the matrix elements. Such problems are what the mathematicians call "exponential" (as compared to problems containing n entities in which the computational work is of a "power" character, ie the number of terms rises as n^β with β a constant).

In this note we partition the permutation group into classes. The classes can be grouped together into sets of corresponding "order", eg 2-exchanges, $2 \otimes 2$- and 3-exchanges, $2 \otimes 2 \otimes 2$- ,$3 \otimes 2$- and 4-exchanges etc. We provide a general formula for the Bose-Einstein weights and one major finding is that they decrease in size wrt order.

The result is that each state in the Lund Fragmentation Model should be endowed with a Bose-Einstein weight, which may be calculated as soon as we have provided the state (which also defines all the $(n_1)!(n_2!)(n_3)!$ possible symmetrical states obtained in the case we have three different bosons, $j = 1, 2, 3$ with the multiplicities n_j).

In this report I will be satisfied to discuss the results from a "toy model" in which the Lund Fragmentation process only contains two flavors (u, d), which may be used to make up just the three identical bosons (which we will call (π^+, π^0, π^-) in order to connect them to the three pion species), all of them with the same mass m_0 and one further species (to be called an η) with the mass m_1. We will assume the η to be non-bosonic, ie we do not symmetrise the states wrt to this meson.

The Bose-Einstein correlation length that is dynamically implemented by this model can most easily be described as "the correlation length in flavor", or " the region inside which a particular flavor is neutralised". We note the close correlation between flavor and color and anti-flavor and anti-color. The QCD force fields as implemented in the Lund Model by the Massless Relativistic String will drag the colored (anti-colored) charges in different directions and therefore also the "observable" flavor states are correlated in the same way.

I will in Section 2. provide a brief discussion of the Lund Fragmentation Model including some considerations on the parameter values of the model. In Section 3. we consider the model for the Bose-Einstein effects and finally in Section 4. a few comments on the calculational methods and set of predictions for the toy model are gathered.

In particular we find an interesting difference between the production of the π^0's (note that two or more π^0 may be produced as neighbours in rank) and the $\pi^\pm$ (which for the same charge may not be produced as neighbours in rank). We also find irregularities in the distributions for small rapidity bins of the kind called "intermittency". Actually if we chose the parameters of the model to be those of JETSET (although still with only the above-mentioned meson species) then we find that the present trend of the data are just in agreement with our results.

2. Some Properties of the Lund Fragmentation Process

The Lund Model contains a non-trivial description of a color force field in terms of the Massless Relativistic String with the color 3 quarks, q, color $\bar{3}$ anti-quarks, $\bar{q}$ at the endpoints of the string field and the color 8 gluons, g, as internal excitations on the string. There is unique breakup rule for the string [3], corresponding to the production of $(q, \bar{q})$-pairs (ie new "end-points") along the space-time (or equivalently the energy-momentum space) surface spanned by the string during its history. In this way there is a production of "hadrons", composed of a q from one vertex (breakup-point), a $\bar{q}$

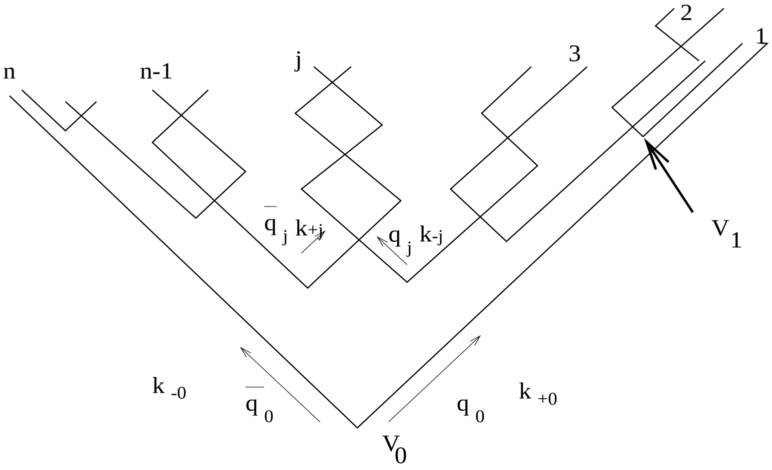

Figure 1: The decay of a Lund Model String with a few "vertices" and hadrons marked out according to the text

from an adjacent one and the string piece in between.

