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Common origin of the shoulder in multiplicity distributions and of oscillations in the factorial cumulants to factorial moments ratio *

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Abstract

The shoulder structure of charged particles multiplicity distributions (MD's) in full phase space in e^+e^- annihilation at the Z_0 peak and the quasi-oscillatory behavior of the ratio of factorial cumulants over factorial moments, H_q, as a function of the order q, are quantitatively reproduced within a simple parametrization of the MD in terms of a weighted superposition of two Negative Binomial Distributions, associated to two- and multi-jet production, i.e., to hard gluons radiation.

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1. Introduction

A definite understanding of the multiparticle production process in full phase space in e^+e^- annihilation at the Z_0 peak is still lacking. Experimental data show two interesting features: a clear shoulder is visible in the intermediate multiplicity range in charged particles multiplicity distributions (MD's)[1, 2, 3]; the ratio of factorial cumulants over factorial moments of the MD, H_q, when plotted as a function of its order q, decreases sharply to a negative minimum at q=5 and follows then a quasi-oscillatory behavior[4, 5]. In order to interpret these results, two complementary approaches can be followed. First, properties of multiparton final states can be computed in the framework of perturbative QCD by exploiting its branching structure[6, 7]; partonic and hadronic distributions are then directly compared by assuming the Local Parton Hadron Duality[8] or its generalized version[9] as hadronization prescription. Second, a phenomenological approach can be taken and data compared with simple parametrizations, like for instance the Negative Binomial (NBD)[10] or the log-normal (LND)[11] distributions.

Within the first approach, the partons' MD has been computed in Double Log Approximation (DLA)[7], getting a much wider MD than the experimental one. By adding an approximate treatment of the energy-momentum conservation law[12], the predicted MD becomes narrower at non asymptotic energies, but it can quantitatively reproduce neither the general shape of the experimental MD nor its shoulder. Concerning the ratio H_q, QCD predictions beyond DLA have been considered[13]. They qualitatively show the same behavior of experimental data; the result supports the general picture of LPHD, but it is quite far from being quantitatively satisfactory.

Within the second more phenomenological approach, let us consider the two most popular parametrizations of MD's, the NBD and the LND. Both of them are related to QCD. The NBD is the exact solution for the gluon MD in a quark-jet in Leading Log Approximation[14] and more detailed QCD calculations in DLA plus recoil corrections turn out to be close to NB behavior[12, 17]. It reveals the self-similar and Markoffian structure of parton shower evolution; it is usually interpreted in the framework of the clan model[10] as an indication of the two-step nature of the production process. The LND results from the application of the central-limit theorem to self-similar parton showers[11]. Both parametrizations quantitatively describe e^+e^- annihilation data in full phase space at c.m. energies below the Z_0 peak[15, 16] and qualitatively reproduce the overall shape of the MD at the Z_0 peak, but neither the NBD nor the LND reproduces the shoulder structure, although the LND fares generally better in terms of χ^2/NDF. Concerning the ratio H_q of factorial cumulants over factorial moments, parametric models give a poor description of experimental results. A full NBD predicts indeed positive definite and monotonically decreasing H_q's. If one truncates a NBD in the high multiplicity tail to properly take into account the experimental cut due to finite statistics, one recovers a quasi-oscillatory behavior qualitatively consistent with experimental results[18]; the size of the experimental effect is however strongly underestimated[4]. Predictions of a truncated LND are in better agreement with data[4], even though they do not provide a quantitative description of the experimental effect.

In this letter, we propose a simple phenomenological parametrization of the MD in e^+e^- annihilation in full phase space in terms of a weighted superposition of two NBD's; both the shoulder structure of the MD and the quasi-oscillatory behavior of the ratio H_q are quantitatively reproduced. These results suggest to explain both effects in terms of the superposition of samples of events with a fixed number of jets, where the standard NB parametrization is indeed recovered.

