LU TP 96–5
MC–TH–96/04
January 1996

Parton distributions of real and virtual photons [1]

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Abstract

Recent progress on the parton distribution functions of the photon, both real and virtual, is briefly reviewed and experimental possibilities at HERA are discussed.

1. Introduction

Before the advent of HERA, the almost only experimental information on the parton structure of the photon was obtained from studies of the structure function of the photon, $F_2^\gamma(x, Q^2)$, in two photon collisions at e^+e^- colliders. Theoretically, this is a very interesting area as, at large x and asymptotically large Q^2, the parton distribution functions (pdfs) of the photon, and hence F_2^γ, are predicted from perturbative QCD (pQCD) [1, 2]. However, in the range of Q^2 experimentally accessible at present and in the foreseeable future, some non-perturbative input is required. Here different groups make different assumptions, all include parameters, and nearly all existing pdfs are constrained by fits to F_2^γ data. There are many competing sets; in sect. 2 we review the various possibilities, discuss the basic underlying physics choices, and also discuss the difficulties in comparing photon pdfs in leading order (LO) and next-to-leading order (NLO) pQCD.

These remarks only apply to real photons. For virtual photons, there is very little experimental information from two photon collisions, because of the difficulties of doing double-tag measurements. However, there have been some theoretical attempts, which we discuss in sect. 3.

Hard photoproduction at HERA offers further possibilities of exploring the structure of the photon, including the gluon content. In hard photoproduction processes, there is a

[1]To appear in the proceedings of the Workshop on HERA Physics, Durham, England, Sept. 1995.
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contribution from the **resolved** processes, where the photon is resolved into its partons which then take part in the hard partonic subprocess [3, 4]: this contribution is sensitive to the pdfs of the photon as well as the proton. There is also a (calculable) background to this from the **direct** processes, in which the photon takes part directly in the hard subprocess: this depends on the proton pdfs but not on those of the photon.

With untagged electrons at HERA the photons are mainly real and have a known spectrum of energies given by the equivalent photon approximation; hence the pdfs of the real photon are measured. With tagged electrons the photons have known energy and virtuality and so it will become possible to study the parton content of virtual photons for the first time. This is why the structure of real and virtual photons is an important physics issue for HERA.

2. The parton distributions of the real photon

In this section we briefly review the different sets of pdfs for the real photon currently available: for more detail on this subject the reader is referred to ref. [5]. For a recent general review see ref. [6]. The special role of the photon in QCD is due to the fact that, at asymptotically large Q^2, the quark and gluon distribution functions are calculable at large x, i.e.

$$q_i^\gamma(x,Q^2)/\alpha \;\simeq\; \frac{a_i(x)}{\alpha_s(Q^2)} \;+\; b_i(x) \quad , \tag{1}$$

with a similar expression for $g^\gamma(x,Q^2)$. The first term is the LO result of Witten [1] and the second term its NLO correction in pQCD [2]. The functions $a_i(x)$ and $b_i(x)$ are **calculable**, but singular at $x=0$.

This point-like part contribution is dominant at large Q^2, where the incalculable hadronic piece is small. However to avoid the unphysical small-x singularities in eq. (1) one must retain the hadronic part by including a boundary condition at a reference scale $Q^2 = Q_0^2$ [7]. If we do that in n-moment space we have (confining ourselves to LO for simplicity)

$$q_i^\gamma(n,Q^2) = \frac{\alpha\, a_i(n)}{\alpha_s(Q^2)}\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{1+d(n)}\right] + q_i^\gamma(n,Q_0^2)\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{d(n)} \tag{2}$$

where the second term in the square brackets regularizes the singularity at $x=0$. The last term is hadronic in the sense that for the case of the pdfs of a hadron, it would be the **only** contribution.

Eq. (2), although a good approximation at large x, is strictly true only for non-singlet combinations of quark densities. The singlet quark $\Sigma^\gamma = \sum_i(q_i+\overline{q}_i)$ and gluon distributions obey similar equations with the important difference that in these sectors the Altarelli-Parisi (AP) equations are **coupled**, i.e. to determine $\Sigma^\gamma(x,Q^2)$ (or $g^\gamma(x,Q^2)$) we need both $\Sigma^\gamma(x,Q_0^2)$ **and** $g^\gamma(x,Q_0^2)$. The bottom line is that, as for the case of hadrons, we need input distributions at a reference scale $Q^2=Q_0^2$.