In this way the Lund String Fragmentation Model contains a unique stochastical process. It is derived by means of (semi)-classical probability arguments stemming from Lorentz covariance and causality (which imply that no vertex is different from any other and therefore the stochastical process must be independent of the ordering of the vertices) . The resulting probability for a string with the energy-momentum P_{tot} to decay into the hadrons $(p_1, ..., p_n)$ is given by

$$dP(p_1, ..., p_n) = \prod (N dp_j \delta(p_j^2 - m_j^2)) \delta(\sum p_j - P_{tot}) \exp(-bA) \qquad (1)$$

ie by the final state hadronic phase space multiplied by the negative exponential of the area A, exhibited in Fig 1. The parameters N and b correspond intuitively to the scales of the "phase space density" and the string surface area, respectively.

Although it stems from semi-classical probability concepts the resulting formula in Eq (1) can be given a quantum mechanical interpretation inside a non-abelian gauge field theory [1].To see that we note the similarity to Fermi's Golden Rule, ie that the (non-normalised) transition probability should be given by the final state phase space multiplied by the absolute square of a transition matrix element, $\mathcal{M}$. (To normalise to a cross-section we need some flux-factors and for the normalisation to a probability we need to divide by the relevant "total cross-section").

In order that the hadron denoted 1 in Fig 1, which is composed of the "original" q_0 (from the vertex V_0) and the $\bar{q}_1$ (from the vertex V_1) should be a color singlet it is necessary that "the two production vertices are color-connected", ie the color from one must match the color from the other. The way to get such a connection was invented by Schwinger a long time ago and his prescription means that the production matrix contains at least a gauge connector factor (with A_μ the gaugefield and g the coupling constant):

$$M_{01} = \exp(ig \int_{V_0}^{V_1} A_\mu dx^\mu) \tag{2}$$

In the same way the matrix element must also contain the factor M_{12} to make the hadron indexed 2 a color singlet etc and in this way we obtain as the minimum ansatz a Wilson Loop Operator:

$$\mathcal{M} = \exp(ig \oint A_\mu dx^\mu) \tag{3}$$

with the integral around the decay surface (note that the fields are singular along the border in the sense that there is a constant force "inside" the surface and no force "outside"; as we are integrating along the border-line we are not allowed to "shrink" the integration contour). According to Wilson's confinement criteria such a Loop Operator should behave like an exponential of the area and we then obtain in terms of the decay area A and the string constant $\kappa \simeq 1\ GeV/fm$:

$$\mathcal{M} = \exp(i\xi A) \quad \text{with} \quad Re(\xi) = \kappa \tag{4}$$

The imaginary part of the quantity ξ is related to the pair-production probability and if we call it $b/2$ we obtain back the result in Eq (1) by the identification

$$|\mathcal{M}|^2 = \exp(-bA) \tag{5}$$

It is worth while to continue the discussion a bit further to show that the interpretation of $b/2$ is consistent also in a different quantum field theoretical setting. In case we consider the persistence probability of the vacuum [4] under the onset of a classical constant electric field, $\mathcal{E}$, we obtain in terms of the space region V where the field is on during the time T the result

$$\exp(-C\kappa^2 VT) \tag{6}$$

where we again use the force on the charges as "the tension" in the field $\kappa = g\mathcal{E}$. The constant C depends upon the field quanta which are assumed to be coupled to the field. For two mass-less flavors and spin 1/2-quanta we obtain

$$C = \frac{\pi}{24} \tag{7}$$

This result stems from a basic tunneling argument, firstly made by Euler and Heissenberg in the 30's, by Schwinger in the 50's and a lot of phenomenologists including the Lund Model builders in the late 70's and early 80's. (It is a good example of the Law of the Conservation of Useful Dynamics: "All the good results of an earlier generation of theorists are reinvented, reused and usually renamed by the next generation").