2. The shoulder structure

The DELPHI Collaboration has shown in [19] that the shoulder structure in the MD in e^+e^- annihilation can be explained by the superposition of the MD's coming from events with 2, 3 and 4 jets, as identified by a suitable jet-finding algorithm, and that the MD's in these classes of events are well reproduced by a single NBD for every value of the jet-finder parameter, y_{cut}. It is thus suggested that the shoulder is associated with the radiation of hard gluons resulting in the appearance of one of more extra jets in the hadronic final state.

Let us also remind that a shoulder structure similar to the one observed in e^+e^- annihilation has been observed in $p\bar{p}$ collisions at high energies[20] and was shown to be well described by a 5-parameter parametrization in terms of the weighted superposition of two NBD's[21].

Inspired by these experimental results and NB universality for all classes of reactions, we propose now to parametrize experimental data on MD's in full phase space in e^+e^- annihilation at the Z_0 peak in terms of the superposition of 2 NBD's; according to DEL-

PHI's result, we argue that the two MD's should be associated with the contribution of two- and multi(≥ 3)-jets events respectively. We fix therefore the weight parameter to be equal to the fraction of 2-jet events. Formally, we perform a fit on available experimental data obtained at LEP[1, 2, 3][1] and SLC[4] with the following 4-parameter distribution:

$$P_n(\alpha; \bar{n}_1, k_1; \bar{n}_2, k_2) = \alpha P_n^{NB}(\bar{n}_1, k_1) + (1-\alpha) P_n^{NB}(\bar{n}_2, k_2) \tag{1}$$

where $P_n^{NB}(\bar{n}, k)$ is the NBD, expressed in terms of two parameters, the average multiplicity $\bar{n}$ and the parameter k, linked to the dispersion by $D^2/\bar{n}^2 = 1/\bar{n} + 1/k$, as:

$$P_n^{NB}(\bar{n}, k) = \frac{k(k+1)\dots(k+n-1)}{n!} \left(\frac{k}{\bar{n}+k}\right)^k \left(\frac{\bar{n}}{\bar{n}+k}\right)^n \tag{2}$$

The weigth α gives the 2-jet events fraction: since the latter depends on the jet-finder parameter, y_{cut}, different values of α corresponding to different values of y_{cut} have been considered.

As far as MD's in full phase space are concerned, one has also to take care of the "even-odd" effect, i.e., of the fact that the total numer of final charged particles must be even due to charge conservation; accordingly, the actual form used in the fit procedure is given by:

$$P_n = \begin{cases} A P_n(\alpha; \bar{n}_1, k_1; \bar{n}_2, k_2) & \text{if } n \text{ even} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where A is the normalization parameter, so that $\sum_{n=0}^{\infty} P_n = 1$.

The parameters of the fits and the corresponding χ^2/NDF for different values of α are given in Table 1; for comparison, the parameters of the NBD's fitted to 2- and 3-jet data samples by DELPHI Coll.[19] are also shown. As an example, Figure 1 compares the experimental MD's with the parametrization (3) with best-fit parameters given in Table 1 for $\alpha = 0.767$; the two NBD contributions are plotted with a dotted line. The residuals, i.e., the difference between data and theoretical predictions, are also shown below each MD in units of standard deviations. One concludes that the proposed parametrization can reproduce the experimental data very well; no structure is visible in the residuals. The agreement holds for all considered values of α, even though the parameters of the two NBD's obviously depend on the choice of the weight parameter. To further support the interpretation of the two components of eq. (1) as the two- and multi-jet contribution respectively, we notice that the parameters of the two NBD's extracted from the fit at

[1]Data on MD by L3 Coll.[22] have not been analyzed due to their large systematic error.

a given α are close to the fitted values extracted by DELPHI Coll. for 2- and 3-jet MD's at the corresponding value of y_{cut}; in fact for these values of y_{cut} the contribution of ($\geq$ 4)-jet events is negligible (less than 10%). Finally, let us also notice that the fit parameters present large errors; their determination would be strongly improved by direct experimental analyses which could take properly into account the full covariance matrix of the MD's. These analyses are eagerly awaited.