We note here that the anomalous $[\alpha_s(Q^2)]^{-1}$ behaviour of the pdfs of the photon arises because of the direct $\gamma \to q\bar{q}$ coupling which gives inhomogeneous terms in the AP evolution equations [8]. This behaviour of the quark distributions is confirmed by data on $F_2^\gamma(x, Q^2)$, over a wide range of Q^2, see e.g. ref. [9].

In order to obtain photon pdfs at all Q^2, one has to choose a reference scale Q_0^2 and fix the input pdfs there, using some ansatz (usually vector meson dominance, VMD) and employing F_2^γ data to fix some free parameters. The (anti)quark distributions $q_i^\gamma(x, Q_0^2)$ are reasonably well determined by the data, since in LO

$$F_2^\gamma(x, Q^2) = \sum_i e_i^2 \, x q_i^\gamma(x, Q^2) \quad . \tag{3}$$

On the other hand, fixing the gluon distribution is a problem because of the lack of a momentum sum rule [5, 10]: one is completely dependent on the ansatz. Note however that a substitute for the usual hadronic parton momentum sum rule has been proposed recently [11]. For the evolved pdfs the fact that the coupling of the AP equations is weak works two ways: (a) the output $F_2^\gamma(x, Q^2)$ is insensitive to the input gluon (except at small x) but (b), consequently, a comparison with present F_2^γ data does not provide any restriction on $g^\gamma(x, Q_0^2)$.

VMD provides a connection between the photon and ρ meson pdfs, and since the latter satisfy a momentum sum rule, we have a constraint on the VMD part of the gluon pdf of the photon. This is particularly useful if we use SU(6) to relate the pion and ρ pdfs as there are experimental constraints on the pion pdfs from Drell-Yan lepton pair and direct-photon production data [12]. However, another problem arises here. For the traditional input scales, $Q_0^2 \geq 1$ GeV^2, a pure VMD input is known to be insufficient to fit the data at higher Q^2 [7, 13]. Two approaches have been adopted to circumvent this: the first is to maintain the VMD idea and start the AP evolution at a very low scale $Q_0 < 1$ GeV [14, 15, 16, 17]. The second is to keep $Q_0 \geq 1$ GeV and fit the quark densities to F_2^γ data, e.g. supplementing the VMD values with a point-like component, seemingly naturally provided by the Born-Box diagram. Unfortunately there is no corresponding natural choice for the gluon density and a guess must be made here. This method was adopted in refs. [17, 18, 19, 20, 21]. The result of all this is that the different distributions agree reasonably well as regards the quark distributions in the region $0.05 \lesssim x \lesssim 0.8$, which must reproduce F_2^γ data, but not as regards the gluon densities. This can be seen in figs. 1 and 2, where we have plotted a representative set of quark and gluon distributions in LO and NLO.

The reader will note that there seems to be very little resemblance between the quark distributions in LO and NLO. This is because of a subtlety peculiar to the photon in the $\overline{\mathrm{MS}}$ scheme, and has been discussed in detail in ref. [5]. It arises in the lowest order QCD process, the Born-Box diagram, where the term leading in $\ln Q^2$ gives the inhomogeneous term in the AP equations and the non-leading term C_γ is negative and divergent as $x \to 1$. In the usual $\overline{\mathrm{MS}}$ scheme this C_γ is not absorbed into the quark densities. However, in NLO, it reappears as a Wilson coefficient for a subleading 'direct' contribution to F_2^γ.