The results of Eqs (5) and (6) can be compared by the identification $VT = AA_\perp$ with $A_\perp$ the transverse size of the string field. (For definiteness we will use space-time dimensions for the areas but we note that in most applications of the Lund Model the decay area A is calculated in energy momentum space. There is, however, a simple way to transform between the spaces just by applying κ^2).

In this way we may identify

$$b = C\kappa^2 A_\perp = \frac{\alpha\kappa}{6} \tag{8}$$

where in the third line we have used the result in Eq (7) and applied Gauss theorem for the constant electric field over the transverse string area. We have also introduced the (dimension-less) coupling α. This procedure may be questioned in a non-abelian field theory and the result may depending upon the precise application (and also remember eg the vacuum fields for the confinement effects) contain a "fudge-factor" of maybe 2 up or down. Besides the dimensional factor κ this is, however, just the expected (perturbative) pair production rate.

It is possible to take even one further step and to compare the predicted b value to the one obtained by comparisons to the experimental data. This was done in Ref [5], where we investigated the infra-red stability of the Lund Model.

Consider a partonic cascade down to some cut-off "virtuality", eg $k_{\perp min}$, and then use the Lund Model Fragmentation formulas to obtain the final state hadrons. In Ref [5] we used the Monte Carlo simulation programs ARIADNE for the cascade and JETSET for the fragmentation. Then we fit the Lund Model parameters (a, b, σ) in such a way that we obtain the experimentally observed distributions (the parameter a is a "derived" quantity, similar to a Regge intercept, dependent upon (b, N) in Eq (1) and σ is the width of the transverse momentum distribution for the hadrons).

This means that we obtain the parameters (a, b, σ) as functions of the cut-off $k_{\perp min}$. The fact that it is possible to find fragmentation parameter values for all values of $k_{\perp min}$ from just above Λ_{QCD} all the way to at least 7 GeV/c means that the procedure is very stable.

The interesting results are that while the parameter a stays around 0.5 (the intercept of the ρ-trajectory !) essentially independent of $k_{\perp min}$ and σ increases slightly for increasing $k_{\perp min}$ (which of course is reasonable as we "lose out" on the gluonic $k_\perp$-contributions), we find that the parameter b behaves just as the running coupling in QCD with the argument $k_{\perp min}$ and the size determined by Eq (8).

The fact that b decreases with $k_{\perp min}$ is to be expected because increasing $k_{\perp min}$ implies in quantum mechanics better resolution in transverse coordinate (impact parameter) space, ie a decreasing value of the string "width" as measured by $A_\perp$.

One should, however, note that there are correlations among the parameters, in particular between the parameters b and a, ie there are other almost as good choices with different properties. The fact that there is one"very good" choice which fits

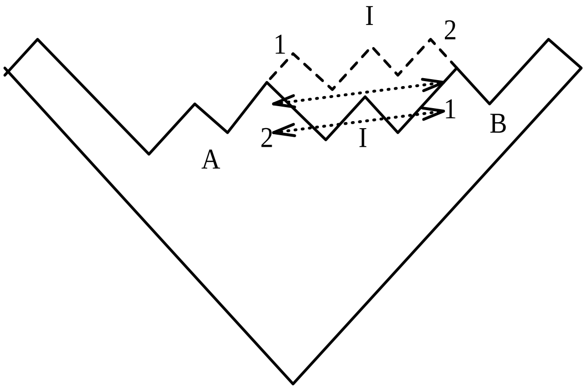

Figure 2: The two ways possible to produce two identical bosons together with the (same) other hadrons "before", "in-between" and "after'. We also show the estimate of the (symmetrical) space-time difference called δx in the main text.

to the predictions is nevertheless reassuring for the statements above. We note as a major fact that b is both from theory and phenomenology essentially smaller than κ.

The main conclusions from this section is that it is possible to interprete the Lund Model Fragmentation function in terms of a quantum mechanical scenarium with the Lund area-law distribution stemming from a transition matrix element containing a Wilson Loop Operator. The phase factor of the matrix element is determined by the decay area A multiplied by the string constant, κ. The decay scale b is in a consistent way determined to be essentially smaller than the phase scale κ.

3. Bose-Einstein According to the Lund Fragmentation Model

In order to see the main mechanism we consider the Fig 2, in which two of the produced hadrons, denoted $(1,2)$ are assumed to be identical bosons. We note that there are two ways to produce the whole state, ie not only the $(1,2)$ and the state in between them, denoted I, but also all the hadrons "before" and "after". The two ways corresponds to exchanging the two identical bosons and both of the production mechanims are exhibited in the figure. The major observation is that the two procedures in general correspond to different decay surface areas!

There is an area-difference to be called δA (a formula for δA is in a more general context provided in Eq (16)). It depends not only upon the energy-momentum vectors (p_1, p_2) and the energy-momentum vector of the intermediate state, p_I. In an intuitively useful way we may write the area difference (in a dimension-less way) as

$$\kappa \delta A = \delta p \delta x \quad \text{with} \quad \delta p = p_1 - p_2 \tag{9}$$

and δx similarly (a reasonable estimate of) the space-time difference (along the string surface, note that it it is always a space-like vector, for more precise discussion cf below the Eq (16)) between the production points of the two identical bosons.

According to quantum mechanics we are expected to symmetrize the transition matrix element so that we obtain in easily understood notation:

$$\mathcal{M} = \mathcal{M}_{12} + \mathcal{M}_{21} \tag{10}$$

This implies that we obtain for the squared matrix element in the Lund Model, cf Eqs (1) and (5):

$$|\mathcal{M}|^2 = (|\mathcal{M}|_{12}^2 + |\mathcal{M}|_{21}^2)(1 + \frac{2Re(\mathcal{M}_{12}\mathcal{M}_{21}^*)}{|\mathcal{M}|_{12}^2 + |\mathcal{M}|_{21}^2}) \tag{11}$$

It is easy to see, using the formula for the matrices $\mathcal{M}$ in Eq (4) that the last term in the parenthesis of Eq (11) only depends upon the area-difference δA, in particular

$$\rho = \frac{2Re(\mathcal{M}_{12}\mathcal{M}_{21}^*)}{|\mathcal{M}|_{12}^2 + |\mathcal{M}|_{21}^2} = \frac{\cos(\kappa\delta A)}{\cosh(b\delta A/2)} \tag{12}$$

Before we consider the generalisation to multi-boson states and the corrections from the transverse momentum generation we note that the implications of Eq (11) is that we may produce the particular final state either by using the classical stochastical Lund Model process to obtain the state indexed (12) or the state indexed (21). But *it is then necessary to weight the results from each one of them by means of the factor* $(1+\rho)$ *to keep to the quantum-mechanical symmetrisation prescription for identical bosons.*

Using Eq (9) we may relate the result to the ordinary way to think about the Bose-Einstein correlations, ie to the Fourier transform of "a source density", $\gamma(x)$:

$$\rho_O = \int dx \gamma(x) \exp(\delta p x) \tag{13}$$

(with the index O for "ordinary). It is obvious that the Bose-Einstein factor obtained in the Lund Model also has an intuitive meaning related to such a source density.

We further note that if there are several identical bosons, called (j_m), $m = 1, ...n$ then there is a corresponding result from the square of the symmetrised transition matrix element

$$|\mathcal{M}|^2 = (\sum_{\mathcal{P}} |\mathcal{M}|_{\mathcal{P}}^2 (1 + \sum_{\mathcal{P}' \neq \mathcal{P}} \frac{2Re(\mathcal{M}_{\mathcal{P}}\mathcal{M}_{\mathcal{P}'}^*)}{|\mathcal{M}|_{\mathcal{P}}^2 + |\mathcal{M}|_{\mathcal{P}'}^2}) \tag{14}$$

where the sums go over the $n!$ possible permutations $\mathcal{P}$ ($\mathcal{P}'$) of the identical bosons. Once again the result can be formulated so that we may according to the classical Lund Model stochastical process produce the state containing the identical bosons together with the remaining particles in accordance with any order of "the identicals". But it is then necessary for quantum mechanics to correct with a Bose-Einstein weight factor which for each such permutation, ie ordering, $\mathcal{P}$ is (note that for the simple case of two identical bosons discussed before there is only one permutation available and therefore only one common factor)

$$(1+\rho)_{\mathcal{P}} = 1 + \sum_{\mathcal{P}' \neq \mathcal{P}} \frac{\cos \kappa \delta A(\mathcal{P}, \mathcal{P}')}{\cosh(b\delta A(\mathcal{P}, \mathcal{P}')/2)} \tag{15}$$

It is easily seen that it is always the area differences which will come in. In this case the area differences will depend not only upon the energy-momentum vectors of the "identicals" but also upon all those vectors which are "in between" the bosons which are permuted.

The "intuitive" formula corresponding to Eq (13) for the multiparticle correlations can be written in different ways (we note in particular that if more than two bosons are involved the area difference does not vanish if only two of the bosons have identical energy-momentum vectors). As an example of the result consider the particular term obtained when (at least) three identical bosons (to be called j, $j = 1, 2, 3$) are produced in the state $S_{123} \equiv (1, I, 2, II, 3)$ ((I, II) are the states "in between") and when we permute $1 \to 3 \to 2 \to 1$). This means the weight-term obtained between the states S_{123} and S_{231}.

What actually happens is that 2 is in the permutation"moved forward" over the "between-state" I, 3 is moved forward over the between-state II and finally 1 is "moved backwards" over the between-state $(I \oplus 2 \oplus II \oplus 3)$. The result is (using the two-dimensional vectors, written in lightcone component language) $\hat{x} = (x_+, -x_-)$ "dual" to $(p_+, p_-)/\kappa \equiv x = (x_+, x_-)$:

$$\kappa \delta A(S_{123}, S_{231}) = (p_2 \hat{x}_I + p_3 \hat{x}_{II} - p_1(\hat{x}_I + \hat{x}_2 + \hat{x}_{II} + \hat{x}_3) \tag{16}$$

(Note that with this definition $p\hat{x} = \kappa x \hat{x} = 0$ and that $\hat{x}^2 \leq 0$, ie $\hat{x}$ is a space-like vector). Therefore we may write eg

$$\kappa \delta A(S_{123}, S_{231}) = (p_2 - p_1)(\hat{x}_I + \hat{x}_2) + (p_3 - p_1)(\hat{x}_{II} + \hat{x}_3) \tag{17}$$

(although also other forms containing energy-momentum vector differences are easily constructed). The $\hat{x}$-combinations correspond to the estimates of the (space-like) space-time difference vectors between the production points, which where mentioned in connection with Eq (13).

We have up to now neglected the transverse momentum generation in the string breakup. A mass-less $(q\bar{q})$ pair without transverse momentum can be produced in a pointlike way anywhere along the string. But this is no longer the case for such a pair produced with transverse momentum, $k_\perp$. Then it is necessary in order to conserve energy-momentum to take some energy from the string (note that the $(q\bar{q})$ has then each a non-vanishing "transverse mass" $m_\perp = k_\perp$).

In the Lund Model we have treated this production as a tunneling process, Refs [6] [7], similar to the so-called Schwinger mechanism discussed in connection with Eq (6) above. For a single pair with the compensating transverse momenta $\pm\vec{k}_\perp$ one obtains from the tunneling process a gaussian suppression

$$\propto \exp(-k_\perp^2/2\sigma) \quad \text{with} \quad 2\sigma = \pi/\kappa \tag{18}$$

The value of σ, used as default in JETSET, is a bit larger than the one quoted in Eq (18) but this can be understood as an effect from (soft) gluon generation along the string field.

Note that these "zero-point fluctuations" of the field obtain contributions both from the field extensions (ie there are contributions which behave inversely to the size of $A_\perp$, cf the results for the b-parameter in connection with the Eqs (7) and (8)) and from the transverse cms-motion of the string (necessary in quantum mechanics). Due to the smallness of b compared to κ it is the last effect which will provide the main contribution in general. But the fact that $A_\perp$ diminishes with increasing $k_{\perp min}$ implies that when we do not account for the gluons "softer" than $k_{\perp min}$ then we have to introduce larger transverse fluctuations from the diminishing transverse size of the string field. Therefore the parameter σ increases with $k_{\perp min}$ as mentioned before.

A hadron produced from a q_1 and a $\bar{q}_2$ is then in the Lund Model endowed with the transverse momentum $\vec{p}_\perp = \vec{k}_{\perp 1} - \vec{k}_{\perp 2}$ (in easily understood notations). In Ref [7] we have shown that the generation process actually can be treated as a kind of Brownian motion (which was the way it has been treated for a long time in connection with Regge concepts, cf [8]). Actually it is the contributions from these transverse fluctuations, which produce the "chaotic" noice in our model, corresponding to the assumption of chaoticity in the sources in connection with the ordinary treatment of Bose-Einstein fluctuations.

There is a difference, however, because in the tunneling process we do not expect to obtain any phase factors, i.e. the transition matrix is expected to be real in this case. If we go back to the two-particle permutations in connection with Fig 2 we note that while the transverse momenta "ingoing" at the vertex A and "outgoing" at the vertex B are fixed by the properties of the states "before" and "after" the symmetrisation state defined by (p_1, p_I, p_2), there is the possibility to rearrange the transverse momentum generation at the vertices around the intermediate state I in such a way that we may conserve the transverse momentum of all the particles.

To be more precise, suppose that we introduce a fixed "incoming" $\vec{k}_{\perp 1}$ and "outgoing" $-\vec{k}_{\perp 2}$ at the vertices A and B and that the transverse momenta of the vectors (p_1, p_I, p_2) are fixed as ("the observed") $\vec{p}_{\perp j}$. Then to conserve $\vec{p}_{\perp j}$ going from the order (12) to (21) it is necessary to exchange the production of transverse momentum at the vertices around I so that (in easily understood notations)

$$\begin{aligned} \vec{k}_{\perp I} &\to \vec{k}_{\perp 1} + \vec{k}_{\perp 2} - \vec{k}'_{\perp I} \\ \vec{k}'_{\perp I} &\to \vec{k}_{\perp 1} + \vec{k}_{\perp 2} - \vec{k}_{\perp I} \end{aligned} \tag{19}$$

But it is then necessary to endow the real exponential factors with this change in transverse momentum generation and we therefore must exchange the formula for the (two-particle) correlation ρ from Eq (12) into

$$\rho = \frac{\cos(\kappa\delta A)}{\cosh((b\delta A)/2 + \delta k_\perp^2/4\sigma)} \tag{20}$$

with $\delta k_\perp^2$ corresponding to the change implied in Eqs (19) and (18). It is straightforward to obtain the corresponding transverse momentum contributions also in connection with the multi-boson permutations.

The general finding is that, due to the smallness of the parameter b compared to κ, it is for each of the contributions to Eq (15) the cos-function decrease with increasing δA that governs the behaviour. Note that the cos argument contains κ as the basic scale. When this function is close to 0 then the transverse momentum contributions, which once again are governed by essentially κ, in the cosh-denominator takes over to damp the possibly negative cos-contributions for a larger δA.

Consequently the main parameter in this model for Bose-Einstein correlation weights is the string tension, κ, which in a well-known way is related to the only hadronic scale of the old Regge theory, ie the slope of the trajectories. It also occurs as the scale of the transverse momentum fluctuations in hadronic physics, generally.

We also note that for the simple two-boson correlation weight in Eqs (12) and (9) the area difference δA vanishes with the vector difference $(p_1 - p_2)$. But for small vector differences, δp, the area difference is governed by the size of the in-between-state I so that δA increases (and $\cos(\kappa\delta A)$ decreases) quickly with this size.

4. Results of the Toy Model

In this report I will not go into any details of the way we calculate the weight distribution, which is defined in connection with Eq (15). It is sufficient to say that it is possible to subdivide the permutation group and to calculate the contributions (for a given multiplicity of identical bosons) iteratively by grouping the contributions.

The result is that the overwhelming amount of weight-factors are centered around 1 with a tiny width, but there are some states (on the percentage level), generally with large multiplicites, which provide weights which may be essentially above 1 or even may be negative. In Ref [2] we will provide more details (the toy model is meant to provide a simple scenarium in which we may calculate the parametric dependence independently of the many disturbances occurring in a more realistic model).

As a first example of what we obtain in the toy model (this is also essentially the same in the general model implemented in JETSET in case we neglect the phase contributions from the resonance decays just as we did in Ref [1]) we show in Fig 3 the correlation function of both charged and neutral pions.

The plot is the result of a simple Monte Carlo simulation of the Lund Model fragmentation formulas (in case we neglect gluons). We have chosen a two-jet state using the Lund Model fragmentation function with the values of the pion mass $m_0 \simeq 0.2$

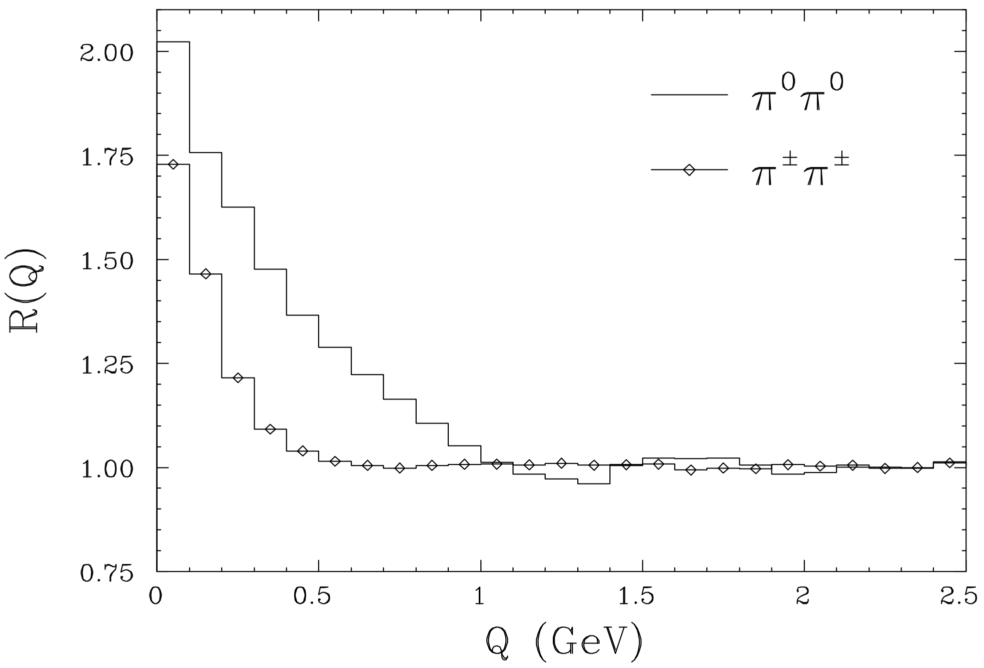

Figure 3: The correlation distributions of both charged and neutral pions in the toy model expressed in terms of the invariant Q, defined in the text.

GeV and with the JETSET standard values of the multiplicity density as well as the transverse momentum width in two-jet events. We note that there are different (two-particle) correlation distributions in the invariant Q-variable, ($Q = \sqrt{-(p_1 - p_2)^2}$) for the charged (the $\pi^\pm$ are the same of course) and the neutral pions.

In a model of this kind the Bose Einstein correlation length can most easily be described as the "flavor compensation length". The (same-)charged pions cannot be produced as neighbours in rank and then it is always necessary to have a non-vanishing "in-between" state I. Consequently the average distance, as measured by the inverse Q-width of the correlation distribution, is larger for the charged pions than for the neutral pions, which are possible to produce (even several in a row) as neighbours in rank (note that the correlation function value at $Q = 0$ is much larger for the neutrals than for the charged pions!). *This prediction of our model is, independently of whatever else one may invent or do, very difficult to get away from.*

My next prediction (the last, due to the space-time limits) from the toy model (and the same is again essentially true inside the full JETSET scenarium) has a similar property, cf Fig 4, ie that it does explain the present data(-trends). But the main prediction is that there should be major changes in the data in case our valiant experimental friends would be able to do an even more sophisticated analysis. I am sorry about the situation but I take recourse to the general Lutheran Rule of High Energy Physics: " There are very few free luncheons (or available positions) and any

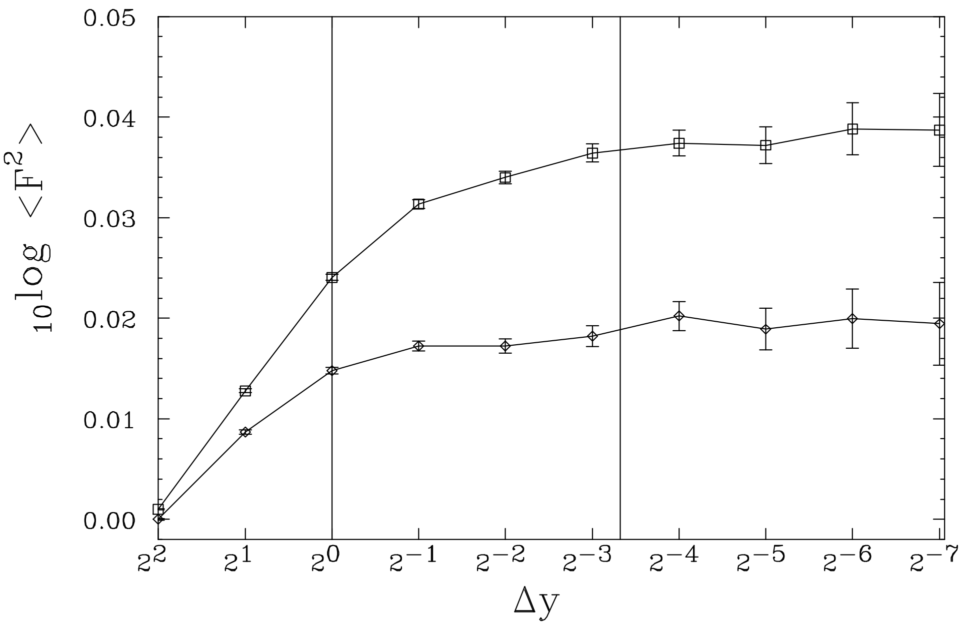

Figure 4: The (log) of the factorial moment, F_2, in terms of the rapidity bin width

time you may feel that you are close to provide a basic dynamical prediction there is a very large noice level, usually stemming from a set of little green men. The result is unfortunately always that all of us must work harder".

In Fig 4 we show the factorial moment F_2 as a function of the rapidity bin-width in case there are no Bose-Einstein correlations (dots) and when we include the Bose-Einstein correlations obtained in this model (the squares). The most noticable difference is that inside just the region where our experimental friends are able to measure (marked by the two lines) there is in the present BE-scenarium a rise and that at smaller Δy-values (than those presently attainable) there is again a saturation (although at a larger level than "plain Lund".

I would end by thanking the arrangers of this very pleasant meeting. It is my feeling also that there is a lot more to do in Multiparticle Dynamics!

1. B.Andersson, W.Hofmann, Phys Lett **B169**(1986)364
2. B.Andersson, M. Ringnér, to be published
3. B.Andersson, G.Gustafson, B.Söderberg, Z. Phys **C20**(1983)317
4. W.Heisenberg, H. Euler, Z. Phys **98**(1936)714, J.Schwinger, Phys.Rev. **82**(1951)664, N.K.Glendenning,T.Matsui, Phys Rev **D28**(1983)2890
5. C. Sjögren Parton Cascades and Multiparticle Production, Ph D thesis Lund University, Dept of Theoretical Physics (1992)
6. B. Andersson, G.Gustafson, T.Sjöstrand, Z. Phys **C6**(1980)235
7. B.Andersson, G.Gustafson, J.Samuelsson, Z. Phys **C64**(1994)653
8. M. Baker, K.A.Ter-Martirosyan, Phys. Rep **28C**(1976)1