3. The ratio H_q

Let us consider now the ratio of factorial cumulant over factorial moments

$$H_q = K_q/F_q \tag{4}$$

as a function of the order q. The factorial moments, F_q, and factorial cumulant moments, K_q, can be obtained in general from the MD, P_n, through the relations:

$$F_q = \sum_{n=q}^{\infty} n(n-1)\dots(n-q+1)P_n, \tag{5}$$

and

$$K_q = F_q - \sum_{i=1}^{q-1} \binom{q-1}{i} K_{q-i} F_i. \tag{6}$$

A direct experimental analysis of the ratio H_q with correlations and systematics effects fully taken into account has been so far performed by SLD Coll.[4] only. To extend the analysis to LEP experiments, we extracted *a posteriori* the values of the H_q from published MD's. Due to the lacking of the full covariance matrix for MD's, the statistical error on H_q cannot be properly calculated; in order to estimate it, for each experimental MD we calculated the ratio H_q from 1000 MD's obtained from the original one by allowing gaussian fluctuations around the measured value of each P_n the width of the gaussian being given by the experimental statistical error. The standard deviation of the H_q distribution for each order q was then taken as an estimate of the error associated with the ratio H_q itself. As a cross-check, this method has been applied to SLD data, since both MD and the ratio H_q are available in this case; a good agreement between the errors extracted with this procedure and the sum of statistical and systematic errors measured by SLD has been found. It should be stressed of course that this method can give just an approximate value of the size of the errors and direct experimental analyses by LEP Collaborations are awaited. Let us now look at the ratio H_q as predicted by the parametrization proposed in Section 2. Since the H_q's were shown to be sensitive to the truncation of the tail due to the finite statistics of data samples[18], moments were extracted from a truncated MD defined as follows:

$$P_n^{trunc} = \begin{cases} A' P_n(\alpha; \bar{n}_1, k_1; \bar{n}_2, k_2) & \text{if } (n_{min} \le n \le n_{max}) \text{ and } n \text{ even} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Here n_{min} and n_{max} are the minimum and the maximum observed multiplicity and A' is a new normalization parameter[2], so that $\sum_{n=n_{min}}^{n_{max}} P_n^{trunc} = 1$.

Figure 2 shows the ratio H_q as a function of the order q for SLD[4], ALEPH[3], DELPHI[1] and OPAL[2]. It should be noticed that SLD, DELPHI and OPAL give very similar results, whereas ALEPH shows a different behavior, in particular at low orders of q and after, when oscillations start. However, due to the large errors and to the uncertainties in their estimate, a definite answer on the consistency among different set of data requires a direct experimental determination of errors. In Figure 2 predictions of the parametrization (7) with parameters fitted to reproduce the MD as given in Table 1 for different values of α are also shown (solid lines). It turns out that this parametrization reproduces not only the shoulder observed in experimental MD's but also quantitatively describes the experimental behavior of the ratio H_q. Results are essentially independent of α; the small spread among theoretical predictions for different values of α can be taken as an estimate of the theoretical error. The plot referring to SLD shows in addition predictions of eq. (3), i.e., of the same parametrization, but without taking into account the effect of truncation (dashed lines): the quasi-oscillations become smaller. To further support the interpretation of this oscillatory behavior as due to the superposition of two- and multi-jet events, it should be noticed that oscillations in samples of isolated 2- and 3-jet events are much smaller than in the full sample of events[5]. In conclusion, the observed behavior of H_q's results from the convolution of two different effects, a physical one, i.e., the superposition of two components, and a statistical one, i.e., the truncation of the tail due to the finite statistics of data samples.

[2] We found that a small (0.1%) error in the normalization leads to very different values for the H_q's

4. Conclusions

Two experimental features of multiparticle production in full phase space in e^+e^- annihilation at the Z_0 peak, i.e., the shoulder in the MD and the oscillatory behavior of the ratio of factorial cumulants over factorial moments, H_q, as a function of the order q, have been adressed: a simple phenomenological parametrization of the MD in terms of a weighted superposition of two components, each one being distributed according to a NBD, has been proposed. The weight of the first component was taken equal to the fraction of 2-jet events, i.e., the two components were identified with the two- and the multi-jet contribution respectively. The shoulder structure is found to be quantitatively reproduced by this parametrization; the behavior of the ratio H_q is also quantitatively described, after taking properly into account the effect due to the truncation of the tail. A consistent picture seems then to emerge: both experimental effects which deviate by a single NBD behavior are explained in terms of a common mechanism, linked to the emission of hard gluons. The simple NB parametrization is reestablished at the level of events with fixed number of jets. Further tests of the above mentioned picture can be provided by direct experimental analyses which take into account the effects of correlations and systematics; these analyses are eagerly awaited.