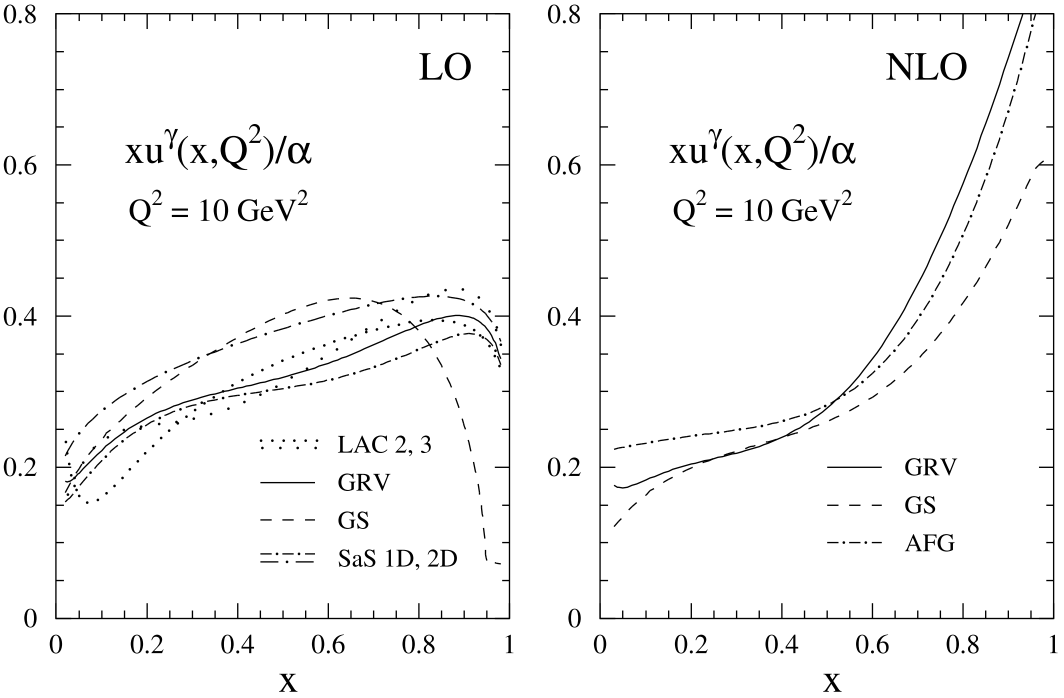

Figure 1: Photonic u-quark parametrizations at LO [14, 17, 19, 20] and NLO [14, 16, 20]. The NLO results are presented in the $\overline{\mathrm{MS}}$ scheme. For a discussion of the different assumptions and F_2^γ data sets employed see [5].

Thus if we are to require approximately the same F_2^γ in LO and NLO, then the NLO pdfs must be substantially modified accordingly and this is what we are seeing in fig. 1. An alternative approach is to work in the DIS$_\gamma$ scheme [22], where C_γ is absorbed into the definition of the quark density and does not appear in the NLO expression for F_2^γ. Hence in this scheme perturbatively stable, physically motivated inputs for the photon pdfs, such as VMD, can be used in NLO as well as in LO.

We conclude this section with a few comments on what has been learnt from experimental data since most of these pdfs were proposed. We start with two-photon data. There have been new F_2^γ data from TOPAZ [23] and AMY [24] at TRISTAN and from OPAL [25] and DELPHI [26] at LEP, which are shown in fig. 3. As can be seen the data are of limited statistics. These results seem to indicate some offset at x around 0.2 with respect to the average of earlier data from lower energy machines, as can be seen by comparing to the LAC [19] and GRV [14] parametrizations which were fitted to all F_2^γ data available in 1991. Moreover, at small x the recent TOPAZ [23] results are at variance with the LEP data [25, 26]. If anything, the recent measurements confuse the situation slightly as regards the quark distributions. In addition, there have been measurements of the one- and two-jet inclusive jet cross sections at TRISTAN [27, 28] which show some sensitivity to the gluon distribution. One can conclude from these jet data that there is now evidence from $\gamma\gamma$ collisions that the gluon density is non-zero [10, 27, 28]. They also rule out the LAC3 distribution with its large gluon component at large x, which considerably overestimates the cross section.

