CERN–TH/96–04
LU TP 96–2
January 1996

Parton Distributions of the Virtual Photon

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Abstract

We propose a generic ansatz for the extension of parton distributions of the real photon to those of the virtual photon. Alternatives and approximations are studied that allow closed-form parametrizations.

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1 Introduction

It is well known that the real photon has a partonic substructure, induced by virtual fluctuations into $\mathrm{q\overline{q}}$ pairs. These fluctuations are in part non-perturbative, and so cannot be calculated from first principles (at least not without major advances in lattice-gauge theory). Only if the parton distributions are specified by hand at some sufficiently large input scale Q_0 can the continued evolution with Q^2 be described perturbatively. Here Q^2 is the scale of the "probing" hard process. Such a partonic substructure also exists for a space-like photon, $P^2 = -p_\gamma^2 > 0$. Only if P^2 is in the deeply inelastic scattering region can the substructure be neglected; the effects here die away like a higher-twist contribution. For the experimentally accessible and theoretically challenging region $\Lambda_{QCD}^2 \lesssim P^2 \lesssim 2\,\mathrm{GeV}^2$ evolution equations for the parton distributions and their boundary conditions cannot be derived from perturbative QCD. In this letter we propose a theoretical ansatz to prescribe the modification of the Q^2-evolution equations of the parton distributions with changing P^2 and the input at Q_0 as a function of P^2.

While the parton distribution functions (pdf's) of the real photon have been studied in some detail, both experimentally and theoretically (for a recent survey see e.g. [1]), much less is known about the virtual photon. The only published data are by the PLUTO Collaboration [2]. However, recently the ZEUS Collaboration presented new data from HERA [3]. The observed x_γ distribution has been constructed for events with two jets above 4 GeV transverse momentum. As P^2 is increased, this distribution is gradually suppressed at small x_γ, in agreement with theoretical expectations. These first results will be followed by more with increasing precision. Also LEP 2 should contribute in the future [1].

In a previous publication we presented several parametrizations for the parton distributions of the real photon [4], with a proposed extension to virtual photons. This is one of the very few studies that give explicit predictions. Drees and Godbole [5], following an analysis of Borzumati and Schuler [6], have proposed a method based on simple multiplicative factors relative to the parton distributions of the real photons. A similar recipe has been used by Aurenche and collaborators [7]. This may be useful for estimating the effects of mildly virtual photons in a sample of almost real photons, but does not appear well suited for QCD tests of the virtual-photon distributions. A detailed study is performed by Glück, Reya and Stratmann [8], but a main disadvantage here is that there exists no explicit parametrizations of the resulting distributions. The strategy adopted is also only one possibility.

In this letter we extend our previous study to a few alternative approaches for the virtual photon, allowing a study of the limits of our current understanding. Numerical approximations are introduced, which permits the parametrizations of real-photon pdf's to be easily extended to virtual-photon dittos. Section 2 contains a brief summary on the real photon, the virtual one is covered in section 3 and some comparisons are shown in section 4.

2 The Real Photon

The parton distributions of the real photon obey a set of inhomogeneous evolution equations:

$$\frac{\partial f_a^{\gamma}(x,Q^2)}{\partial \ln Q^2} = \int_x^1 \frac{\mathrm{d}y}{y}\, f_b^{\gamma}(y,Q^2)\, \frac{\alpha_{\mathrm{s}}}{2\pi}\, P_{a/b}\left(\frac{x}{y}\right) + 3\, e_a^2\, \frac{\alpha_{\mathrm{em}}}{2\pi} \left(x^2 + (1-x)^2\right) \; , \tag{1}$$

to leading order. The first term is the one present in standard evolution equations, e.g. for the proton. The second term, the so-called anomalous one, comes from branchings $\gamma \to \mathrm{q}\overline{\mathrm{q}}$, and is unique for the photon evolution equations.

The solution can be written as the sum of two terms,

$$f_a^{\gamma}(x,Q^2) = f_a^{\gamma,\mathrm{NP}}(x,Q^2;Q_0^2) + f_a^{\gamma,\mathrm{PT}}(x,Q^2;Q_0^2) \; . \tag{2}$$

The former term is a solution to the homogeneous evolution (i.e. without the second term in eq. (1)) with a non-perturbative input at $Q = Q_0$, and the latter is a solution to the full inhomogeneous equation with boundary condition $f_a^{\gamma,\mathrm{PT}}(x,Q_0^2;Q_0^2) \equiv 0$. One possible physics interpretation is to let $f_a^{\gamma,\mathrm{NP}}$ correspond to $\gamma \leftrightarrow V$ fluctuations, where $V = \rho^0, \omega, \phi, \mathrm{J}/\psi, \ldots$, is a set of vector mesons, and let $f_a^{\gamma,\mathrm{PT}}$ correspond to perturbative ("anomalous") $\gamma \leftrightarrow \mathrm{q}\overline{\mathrm{q}}$ fluctuations, q = u, d, s, c and b. The discrete spectrum of vector mesons can be combined with the continuous (in virtuality k^2) spectrum of $\mathrm{q}\overline{\mathrm{q}}$ fluctuations, to give

$$f_a^{\gamma}(x,Q^2) = \sum_V \frac{4\pi\alpha_{\mathrm{em}}}{f_V^2} f_a^{\gamma,V}(x,Q^2;Q_0^2) + \frac{\alpha_{\mathrm{em}}}{2\pi} \sum_{\mathrm{q}} 2 e_{\mathrm{q}}^2 \int_{Q_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2}\, f_a^{\gamma,\mathrm{q}\overline{\mathrm{q}}}(x,Q^2;k^2) \; , \tag{3}$$

where each component $f^{\gamma,V}$ and $f^{\gamma,\mathrm{q}\overline{\mathrm{q}}}$ obeys a unit momentum sum rule. Although each component formally depends on two scales, Q_0^2/k^2 and Q^2, it is the combination

$$s = \int_{Q_0^2/k^2}^{Q^2} \frac{\alpha_{\mathrm{s}}(r^2)}{2\pi} \frac{\mathrm{d}r^2}{r^2} \tag{4}$$

that sets the length of the evolution range and thus gives the full scale dependence.

Beyond this fairly general ansatz, a number of choices has to be made. The approach adopted in the SaS sets [4] is described in the following.

What is Q_0? A low scale, $Q_0 \approx 0.6$ GeV, is favoured if the V states above are to be associated with the lowest-lying resonances only. Then one expects $Q_0 \sim m_\rho/2$–m_ρ. Furthermore, with this Q_0 one obtains a reasonable description of the total γp cross section, and continuity, e.g. in the primordial $k_\perp$ spectrum [9]. Against this choice there are worries that perturbation theory may not be valid at such low Q, or at least that higher-twist terms appear in addition to the standard ones. Alternatively one could therefore pick a larger value, $Q_0 \approx 2$ GeV, where these worries are absent. One then needs to include also higher resonances in the vector-meson sector, which adds some arbitrariness, and one can no longer compare with low-Q data. We have chosen to prepare sets for both these (extreme) alternatives.

How is the direct contribution to be handled? Unlike the p, the γ has a direct component where the photon acts as an unresolved probe. In the definition of F_2^{γ} this adds a component C^{γ}, symbolically

$$F_2^{\gamma}(x,Q^2) = \sum_{\mathrm{q}} e_{\mathrm{q}}^2 \left[f_{\mathrm{q}}^{\gamma} + f_{\overline{\mathrm{q}}}^{\gamma} \right] \otimes C_{\mathrm{q}} + f_{\mathrm{g}}^{\gamma} \otimes C_{\mathrm{g}} + C^{\gamma} \; . \tag{5}$$

Since $C^\gamma \equiv 0$ in leading order, and since we stay with leading-order fits, it is permissible to neglect this complication. Numerically, however, it makes a non-negligible difference. We therefore make two kinds of fits, one DIS type with $C^\gamma = 0$ and one $\overline{\mathrm{MS}}$ type including the universal part of C^γ[10].

How are heavy flavours, i.e. mainly charm, to be dealt with? When jet production is studied for real incoming photons, the standard evolution approach is reasonable, but with a lower cut-off $Q_0 \approx m_\mathrm{c}$ for $\gamma \to \mathrm{c\overline{c}}$. Moving to deep inelastic scattering, $\mathrm{e}\gamma \to \mathrm{e}X$, there is an extra kinematical constraint: $W^2 = Q^2(1-x)/x > 4m_\mathrm{c}^2$. It is here better to use the "Bethe-Heitler" cross section for $\gamma^*\gamma^* \to \mathrm{c\overline{c}}$. Therefore two kinds of output are provided. The parton distributions are calculated as the sum of a vector-meson part and an anomalous part including all five flavours, while F_2^γ is calculated separately from the sum of the same vector-meson part, an anomalous part and possibly a C^γ part now only covering the three light flavours, and a Bethe-Heitler part for c and b.

Should ρ^0 and ω be added coherently or incoherently? In a coherent mixture, $\mathrm{u\overline{u}} : \mathrm{d\overline{d}} = 4 : 1$ at Q_0, while the incoherent mixture gives $1 : 1$. We argue for coherence at the short distances probed by parton distributions. This contrasts with long-distance processes, such as $\mathrm{p}\gamma \to \mathrm{p}V$.

What is Λ_QCD? The F_2^γ data are not good enough to allow a precise determination. Therefore we use a fixed value $\Lambda^{(4)} = 200$ MeV, in agreement with conventional results for proton distributions.

In total, four distributions are presented [4], based on fits to available data:

- SaS 1D, with $Q_0 = 0.6$ GeV and in the DIS scheme.
- SaS 1M, with $Q_0 = 0.6$ GeV and in the $\overline{\mathrm{MS}}$ scheme.
- SaS 2D, with $Q_0 = 2$ GeV and in the DIS scheme.
- SaS 2M, with $Q_0 = 2$ GeV and in the $\overline{\mathrm{MS}}$ scheme.

The VMD distributions and the integral of the anomalous distributions are parametrized separately and added to give the full result; this is of importance for the following.

3 The Virtual Photon

The evolution equations (in Q^2) of the pdf's of the virtual photon (and its solutions) can be exactly calculated in perturbative QCD for a restricted P^2 range, namely [11]

$$Q_0^2 \ll P^2 \ll Q^2 \; . \tag{6}$$

Experimentally accessible, and theoretically challenging is, however, the low-P^2 range $\Lambda_\mathrm{QCD}^2 \lesssim P^2 \lesssim Q_0^2$ where evolution equations cannot be derived from perturbative QCD. We propose pdf's that are valid for all $0 \le P^2 \le Q^2$. These are arrived at as follows. We start from the observation that the moments of the pdf's are analytic in the P^2 plane. A natural way to make use of this property is [12] to express them in terms of a dispersion-integral in the (time-like) mass square k^2 of the $\mathrm{q\overline{q}}$ fluctuations. This links perturbative and non-perturbative contributions and allows the smooth limit $P^2 \to 0$. The model-dependence enters when specifying the necessary weight functions. We choose these in such a way that the resulting expressions possess the correct, known behaviours for both $P^2 \to 0$ and the range (6). The result is

$$f_a^{\gamma^\star}(x,Q^2,P^2) = \int_0^{Q^2} \frac{\mathrm{d}k^2}{k^2} \left(\frac{k^2}{k^2+P^2}\right)^2 \frac{\alpha_\mathrm{em}}{2\pi} \sum_\mathrm{q} 2e_\mathrm{q}^2 f_a^{\gamma,\mathrm{q\overline{q}}}(x,Q^2;k^2) \; . \tag{7}$$

A definite behaviour for $P^2 \to Q^2$ has not yet been imposed.

The non-appearance of a $1/P^2$ contribution in (7) can be argued in two ways. First this is what one expects when applying generalized vector-meson dominance to a continuous (in k^2) spectrum of $\mathrm{q\overline{q}}$ fluctuations. Second, at large P^2 an operator-product expansion of the pdf's in powers of $1/P$ holds, but the first non-vanishing higher-twist contribution comes from the dimension-4 gluon condensate [13].

Associating the low-k^2 part of relation (7) with the discrete set of vector mesons gives a generalization of eq. (3) to

$$f_a^{\gamma^\star}(x, Q^2, P^2) = \sum_V \frac{4\pi\alpha_{\mathrm{em}}}{f_V^2} \left(\frac{m_V^2}{m_V^2 + P^2} \right)^2 f_a^{\gamma,V}(x, Q^2; \tilde{Q}_0^2) + \frac{\alpha_{\mathrm{em}}}{2\pi} \sum_{\mathrm{q}} 2e_{\mathrm{q}}^2 \int_{Q_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \left(\frac{k^2}{k^2 + P^2} \right)^2 f_a^{\gamma,\mathrm{q\overline{q}}}(x, Q^2; k^2) \; . \tag{8}$$

In addition to the introduction of the dipole form factors, note that the lower input scale for the VMD states is here shifted from Q_0^2 to some $\tilde{Q}_0^2 \geq Q_0^2$. This is based on a study of the evolution equation [6] that shows that the evolution effectively starts "later" in Q^2 for a virtual photon. $\tilde{Q}_0$ can be associated with the P_0, P'_0, P_{eff} or P_{int} scales to be introduced below.

Equation (8) is one possible answer, which we will use as a reference in the following. It depends on both Q^2 and P^2 in a non-trivial way, however, so that results are only obtained by a time-consuming numerical integration rather than as a simple parametrization.

In order to obtain a tractable answer, one may note that the factor $(k^2/(k^2 + P^2))^2$ provides an effective cut-off at $k \approx P$, so one possible substitution for the anomalous component is

$$\int_{Q_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \left(\frac{k^2}{k^2 + P^2} \right)^2 \left[\cdots \right] \longmapsto \int_{P_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \left[\cdots \right] \; , \tag{9}$$

with $P_0^2 = \max(Q_0^2, P^2)$. All the Q^2 and P^2 dependence is now appearing in a combination like s in eq. (4), i.e. parametrizations of $f_a^{\gamma^\star}(x, Q^2, P^2)$ are readily available by simple modifications of those for $P^2 = 0$. This gives the approach we adopted in ref. [4].

Some objections can be raised against this substitution. The choice of P_0 means that the anomalous component is independent of P for $P < Q_0$, and that there is a discontinuous change in behaviour at $P = Q_0$. Other simple expressions could have been adopted to solve the problem, such as[1] $P_0'^2 = Q_0^2 + P^2$, but this only illustrates the arbitrariness of the choice.

One guiding principle could be the preservation of the momentum sum. We recall that the components $f^{\gamma,V}$ and $f^{\gamma,\mathrm{q\overline{q}}}$ integrate to unit momentum, i.e.

$$\sum_a \int_0^1 \mathrm{d}x \, f_a^{\gamma^\star}(x, Q^2, P^2) = \sum_V \frac{4\pi\alpha_{\mathrm{em}}}{f_V^2} \left(\frac{m_V^2}{m_V^2 + P^2} \right)^2 + \frac{\alpha_{\mathrm{em}}}{2\pi} \sum_{\mathrm{q}} 2e_{\mathrm{q}}^2 \int_{Q_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \left(\frac{k^2}{k^2 + P^2} \right)^2 \tag{10}$$

is a measure of the probability for a photon to be in a V or $\mathrm{q\overline{q}}$ state. Momentum sum preservation would thus suggest the introduction of a scale P_{eff}^2 according to

$$\int_{Q_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \left(\frac{k^2}{k^2 + P^2} \right)^2 \equiv \int_{P_{\mathrm{eff}}^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \; , \tag{11}$$

[1]In the following the P'_0 prescription implies also the replacement $Q^2 \longmapsto Q^2 + P^2 \, Q_0^2/Q^2$ in (8) so as to ensure sensible behaviours for $Q^2 \to Q_0^2$ and $P^2 \to Q^2$.

which gives

$$P_{\rm eff}^2 = Q^2 \frac{Q_0^2 + P^2}{Q^2 + P^2} \exp\left\{ \frac{P^2(Q^2 - Q_0^2)}{(Q^2 + P^2)(Q_0^2 + P^2)} \right\} . \tag{12}$$

For $Q^2 \gg P^2$ this simplifies to $P_{\rm eff}^2 = (Q_0^2 + P^2)\exp(P^2/(Q_0^2+P^2))$. A simple recipe is then to use $P_{\rm eff}$ as a lower cut-off for the anomalous components (and also as the expression for $\tilde{Q}_0$ in $f_a^{\gamma,V}$ in (8)).

While $P_{\rm eff}$ gives the same normalization of parton distributions as does eq. (8), it does not give the same average evolution range and therefore not the same x shape. That is, eq. (8) receives contributions from components $f^{\gamma,\mathrm{q\overline{q}}}(x, Q^2, k^2)$ with k down to Q_0, and thus corresponds to a larger average evolution range s than a procedure with a sharp cut-off $k > P_{\rm eff}$. In order to reproduce the x shape better, an intermediate $P_{\rm int}$ in the range $Q_0 < P_{\rm int} < P_{\rm eff}$ is to be preferred. We found no simple formula that defines an optimal $P_{\rm int}$, so will use $P_{\rm int}^2 = Q_0 P_{\rm eff}$ as a pragmatic choice. The momentum sum can be preserved by a simple prefactor, i.e. in total the anomalous component is changed according to

$$\int_{Q_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \left(\frac{k^2}{k^2 + P^2} \right)^2 \left[\cdots \right] \longmapsto \frac{\ln(Q^2/P_{\rm eff}^2)}{\ln(Q^2/P_{\rm int}^2)} \int_{P_{\rm int}^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \left[\cdots \right] . \tag{13}$$

One should note that the above approaches correspond to different evolution equations. The pdf's of the virtual photon given by our ansatz (8) obey evolution equations different from those of the real photon, eq. (1): the homogeneous term is the same but the inhomogeneous one is multiplied by a factor $(Q^2/(Q^2+P^2))^2$. That is, the branchings $\gamma^\star \to \mathrm{q\overline{q}}$ are suppressed relative to those of a real photon, in accordance with the relation (7). Approximately this also holds for the $P_{\rm eff}$ and $P_{\rm int}$ prescriptions. The introduction of P_0 or P_0' according to eq. (9) removes the $(Q^2/(Q^2+P^2))^2$ factor, i.e. restores evolution to be fully according to eq. (1). Hence differences between the pdf's of the real photon and those of the virtual photon in the P_0 and P_0' schemes (and also the difference between the latter two) arise solely from different input distributions. Evolution equations for parton distributions are rigorously defined only in the range (6), where $(Q^2/(Q^2+P^2))^2 \approx 1$. Hence, differences of the kind $Q^2/(Q^2+P^2)$ are formally legitimate, but since the evolution normally is started from a Q_0 of the same order as P, they are non-negligible numerically.

So far we have not imposed any constraint on the $P^2 \to Q^2$ behaviours of the pdf's of the virtual photon. For $P^2 \approx Q^2$ power-like terms $\propto (P^2/Q^2)^p$ are more important than the logarithmic ones ($\propto \ln Q^2/P^2$). Then calculations based on fixed-order perturbation theory, where the full P^2 dependence is kept to the order considered, are more appropriate than ones invoking pdf's that sum leading logarithms. Indeed, for $P^2 \to Q^2$ resolved-photon contributions originating from the quark (gluon) content of the virtual photon become part of the $O(\alpha_s)$ ($O(\alpha_s^2)$) corrections[2] to the leading (in α_s) direct-photon contributions. A sensible scheme is therefore arrived at by demanding the pdf's of the virtual photon to approach the respective parton-model expressions, which vanish like $\ln Q^2/P^2$ for the quark distributions and faster for the gluon distribution. Such a behaviour is already respected by the P_0 and P_0' schemes. To ensure the same limiting behaviour also for the other two schemes we modify the $P_{\rm eff}$ and $P_{\rm int}$ schemes as follows (recall $P_0^2 = \max(Q_0^2, P^2)$):

$$P_{\rm eff}^2 \longmapsto \left(1 - P^2/Q^2\right) P_{\rm eff}^2 + \left(P^2/Q^2\right) P_0^2 \ ;$$

[2] That is, the gluon distribution vanishes faster than the quark distributions for $P^2 \to Q^2$ [6].

$$P_{\mathrm{int}}^2 \longmapsto \left(1 - P^2/Q^2\right) P_{\mathrm{int}}^2 + \left(P^2/Q^2\right) P_{\mathrm{eff}}^2 \quad \text{in } \ln(Q^2/P_{\mathrm{int}}^2) \text{ in eq. (13) ;}$$
$$P_{\mathrm{int}}^2 \longmapsto \left(1 - P^2/Q^2\right) P_{\mathrm{int}}^2 + \left(P^2/Q^2\right) P_0^2 \quad \text{in the } k^2 \text{ integration in eq. (13) .} \qquad (14)$$

Eventually one will aime at a complete description of photon-induced reactions where resolved-photon contributions (i.e. those involving the pdf's of the virtual photon) are matched with direct-photon ones by subtracting from the latter those terms that are already included through the pdf's of the photon.

4 Comparisons

The different approaches studied above may be seen as alternatives, indicating a spread of uncertainty caused by our limited understanding. The P_{eff} and P_{int} ones are attempts to obtain simple numerical approximations to eq. (8), while the P_0 and P_0' ones are more loosely related, cf. the comment above on evolution equations.

Further uncertainties come from the assumed pdf's of the real photon and from the conventional scale ambiguity problems. (Is Q set by the $p_\perp$ of a jet or by some multiple thereof?) While very significant in their own right, they are not studied here.

The above prescriptions can be applied to any set of parton distributions, provided that the VMD and anomalous parts have been parametrized separately. For illustrative purposes, we use SaS 1D throughout in the following. The results are similar for set 2D although differences are reduced in magnitude due to the larger Q_0 value.

Fig. 1 compares the u quark distribution at $P^2 = 0$, 0.25 and 1 GeV^2, for $Q^2 = 10$ GeV^2. All approaches have in common that the small-x part of the spectrum is reduced more than the high-x one, proportionally speaking, reflecting the larger suppression of components with longer evolution range s. The P_0 and P_0' approaches are considerably above the integral of eq. (8), reflecting the difference in momentum sum. Among the alternatives with the same momentum sum, the shorter average evolution range for P_{eff} is reflected in more quarks at large x (and less gluons), while P_{int} gives a good approximation to eq. (8). (In principle the integral could be performed with either scale choice for $\tilde{Q}_0$ in the VMD evolution range, but differences are minor and so we choose to use $\tilde{Q}_0 = P_0'$ as a simple alternative.)

ZEUS studies the QCD jet rate for $p_\perp > 4$ GeV, i.e. for $\langle Q^2 \rangle \approx 20$ GeV^2, in the range $0.1 \lesssim x < 0.75$ [3]. Events with $x > 0.75$ are assumed to be dominated by the direct process, which is less sensitive to the P^2 variation and is therefore used to fix the normalization. The ZEUS results cannot be directly compared with pdf parametrizations, since effects of event migration and experimental cuts are non-trivial. However, to give some first impression, we study the quantity

$$\mathcal{I}(P^2; Q^2 = 20\ \mathrm{GeV}^2) = \int_{0.1}^{0.75} \mathrm{d}x \left\{ \sum_{\mathrm{q}} \left[xq(x, Q^2; P^2) + x\overline{q}(x, Q^2; P^2) \right] + \frac{9}{4} xg(x, Q^2; P^2) \right\} , \qquad (15)$$

which is related to the QCD jet rate. The colour factor 9/4 is the standard enhancement of gluon interactions relative to quark ones. The variation of $\mathcal{I}(P^2)/\mathcal{I}(0)$ is shown in Fig. 2. (It can be discussed whether one should have omitted the x factor in eq. (15); one would then have obtained a slightly steeper fall-off.) The integral in eq. (8) and its P_{eff} and P_{int} approximations give a drop by a factor of about 2 between $P^2 = 0$ and $P^2 = 0.5$ GeV^2, in rough agreement with the ZEUS data, while the P_0 and P_0' approaches do not drop at all as much. (Note the discontinuous derivative in the P_0 curve at $P = Q_0$, caused by the way P_0 is defined. There are also smaller kinks visible in the P_0 and P_0' curves for $P \approx m_c = 1.3$ GeV, related to the way the charm threshold is modelled.) With more precise data and acceptance corrections understood, it should therefore be possible to discriminate among the alternatives.

5 Summary

In this letter we have studied the extension of the parton distributions of the real photon to those of the virtual one. Analyticity in P^2 allows us to represent the pdf's of the virtual photon as a dispersion integral in the mass of the $\mathrm{q\overline{q}}$ fluctuations. We have obtained an explicit solution for the pdf's. Under the assumption of a separation of the $\mathrm{q\overline{q}}$ fluctuations in a low-mass, discrete sum of vector-meson states and a high-mass, continuous spectrum, various constraints on the pdf's provide us with a unique result for $P^2 \ll Q^2$. The final expressions for pdf's of the virtual photon $f_a^{\gamma^\star}(x, Q^2, P^2)$ cannot be given a closed form due to their non-trivial dependence on the three variables x, Q^2, and P^2. Therefore we constructed a parametrization (the P_{int} prescription) which allows the $f_a^{\gamma^\star}(x, Q^2, P^2)$ to be very well approximated by simple modifications to the parton distributions of the real photon, i.e. parametrizations are only needed for the x and Q^2 dependence. This extension of real-photon pdf's to those of a virtual photon can be applied to any set of parton distributions, provided that the VMD and anomalous parts are available separately. It also gives $F_2^{\gamma^\star}(x, Q^2, P^2)$.

In order to allow for a test of the model-dependence of the pdf's of the virtual photon we have constructed three prescriptions alternative to P_{int}, all provided in closed form. The various prescriptions correspond to variants of order P^2/Q^2 in the evolution equations and/or boundary conditions. The differences are readily visible in the P^2 dependence of distributions, so HERA and LEP 2 should offer the opportunity to distinguish between alternatives.

A program with the SaS parametrizations modified according to the prescriptions studied in this paper is available on WWW under
http://thep.lu.se/tf2/staff/torbjorn/lsasgam2.

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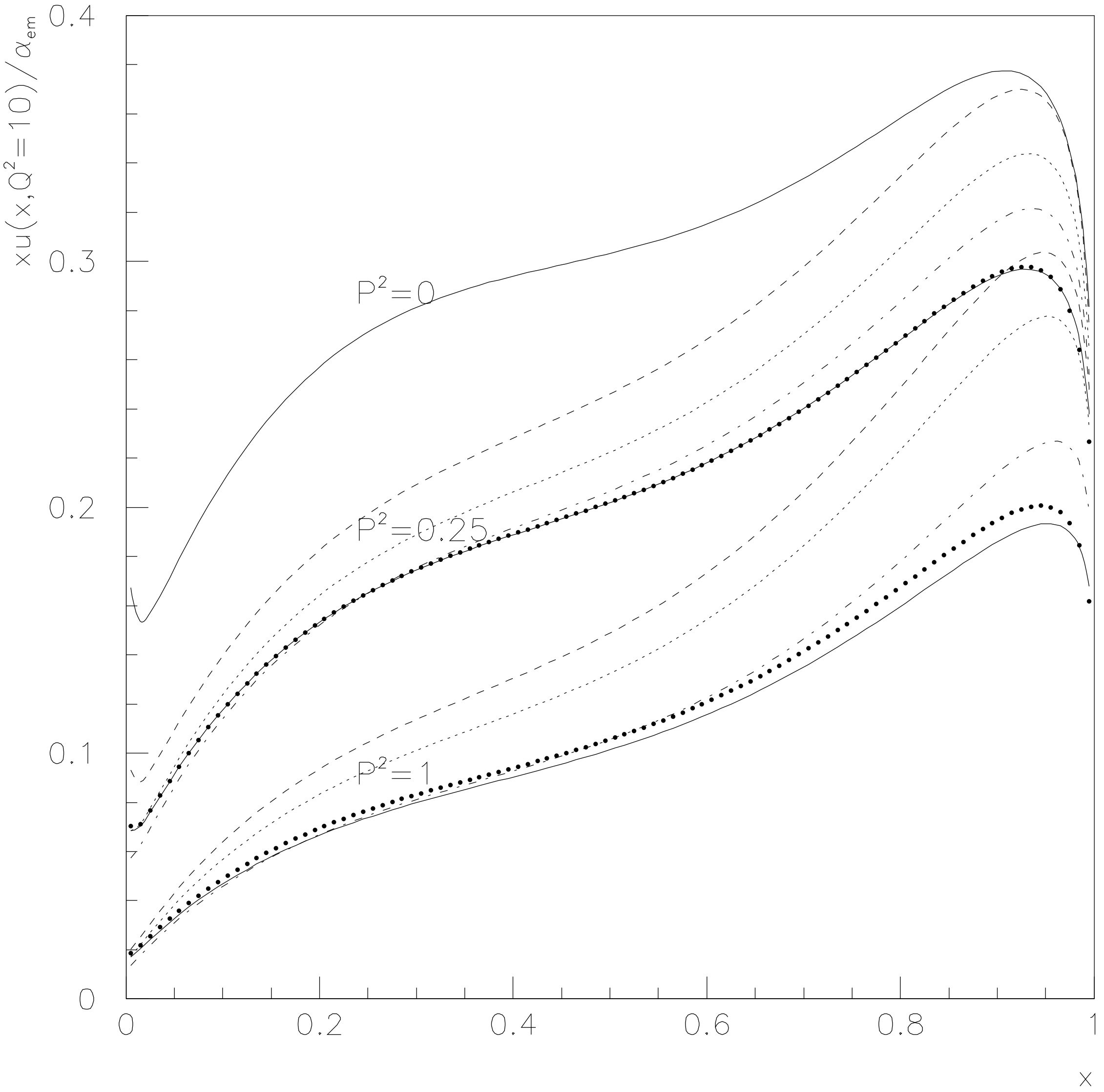

Figure 1: The u-quark parton distribution $xu(x, Q^2 = 10\,\mathrm{GeV}^2)/\alpha_{\mathrm{em}}$. Top full line for $P^2 = 0$, below comparison of five alternatives for $P^2 = 0.25$ and 1 GeV^2. Ordered roughly from top to bottom, dashed is P_0, dotted is P_0', dash-dotted is P_{eff}, large dots is P_{int} and full is the integral in eq. (8).

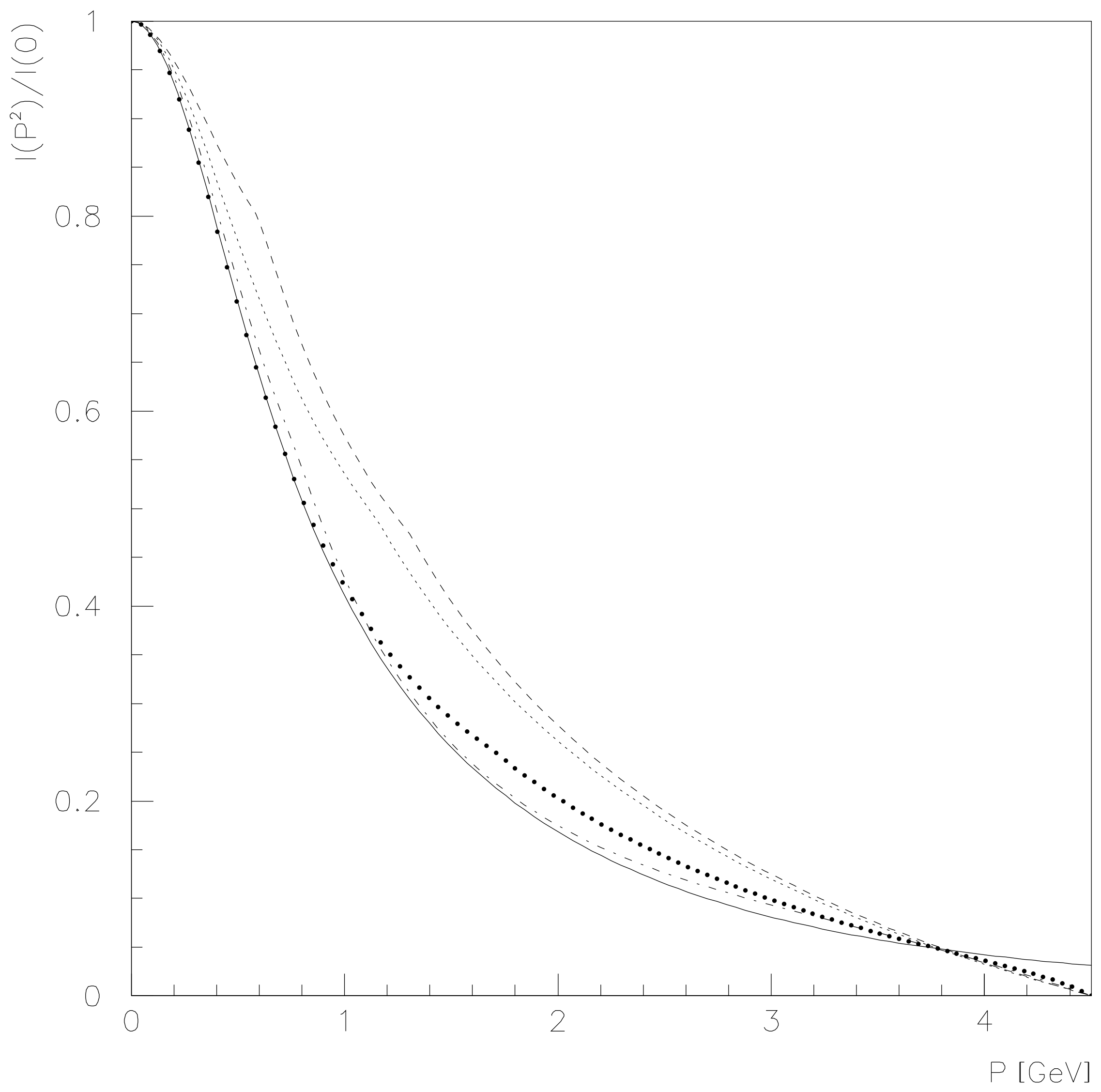

Figure 2: The fall-off of parton distributions with virtuality P (we have chosen P as x scale rather than P^2, so as to better show the small-P region), normalized to the value at $P^2 = 0$, $\mathcal{I}(P^2)/\mathcal{I}(0)$, for $Q^2 = 20.25$ GeV^2. Here $\mathcal{I}$, defined by eq. (15), is the colour-factor-weighted sum of parton distributions in the range $0.1 < x < 0.75$. Curves are labelled as in Fig. 1.