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References

[1] DELPHI Coll., P. Abreu et al., Z. Phys. C52 (1991) 271.

[2] OPAL Coll., P. D. Acton et al., Z. Phys. C53 (1992) 539.

[3] ALEPH Coll., D. Buskulic et al., Z. Phys. C69 (1995) 15.

[4] K. Abe et al., SLD Coll., "Factorial and Cumulant Moments in $e^+e^- \rightarrow$ hadrons at the Z^0 resonance", Preprint SLAC-PUB-95-7027, December 1995, submitted to Phys. Lett. B.

[5] I.M. Dremin et al., Phys. Lett. B336 (1994) 119;
G. Gianini, in Proc. XXIV International Symposium on Multiparticle Dynamics (Vietri sul Mare (SA), Italy, 1994), eds. A. Giovannini, S. Lupia, R. Ugoccioni, World Scientific, Singapore, 1995, p. 67.

[6] K. Konishi, A. Ukawa and G. Veneziano, Nucl. Phys. B157 (1979) 45.

[7] Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, "Basics of perturbative QCD", Editions Frontières, Gif-sur-Yvette, 1991.

[8] D. Amati and G. Veneziano, Phys. Lett. B83 (1979) 87;
G. Marchesini, L. Trentadue and G. Veneziano, Nucl. Phys. B181 (1981) 335;
Ya.I. Azimov, Yu.L. Dokshitzer, V.A. Khoze and S.I. Troyan, Z. Phys. C27 (1985) 65; Z. Phys. C31 (1986) 213.

[9] L. Van Hove and A.Giovannini, Acta Phys. Pol. B19 (1988) 917.

[10] A. Giovannini and L. Van Hove, Z. Phys. C30 (1986) 391.

[11] S. Carius and G. Ingelmann, Phys. Lett. B252 (1990) 647;
R. Szwed, G. Wrochna and A. K. Wróblewski, Mod. Phys. Lett. A5 (1990) 1851.

[12] Yu.L. Dokshitzer, Phys. Lett. B305 (1993) 295.

[13] I. M. Dremin and V. A. Nechitailo, JETP Lett. 58 (1993) 945
I.M. Dremin, Physics Uspekhi 164 (1994) 785.

[14] A. Giovannini, Nucl. Phys. B161 (1979) 429.

[15] M. Derrick et al., HRS Coll., Phys. Lett. B168 (1986) 299.

[16] W. Braunschweig et al., TASSO Coll., Z. Phys. C45 (1989) 193.

[17] E.D. Malaza and B. R. Webber, Nucl. Phys. B267 (1986) 702.

[18] R. Ugoccioni, A. Giovannini and S. Lupia, Phys. Lett. B342 (1995) 387.

[19] DELPHI Coll., P. Abreu et al., Z. Phys. C56 (1992) 63.

[20] R.E. Ansorge et al., UA5 Collaboration, Z. Phys. C43 (1989) 357; Notice that the shoulder structure has also been observed at TEVATRON: F. Rimondi, in Proc. XXIII International Symposium on Multiparticle Dynamics (Aspen, CO, USA, 1993), eds. M. Block and A. White, World Scientific, Singapore, 1994, p. 400.

[21] C. Fuglesang, in Multiparticle Dynamics (Festschrift for Léon Van Hove), La Thuile, Italy, eds. A. Giovannini and W. Kittel, World Scientific, Singapore 1990, p. 193.

[22] L3 Coll., B. Adeva et al., Z. Phys. C55 (1992) 39.