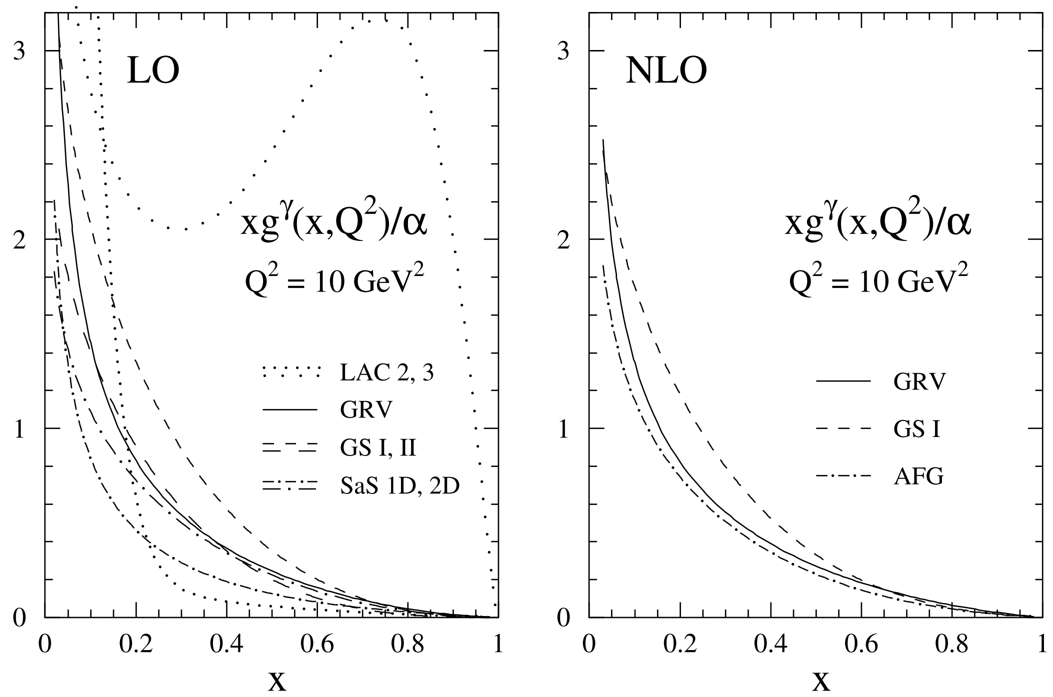

Figure 2: Parametrizations of the photon's gluon distribution at LO [14, 17, 19, 20] and NLO [14, 16, 20]. Note that the similarity of the NLO results is due to common VMD prejudices and not enforced by data.

Turning to jet photoproduction at HERA, already the first measurements [29] enabled LAC3 to be ruled out: the contribution from the quarks virtually saturates the observed cross section, leaving no scope for a large gluon density except at small x [10]. Since then more accurate data have appeared. H1 has extracted a LO gluon distribution from the data [30]: it disfavours the more extreme scenarios for gluons at small-x such as LAC1 and LAC2. These latter data have been compared with NLO calculations, as discussed at this workshop [3]. It appears that in the negative rapidity region, where originally the direct component was expected to dominate, there is some sensitivity to the large-x photon pdfs. A large x quark structure more in accord with the GS distributions than those of GRV or AFG seems to be favoured, although some theoretical questions have to be answered before definite conclusions can be drawn [3]. In the positive rapidity region, a comparison of the NLO calculations with the jet cross sections is complicated by the possibility of multiple hard parton interactions, discussed in a separate contribution to these proceedings [31].

3. The parton distributions of the virtual photon

As with the pdfs of the real photon, the pdfs of a virtual photon have to be based on some ansatz. There is therefore no unique answer. The non-perturbative hadronic (VMD) contribution to the photon structure is expected to go away with increasing P^2, allowing for a purely perturbative prediction for $F_2^\gamma(x,Q^2,P^2)$ at sufficiently high P^2 [32]. Here we use P^2 to denote the virtuality of the photon; Q^2 is reserved for the scale of the

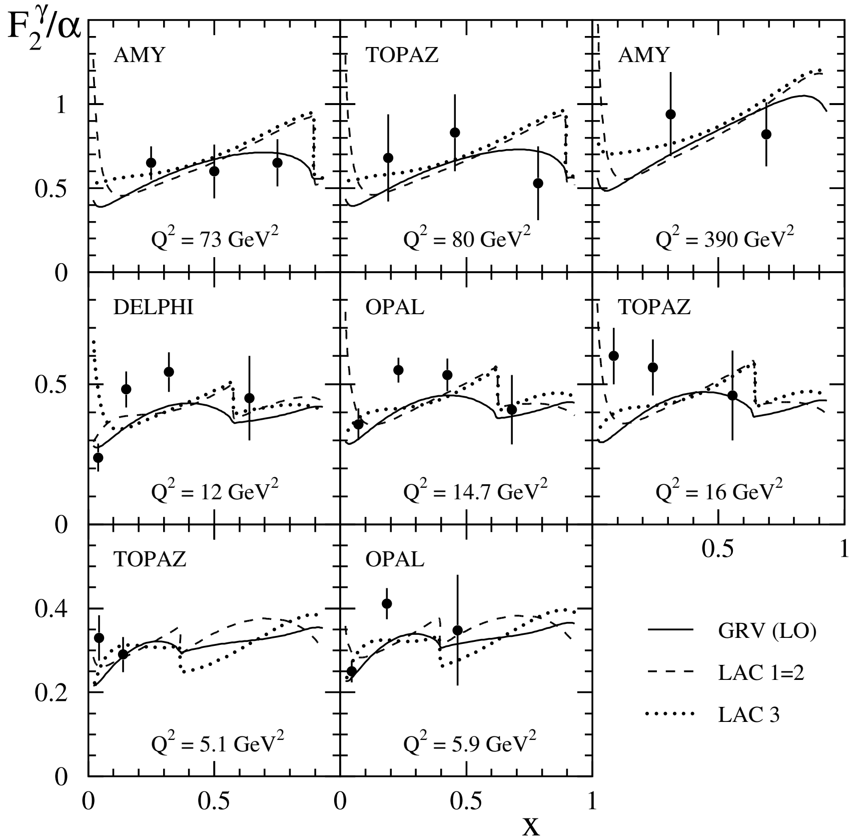

Figure 3: Recent data on $F_2^\gamma(x,Q^2)$ from TRISTAN [23, 24] and LEP [25, 26] compared to the LO fits to all previous data of LAC [19] and GRV [14].

hard $\gamma^{(*)}$ interaction. The fall-off of the non-perturbative part with increasing P^2 is theoretically uncertain; hence experimental clarification is required to pin down models. We will summarize here a few recent studies that together illustrate the spread in current approaches.

Drees and Godbole [33] seek a simple interpolating multiplicative factor, such that parton distributions reduce to the real pdfs for $P^2 \to 0$ and die like $\ln(Q^2/P^2)$ for $P^2 \to Q^2$: at $P^2 = Q^2$ it is natural to attribute the whole cross section to direct processes in order to avoid double counting. Several different forms are studied; one of the main alternatives is to use a scaling factor:

$$r = 1 - \frac{\ln(1 + P^2/P_c^2)}{\ln(1 + Q^2/P_c^2)} , \tag{4}$$

where P_c is some typical hadronic scale such as $P_c^2 \approx 0.5$ GeV^2. The factor r is applied to all quark pdfs. The gluon is expected to be further suppressed, however, since the gluon pdf is generated by the quark ones [34]. For instance, if the scale k^2 of $\gamma \to \mathrm{q}\overline{\mathrm{q}}$ branchings is distributed in the range $P^2 \lesssim k^2 \lesssim Q^2$, the scale k'^2 of the $\mathrm{q} \to \mathrm{qg}$ branching is in the reduced range $k^2 \lesssim k'^2 \lesssim Q^2$. A gluon suppression factor r^2 gives the expected limiting behaviour. The above ansatz does not change the x shape of distributions; for that more complicated forms are proposed. Anyway, the thrust of the study is to estimate how much the photon pdfs in the untagged case, i.e. P^2-averaged pdfs, differ from those of the real photon. The forms studied give a suppression of the order of 10% and 15% for the quark and gluon distributions, respectively.

The study of Glück, Reya and Stratmann [35] is based on the observation that the pdfs $f_i^\gamma(x, Q^2, P^2)$ obey evolution equations in Q^2 similar to those of a real photon. The question is therefore reduced to one of finding suitable boundary conditions at $Q^2 = P^2$. The ansatz used is

$$f_i^\gamma(x, Q^2 = \tilde{P}^2, P^2) = \eta(P^2) f_i^{\gamma,\mathrm{nonpert}}(x, \tilde{P}^2) + \left[1 - \eta(P^2)\right] f_i^{\gamma,\mathrm{pert}}(x, \tilde{P}^2) \; . \tag{5}$$