6. Table Captions

Table 1. Parameters and χ^2/NDF of the fit to experimental data from ALEPH[3], DELPHI[1], OPAL[2] and SLD[4] with the weighted sum of two NBD's. Results are shown for different values of α corresponding to the fraction of 2-jet events, f, experimentally measured by DELPHI Collaboration at different values of the jet-finder parameter y_{cut}. NBD parameters extracted by the DELPHI Collaboration by fitting MD's of samples of events with 2- and 3-jets at different values of y_{cut} are also shown for comparison in the last columns.

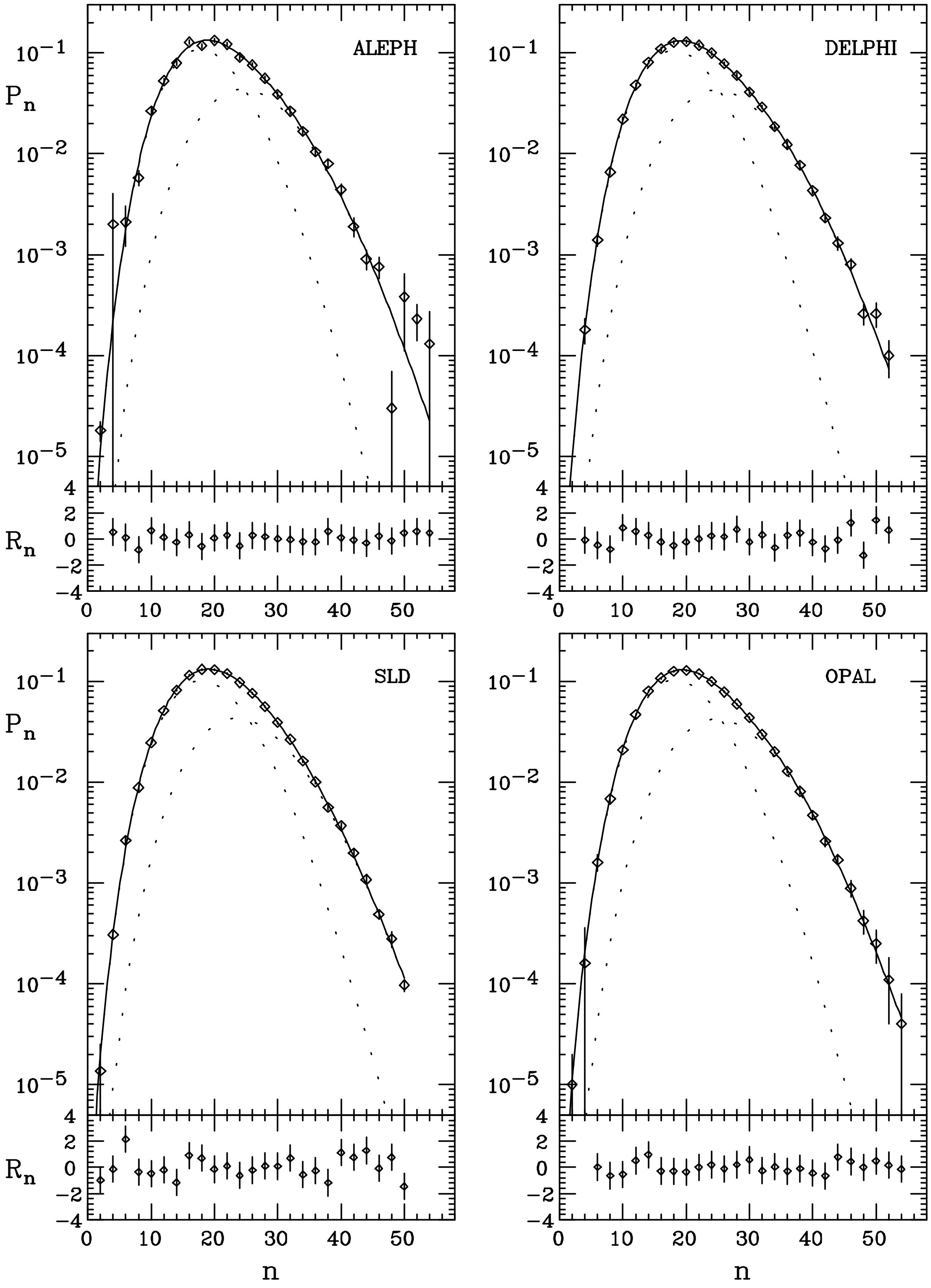
7. Figure Captions

Figure 1. Charged particles' MD in full phase space, P_n, at the Z_0 peak from ALEPH[3], DELPHI[1], SLD[4] and OPAL[2] are compared with equation (3) with $\alpha = 0.767$ (see Table 1 for the values of the corresponding parameters) (solid lines); dotted lines indicate the two separate NBD contributions. The lower part of the figure shows the residuals, R_n, i.e., the difference between data and theoretical predictions, expressed in units of standard deviations.

Figure 2. The ratio of factorial cumulnats over factorial moments, H_q as a function of q; experimental data (diamonds) are compared with equation (7) for different values of α as in Table 1 (solid lines). SLD, ALEPH, DELPHI and OPAL data are explicitly indicated. The dashed lines in the first plot (SLD) show predictions of equation (3), i.e., the same parametrization as the solid lines but without taking into account the effect of truncation. In the figure only statistical errors of SLD data are shown.

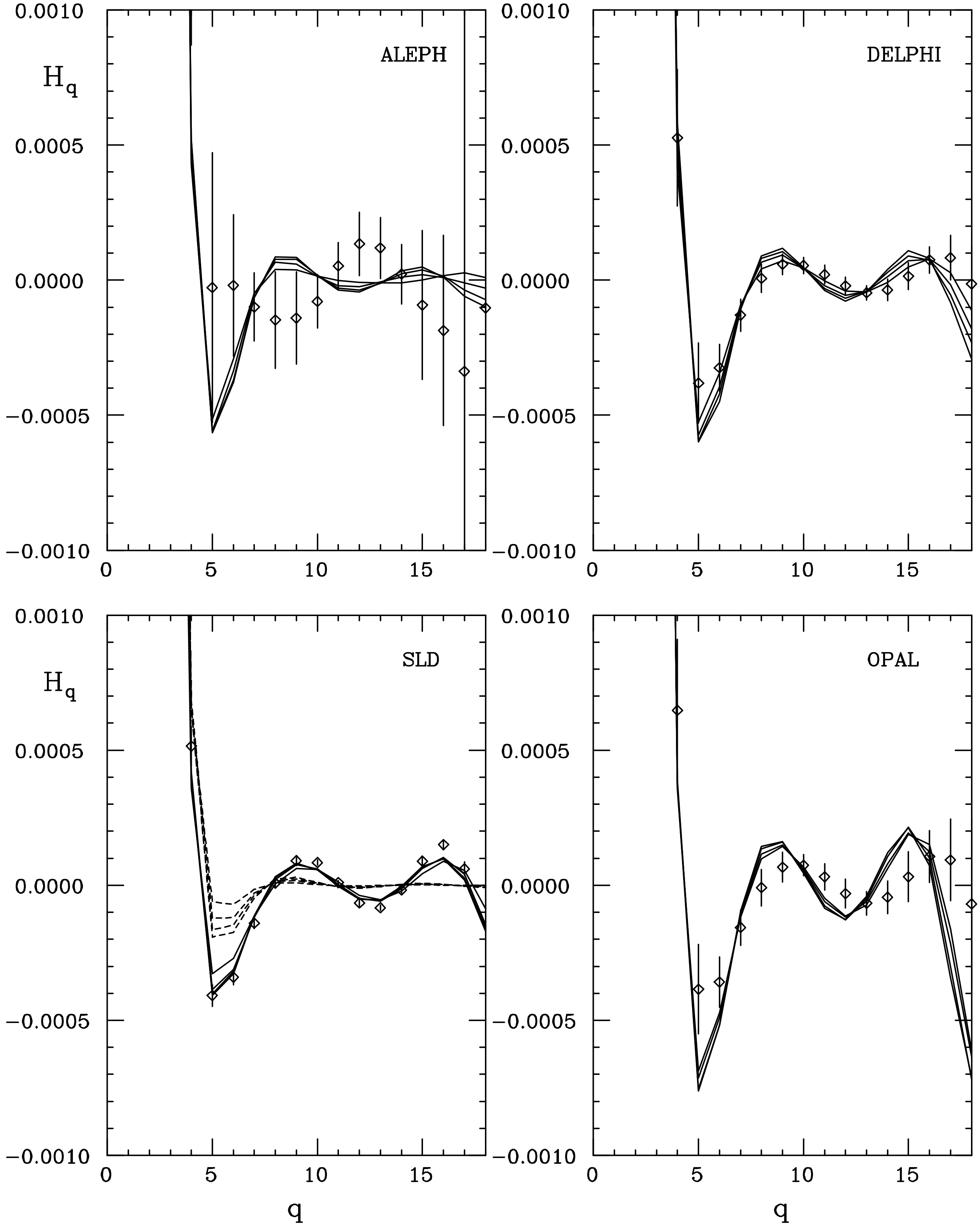
Table 1

	ALEPH[3]	DELPHI[1]	OPAL[2]	SLD[4]	DELPHI[19]	
	$\alpha = 0.463$				$y_{cut} = 0.02$	f=0.463
$\bar{n}_1$	17.7±1.1	18.2±0.2	18.4±0.2	18.4±0.2	$\bar{n}_{2-jet}$	18.5±0.1
k_1	111±168	90±20	71±11	47±4	k_{2-jet}	57±4
$\bar{n}_2$	23.6±0.8	23.9±0.2	24.0±0.2	23.0±0.2	$\bar{n}_{3-jet}$	22.9±0.1
k_2	32±15	31±3	28±2	29±2	k_{3-jet}	44±2
χ^2/NDF	3.56/22	8.95/21	3.32/21	17.6/21		
	$\alpha = 0.659$				$y_{cut} = 0.04$	f=0.659
$\bar{n}_1$	18.5±0.7	18.9±0.2	19.0±0.1	18.9±0.1	$\bar{n}_{2-jet}$	19.4±0.1
k_1	66±46	63±8	54±5	42±3	k_{2-jet}	44±2
$\bar{n}_2$	25.5±1.0	25.8±0.3	25.9±0.2	24.7±0.2	$\bar{n}_{3-jet}$	24.8±0.1
k_2	47±33	44±5	40±5	37±3	k_{3-jet}	42±2
χ^2/NDF	3.72/22	10.1/21	4.40/21	16.3/21		
	$\alpha = 0.767$				$y_{cut} = 0.06$	f=0.767