Here $\tilde{P}^2 = \max(P^2, \mu^2)$, with $\mu \approx 0.5$ GeV the input scale for the evolution of the real photon [14]; and $\eta(P^2) = (1 + P^2/m_\rho^2)^{-2}$ is the standard dipole dampening factor of the ρ meson. The non-perturbative input distribution is taken to be proportional to the pfd's of the pion, $f_i^{\gamma,\mathrm{nonpert}}(x, \tilde{P}^2) = \kappa(4\pi\alpha/f_\rho^2) f_i^\pi(x, \tilde{P}^2)$ [14]. The $f_i^{\gamma,\mathrm{pert}}(x, \tilde{P}^2)$ is perturbatively calculable; in leading order it vanishes. Based on the above ansatz, the evolution equations give the answer for all $Q^2 > P^2$. Closed results can be obtained in moment space, and then a simple numerical Mellin inversion gives actual numbers. A practical limitation is that there exists up to now no simple parametrization, unlike the case of a real photon.

Schuler and Sjöstrand [17] start from an ansatz for the pdfs of a real photon decomposed into VMD and anomalous components:

$$f_i^\gamma(x, Q^2) = \sum_V \frac{4\pi\alpha}{f_V^2} f_i^{\gamma,V}(x, Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \frac{\alpha}{2\pi} \sum_{\mathrm{q}} 2e_{\mathrm{q}} f_i^{\gamma,\mathrm{q\overline{q}}}(x, Q^2, k^2) \; . \tag{6}$$

Here the sum runs over the lowest-lying vector mesons, ρ^0, ω, ϕ and J/ψ, while the integral covers the range of perturbative branchings $\gamma \to \mathrm{q\overline{q}}$ at scales $Q_0 < k < Q$, with $Q_0 \approx 0.6$ GeV (for SaS 1, alternatively 2 GeV for SaS 2) setting the separation between the two components and also the starting value of the evolution. The VMD and anomalous "state" distributions $f_i^{\gamma,V}$ and $f_i^{\gamma,\mathrm{q\overline{q}}}$, respectively, are normalized to unit momentum sum. The VMD distributions and the integral of anomalous distributions are parametrized separately and added to give the full result. In going to a virtual photon, the main change is to introduce a dipole dampening factor for each component, i.e. $(1 + P^2/m_V^2)^{-2}$ for the VMD states and $(1 + P^2/k^2)^{-2}$ for the anomalous ones. Additionally the lower input scale for the VMD states is shifted from Q_0^2 to $P_0^2 \approx \max(P^2, Q_0^2)$ [34].

In order to obtain a tractable answer, one possible approximation for the anomalous component is

$$\int_{Q_0^2}^{Q^2} \frac{1}{(1 + P^2/k^2)^2} \frac{\mathrm{d}k^2}{k^2} \left[\cdots\right] \approx \int_{P_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \left[\cdots\right] , \tag{7}$$

with P_0 as above. Although the VMD and anomalous components still depend on two scales, P_0^2 and Q^2, all the nontrivial dependence comes from the logarithmic integration of the strong coupling constant between the two scales, so the standard pdfs of the real photon can be extended easily to virtual photons, i.e. parametrizations of $f_i^\gamma(x, Q^2, P^2)$ are readily available. The resulting u-quark and gluon densities are dispayed for two photon virtualities P^2 in fig. 4 together with the corresponding LO distributions of ref. [17]. Recently, alternatives to eq. (7) have been studied [36], where the momentum sum and average evolution range of the dipole-dampened version of eq. (6) is preserved. The difference between these procedures can also be viewed as one estimate of the uncertainty.

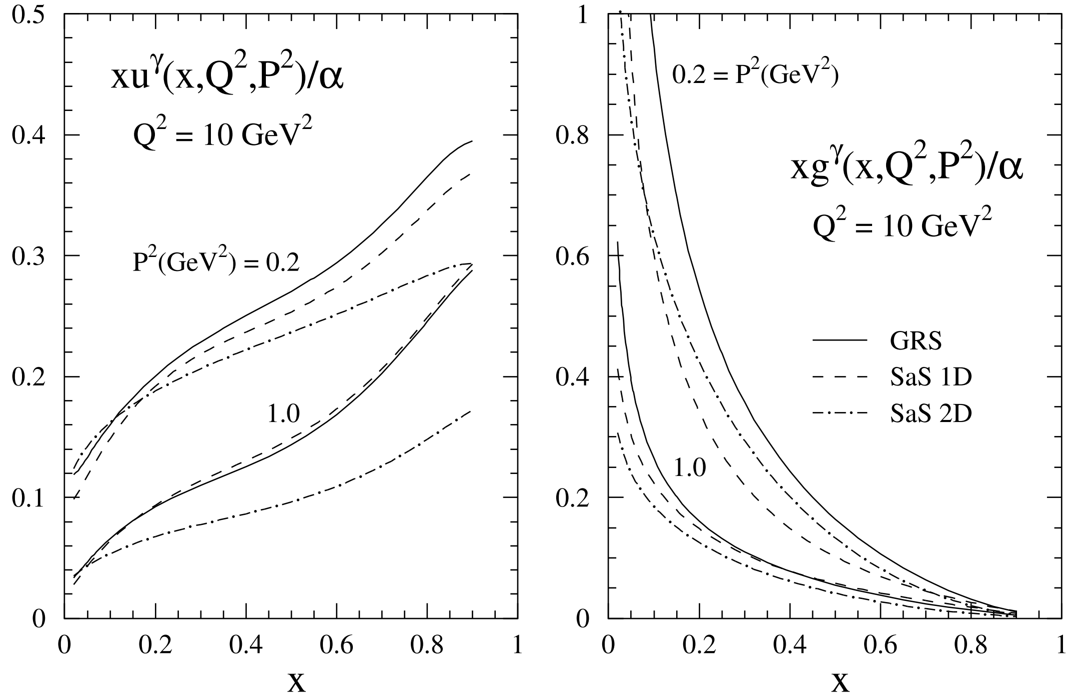

Figure 4: The u-quark and the gluon distributions of the virtual photon in LO as suggested in refs. [17, 35] at two selected values of the photon virtuality P^2.

We now turn to the experimental possibilities at HERA. With their forward electron tagging capabilities, H1 and ZEUS can tag almost real photons, $P^2 \lesssim 0.01$ GeV^2, and virtual photons down to $P^2 \gtrsim 0.1$ GeV^2. This is amply demonstrated by the ZEUS results presented at this meeting [37], shown in fig.7 of ref. [38], where the observed x_γ distribution has been constructed for events with two jets above 4 GeV, i.e. roughly speaking for events with $Q^2 \gtrsim 16$ GeV^2. As P^2 is increased, this distribution is gradually suppressed at small x_γ, where the resolved contribution should dominate. For instance, if we cut between resolved and direct events at $x_\gamma = 0.75$, then the ratio of resolved events (with $x_\gamma < 0.75$) to direct events (with $x_\gamma > 0.75$) drops by about a factor of 2 between $P^2 \approx 0$ and $P^2 \approx 0.5$ GeV^2 [38], in rough agreement with the theoretical arguments of this section.

4. Conclusions

In this note we have briefly reviewed the current phenomenological status of the pdfs of real and virtual photons. As we mentioned earlier, in the case of the real photon the pdfs have been constrained to fit the F_2^γ data. However, given the limitations catalogued in sect. 2, it is difficult to regard a comparison of HERA jet photoproduction data with NLO QCD calculations based on the existing pdfs as a definitive test of anything. We feel that the jet data should be regarded as giving an independent determination of the pdfs of the photon, which at the moment is and in the near future will remain superior to those from two-photon physics. This will be true, certainly for larger x, until high statistics data become available from LEP2. We are already seeing the first signs of this [3] in the comparison of the jet data with NLO calculations. It should also be borne in mind, that jet studies (in both $\gamma\gamma$ and γp) are among the few areas sensitive to the gluon content of the photon. Thus we feel that photoproduction of jets at HERA has much to offer the field of photon structure studies.

As regards virtual photons, HERA offers a unique opportunity to study pdfs. The electron tagging capabilities of H1 and ZEUS offers virtual photons with large event rates. As this area has been regarded to be of theoretical interest for nearly 20 years, but with essentially no experimental input, this is an opportunity not to be missed. In the future, LEP2 may provide complementary information on virtual photons via double-tag events, but at a lower energy and presumably with lower event rates [39].

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