$\bar{n}_1$	19.1±0.5	19.4±0.2	19.5±0.07	19.3±0.09	$\bar{n}_{2-jet}$	20.0±0.1
k_1	53±24	52±6	46±3	39±2	k_{2-jet}	38±1
$\bar{n}_2$	27.0±1.1	27.3±0.3	27.5±0.2	26.0±0.2	$\bar{n}_{3-jet}$	26.0±0.1
k_2	65±62	61±10	55±8	47±5	k_{3-jet}	45±2
χ^2/NDF	3.86/22	11.7/21	6.30/21	15.6/21		
	$\alpha = 0.834$				$y_{cut} = 0.08$	f=0.834
$\bar{n}_1$	19.5±0.4	19.8±0.1	19.9±0.6	19.6±0.1	$\bar{n}_{2-jet}$	20.4±0.1
k_1	45±15	46±3	40±2	37±2	k_{2-jet}	34±1
$\bar{n}_2$	28.2±1.2	28.6±0.3	28.8±0.2	27.1±0.3	$\bar{n}_{3-jet}$	26.8±0.1
k_2	92±121	85±18	76±15	59±7	k_{3-jet}	49±1
χ^2/NDF	3.99/22	13.9/21	8.81/21	15.2/21		



ALEPH
DELPHI
SLD
OPAL
P_n
R_n
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
4
2
0
−2
−4
0
10
20
30
40
50
n

Figure 1:



ALEPH
DELPHI
SLD
OPAL
H_q
H_q
0.0010
0.0005
0.0000
−0.0005
−0.0010
0
5
10
15
q
q

Figure 2: