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The negative binomial distribution in quark jets with fixed flavour *

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Abstract

We show that both the multiplicity distribution and the ratio of factorial cumulants over factorial moments for 2-jet events in e^+e^- annihilation at the Z^0 peak can be well reproduced by the weighted superposition of two negative binomial distributions, associated to the contribution of $b\bar{b}$ and light flavoured events respectively. The negative binomial distribution is then suggested to describe the multiplicity distribution of 2-jet events with fixed flavour.

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1. Introduction

Two different experimental effects in the Multiplicity Distributions (MD's) of charged particles in full phase space in e^+e^- annihilation at the Z^0 peak, i.e., the shoulder visible in the intermediate multiplicity range[1, 2, 3] and the quasi-oscillatory behaviour of the ratio of factorial cumulants over factorial moments of the MD, H_q, when plotted as a function of its order q [4, 5], have been quantitatively reproduced in [6] in terms of a weighted superposition of two Negative Binomial Distributions (NBD's), associated to two- and multi-jet events respectively. A further test of this picture, in which the simple NBD appears at a very elementary level of investigation, is provided by the study of samples of events with a fixed number of jets. In [7], the DELPHI Collaboration has shown that a single NBD can describe the MD's for events with a fixed number of jets for a range of values of the jet resolution parameter $y_{\min}$. This was indeed the starting point of the parametrization proposed in [6].

In this letter, by extracting the ratio H_q from published MD's according to the procedure described in [6], we show that the oscillations observed experimentally are larger than those predicted by a single NBD, even after taking into account the truncation effect, which was shown[8] to be important in the behavior of H_q's. These results suggest that, while hard gluon radiation plays a relevant role in the explanation of the shoulder structure of MD's and of oscillations of the ratio H_q, some other effects should be taken into account for a detailed description of experimental data of events with a fixed number of jets. In this respect, it is worth recalling the interesting results obtained by the OPAL Collaboration[9] on forward-backward correlations and on the increase of transverse momentum of produced hadrons in the intermediate multiplicity range: it has been found indeed that both effects are mainly due to hard gluon radiation, i.e., to the superposition of events with a fixed number of jets. However, a residual positive correlation has been found in a sample of 2-jet events; via Monte Carlo simulations, this effect has been associated to the combined action of superposition of events with different quark flavours and, in the central region, to a residual effect due to resonances' decays. Let us remind, however, that the presence of heavy flavours has been shown not to affect the increase of the transverse momentum of produced hadrons in the intermediate multiplicity range, thus suggesting that not all observables are sensitive to the original quark flavour. A theoretical study based on Monte Carlo simulations has first suggested that the study of MD's can indeed point out interesting features of particle production in $b\bar{b}$ events[10]. Recently the DELPHI Collaboration has established experimentally the sensitivity of MD's to the original quark flavour, by comparing the MD for the full sample of events with a sample enriched in $b\bar{b}$ events[11].

We propose in this letter to associate a NBD to the MD in 2-jet events of fixed flavour. We show, after examining possible alternatives, that the weighted superposition of two NBD's, which we associate to $b\bar{b}$ and light flavoured events, describes very well both the MD's and the ratio H_q for 2-jet events; the two NBD's have the same k parameters and differ in the average multiplicity only. Some consequences of this fact are examined in the conclusions.

Table 1: Parameters and χ^2/NDF of the fit to experimental data[11] on single hemisphere MD's for $b\bar{b}$ sample and for all flavours with a single NBD and with a NBD shifted by one or two units.

	$b\bar{b}$ sample	all flavours
NBD		
$\bar{n}$	11.67±0.07	10.67±0.02
k	24±2	14.5±0.3
χ^2/NDF	118/26	140/28
NBD (shift by 1 unit)		
$\bar{n}$	10.62±0.07	9.63±0.02
k	15.8±0.6	10.42±0.2
χ^2/NDF	60/26	64/28
NBD (shift by 2 units)		
$\bar{n}$	9.55±0.07	
k	10.4±0.3	
χ^2/NDF	20/26	

2. MD's in b-jets

The DELPHI Collaboration has studied the effect of quark flavour composition on the MD in one hemisphere, by comparing the MD for the full sample of events with that for a sample enriched in $b\bar{b}$ events[11]: the MD extracted from the $b\bar{b}$ sample was found to be essentially identical in shape to the MD obtained for the full sample, apart from a shift of one unit, which may be related to the effect of weak decays of B-hadrons[12]. To give a quantitative comparison of the MD's in single hemisphere in the $b\bar{b}$ sample and in the sample with a mixture of all flavours, we have fitted both experimental MD's with a NBD and with a NBD shifted by one or two units. The results of the fit are shown in Table 1. A single NBD gives a poor description of the MD for the $b\bar{b}$ sample; the description improves strongly if one introduces a shift by one unit, and becomes even better with a shift by two units. The reason is that with the shift the NBD is able to reproduce better the head of the distribution; however the tail remains underestimated. In this respect, one should remember that the single NBD cannot give a good description of the MD with all flavours in full phase space, since it cannot reproduce the shoulder structure due to the superposition of events with different number of jets[7]. The fact that this feature should be present for the $b\bar{b}$ sample too should be verified experimentally.

In any case, interesting information on the properties of these MD's can be extracted without using any parametrization at all. In what follows we will consider only 2-jet events, selected with a suitable algorithm, but the same reasoning can be carried out also for the full sample. Let us call p_n, p_n^b and p_n^l the MD's in a single hemisphere for all events, for $b\bar{b}$ events and for light flavoured (non $b\bar{b}$) events respectively, and $g(z) \equiv \sum_{n=0}^{\infty} p_n z^n$, $g^b(z)$ and $g^l(z)$ the associated generating functions. With α the fraction of $b\bar{b}$ events, one has:

$$p_n = \alpha p_n^b + (1-\alpha) p_n^l \tag{1}$$

i.e.,

$$g(z) = \alpha g^b(z) + (1-\alpha) g^l(z) \tag{2}$$

These relations are valid in general; DELPHI data and our analysis shown in Table 1 suggest that p_n^b is given by p_n with a shift of one unit:

$$p_n^b = p_{n-1} \qquad n > 0; \qquad\qquad p_0^b = 0; \tag{3}$$

i.e.,

$$g^b(z) = z g(z) \tag{4}$$

Substituting now eq. (4) in eq. (2), one gets:

$$g(z) = \frac{1-\alpha}{1-\alpha z} g^l(z) \tag{5}$$

i.e.,

$$g^b(z) = \Big[\frac{z(1-\alpha)}{1-\alpha z}\Big] g^l(z) \tag{6}$$

The MD in $b\bar{b}$ events is the convolution of a shifted geometric distribution, of average value $1/(1-\alpha)$, with the MD in light flavoured events. The shifted geometric MD could be related to the MD of the decay products of B hadrons in the framework of [12].

The connection of the MD in a single hemisphere to that in full phase space is not entirely trivial, since one has to take into account additional effects, like for instance charge conservation, which requires that the final multiplicity be even.

Let us use the same notation for MD's as in the previous paragraph, but with capital letters to denote the MD's in full phase space; by taking the two hemispheres as independent (which, as suggested in [9], should be a good approximation at least for light flavours) but applying charge conservation, we obtain:

$$P^i(n_1, n_2) = \begin{cases} 2 p_{n_1}^i p_{n_2}^i & \text{if } n_1 + n_2 \text{ is even} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Here $P^i(n_1, n_2)$ is the probability to produce n_1 particles in one hemisphere and n_2 in the other hemisphere and i denotes either all 2-jet events (no label), or $b\bar{b}$ events ($i = b$) or light flavoured events ($i = l$). The factor 2 is for normalization, assuming that the p_n^i do not privilege the even or odd component; in any case this effect can be easily taken into account and results similar to those given below are obtained.

The MD in full phase space is given by definition by:

$$P_n^i = \sum_{n_1=0}^{n} P^i(n_1, n - n_1) \tag{8}$$

In terms of the generating functions, one obtains

$$\begin{aligned} G^i(z) &= \sum_{n=0}^{\infty} z^n P_n^i = 2 \sum_{\substack{n=0 \\ n \text{ even}}}^{\infty} \sum_{n_1=0}^{n} z^{n_1} p^i(n_1) z^{n-n_1} p^i(n-n_1) \\ &= ([g^i(z)]^2 + [g^i(-z)]^2) \end{aligned} \tag{9}$$

We can see now how the relations which we obtained in the previous paragraph are modified going from a single hemisphere to full phase space: by putting eq. (6) in eq. (9), one has

$$G^b(z) = z^2 \frac{(1-\alpha)^2}{(1-\alpha z)^2}[g^l(z)]^2 + z^2 \frac{(1-\alpha)^2}{(1+\alpha z)^2}[g^l(-z)]^2 \tag{10}$$

For $\alpha = 0$, one obtains that in full phase space the MD in $b\bar{b}$ events coincides with the MD for light flavoured events with a shift of two units, as we get $G^b(z) = z^2 G^l(z)$. The MD for small values of α should not be too far from this limit, as can be easily checked with numerical examples.

In conclusion, going from single hemispheres to full phase space by taking into account only charge conservation, the MD of $b\bar{b}$ events becomes close not to the total MD but to the MD of light flavoured events. The two MD's seem to have the same characteristics, like average value and dispersion, and the only difference should lie in a shift of two units.

3. MD's and H_q's ratio in 2-jet events

The analysis of MD's for events with a fixed number of jets, and in particular 2-jet events, has been performed in [7], where a single NBD has been shown to reasonably describe the MD's for events with a fixed number of jets, for several values of the jet resolution parameter $y_{\rm min}$ (the JADE jet-finding algorithm has been used in [7]). A comparison of DELPHI data with $y_{\rm min} = 0.02$ with a single NBD is shown in Figure 1a together with the residuals, i.e., the normalized difference between data and theoretical predictions, which point out the presence of substructures in experimental data. In view of previous results, it is then interesting to investigate whether these substructures can be explained in terms of the different contribution of quark-jets with different flavours.

We parametrize the experimental data on MD's for 2-jet events in full phase space in e^+e^- annihilation at the Z^0 peak[7] in terms of the superposition of 2 NBD's, associated to the contribution of b- and light flavours (we include the charm among the light flavours to a first extent). We fix therefore the weight parameter to be equal to the fraction of $b\bar{b}$ events, $\alpha = 0.22$[13]. Following the results of the previous section, we ask that the NBD associated to the b flavour be shifted by two units and that both parameters of the two NBD's, $\bar{n}$ and k, be the same. Formally, we perform then a fit with the following 2-parameter distribution:

$$P_n(\bar{n}, k) = \alpha P^{\rm NB}_{n-2}(\bar{n}, k) + (1-\alpha) P^{\rm NB}_n(\bar{n}, k) \tag{11}$$

where $P^{\rm NB}_{n-2}(\bar{n}, k) = 0$ for $n < 2$. $P^{\rm NB}_n(\bar{n}, k)$ is here the NBD, expressed in terms of two parameters, the average multiplicity $\bar{n}$ and the parameter k, linked to the dispersion by $D^2/\bar{n}^2 = 1/\bar{n} + 1/k$, as:

$$P^{\rm NB}_n(\bar{n}, k) = \frac{k(k+1)\dots(k+n-1)}{n!} \left(\frac{k}{\bar{n}+k}\right)^k \left(\frac{\bar{n}}{\bar{n}+k}\right)^n \tag{12}$$

As far as MD's in full phase space are concerned, one has also to take care of the "even-odd" effect, i.e., of the fact that the total number of final charged particles must be even

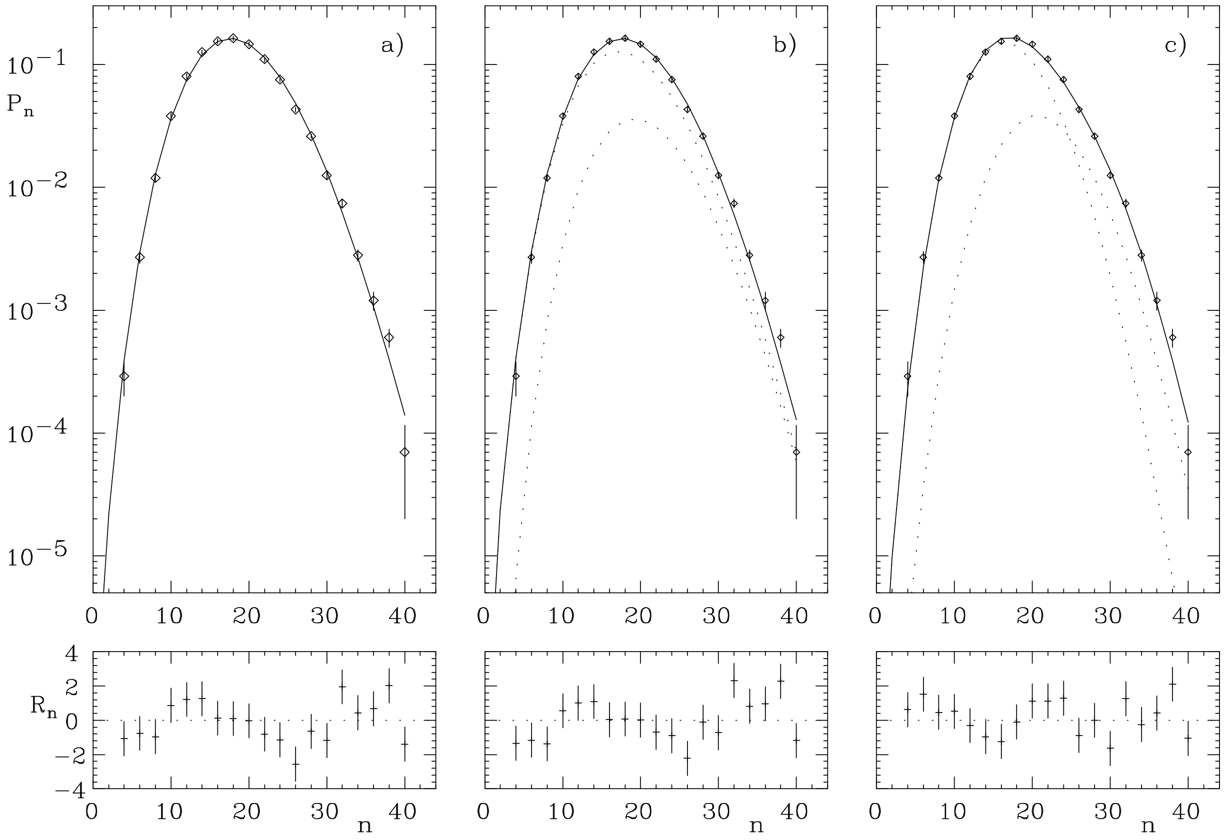

Figure 1: **a)**: charged particles' MD for two-jet events in full phase space, P_n, at the Z^0 peak from DELPHI[7] with $y_{\min} = 0.02$ are compared with the fit with a single NBD as performed by DELPHI Collaboration (solid lines); the lower part of the figure shows the residuals, R_n, i.e., the difference between data and theoretical predictions, expressed in units of standard deviations. The even-odd effect has been taken into account (see eq. (13)). **b)**: Same as in **a)**, but the solid line here shows the result of fitting eq. (11), with parameters given in Table 2. **c)**: Same as in **a)**, but the solid line here shows the result of fitting eq. (14), with parameters given in Table 2.

Table 2: Parameters and χ^2/NDF of the fit to experimental data on 2-jet events MD's from DELPHI[7] with three different MD's: the weighted superposition of a NBD and a shifted NBD with the same parameters (eq. (11)), the weighted superposition of a Poisson plus a shifted Poisson (eq. (14)) and the weighted superposition of two NBD's with the same k (eq. (20)). The weight used is the fraction of $b\bar{b}$ events. The even-odd effect has been taken into account (see eq. (13)). Results are shown for different values of the jet-finder parameter $y_{\min}$.

	$y_{min} = 0.01$	$y_{min} = 0.02$	$y_{min} = 0.04$
NBD + shifted NBD (same $\bar{n}$ and k) eq. (11)			
$\bar{n}$	17.17±0.05	18.01±0.04	18.99±0.05
k	69±5	57±3	44±1
χ^2/NDF	18.2/17	27.9/17	53.9/21
Poisson + shifted Poisson eq. (14)			
$\bar{n}_1$	19.67±0.13	21.10±0.10	22.83±0.09
$\bar{n}_2$	16.37±0.07	16.95±0.06	17.51±0.06
χ^2/NDF	21.7/17	20.75/17	45.05/21
2 NBD (same k) eq. (20)			
$\bar{n}_l$	16.81±0.21	17.22±0.15	17.98±0.15
$\bar{n}_b$	20.26±1.71	21.96±1.57	23.61±1.64
k	124±51	145±53	120±33
χ^2/NDF	17.4/16	12.6/16	27.5/20
δ_{bl}	3.44±0.83	4.6±0.5	5.6±0.5

due to charge conservation; accordingly, the actual form used in the fit procedure is given by:

$$P_n^{fps} = \begin{cases} AP_n & \text{if } n \text{ is even} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

where A is the normalization parameter, so that $\sum_{n=0}^{\infty} P_n^{fps} = 1$.

Table 2 (first group) shows the two parameters $\bar{n}$ and k of eq. (11) for different values of the jet resolution parameter $y_{\min}$; the proposed parametrization gives a rather good description with only two parameters; the agreement is worse for $y_{\min} = 0.04$, which could be due to the contamination of 3-jet events. However, as shown in Figure 1b for $y_{\min} = 0.02$, the oscillatory structure in the residuals does not disappear with this parametrization. Let us remind that the values of χ^2/NDF shown in the Table should be considered just indicative, since we did not know the full covariance matrix and we could not then treat properly the correlations between different channels of the MD. This forbids also a direct comparison of the χ^2/NDF of the present parametrization with the values of χ^2/NDF for a single NBD obtained in [7], where correlations between bins were taken into account. Finally, let us say that we also fit eq. (9) by assuming that p_n^l is a NBD, and we found the same results.

It is interesting at this point to investigate a minimal model where no physical correlations are present at the level of events with fixed number of jets and fixed flavour: we have performed a fit using a weighted sum of a shifted Poisson plus a Poisson distribution (with the correction for the even-odd effect according to eq. (13)):

$$P_n(\bar{n}_1, \bar{n}_2) = \alpha P^{\mathrm{P}}_{n-2}(\bar{n}_1) + (1-\alpha) P^{\mathrm{P}}_n(\bar{n}_2) \tag{14}$$

where

$$P^{\mathrm{P}}_n(\bar{n}) = \frac{\bar{n}^n}{n!} e^{-\bar{n}} \tag{15}$$

Also in this case, we have two free parameters. Results of the fit are shown in Table 2 (second group); also in this case a reasonable fit is achieved, even though the MD at $y_{\min} = 0.04$ shows again some anomaly. The MD with the parameters shown in Table 2 is compared to experimental data with $y_{\min} = 0.02$ in Figure 1c; it should be pointed out that the structure in the residuals is present in this case too. From this analysis, one would then conclude that physical correlations visible in e^+e^- annihilation result trivially from the superposition of samples of events with different quark flavours. A more accurate analysis with full covariance matrix is needed to see which parametrization is preferred by experimental data. However, independent of the chosen parametrization, two different components, which can be associated to b- and light flavours contributions, are visible in the MD of 2-jet events.

A more detailed analysis of the tail of the MD, which can help distinguish different parametrizations, comes from the study of the ratio of unnnormalized factorial cumulant over factorial moments

$$H_q = \frac{\tilde{K}_q}{\tilde{F}_q} \tag{16}$$

as a function of the order q. The factorial moments, $\tilde{F}_q$, and factorial cumulant moments, $\tilde{K}_q$, can be obtained from the MD, P_n, through the relations:

$$\tilde{F}_q = \sum_{n=q}^{\infty} n(n-1)\ldots(n-q+1)P_n, \tag{17}$$

and

$$\tilde{K}_q = \tilde{F}_q - \sum_{i=1}^{q-1} \binom{q-1}{i} \tilde{K}_{q-i}\tilde{F}_i. \tag{18}$$

Since the H_q's were shown to be sensitive to the truncation of the tail due to the finite statistics of data samples[8], moments have to be actually extracted from a truncated MD defined as follows (including again the correction for the even-odd effect as in eq. (13)):

$$P^{trunc}_n = \begin{cases} A' P_n & \text{if } (n_{min} \le n \le n_{max}) \text{ and } \; n \text{ is even} \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

Here n_{min} and n_{max} are the minimum and the maximum observed multiplicity and A' is a new normalization parameter.

In Figure 2 the H_q's extracted from the experimental MD published by DELPHI Collaboration [7] (here $y_{\min} = 0.02$) with the procedure explained in [6] are compared with the predictions of a single NBD as fitted by DELPHI Collaboration [7], and of eq.s (11) and (14). It is clear that all three parametrizations fail to describe the experimental behaviour of the ratios H_q, i.e., the description of the tail of the MD is not accurate. We then conclude that a single NBD cannot describe accurately the MD in 2-jet events, as already suggested by the study of residuals of MD's. The superposition of two NBD's with the same parameters turns out also to be inadequate; one concludes that the imposed constraints are too strong and that some additional differences between $b\bar{b}$ and light flavoured events should be allowed. Finally, the observed deviation from the Poisson-like fit suggests that there are indeed dynamical correlations beyond the purely statistical ones.

4. A new parametrization of MD's in 2-jet events

In the previous paragraph, the difference between the average multiplicity in $b\bar{b}$ and light flavoured events in the parametrization (11) has been fixed to 2; by using the parametrization (14), where this difference is not a priori constrained, larger values have been obtained. Let us also remind that the experimental value of this observable at the Z^0 peak is close to 2.8[14]; theoretical predictions in the framework of Modified Leading Log Approximation plus Local Parton Hadron Duality[15] give even larger values. It is therefore interesting to investigate whether one can reproduce not only the shape of the MD, but also its tail and then the ratio H_q, by using a superposition of two NBD's, but relaxing the constraint on the average multiplicities. The only constraint we impose is that the parameters k of the two NBD's be the same, while we allow a variation of the difference between the average multiplicities. For the sake of simplicity and in order to be more independent from any theoretical prejudice, we do not include any shift in the MD for $b\bar{b}$ events.

Formally, we perform then a fit with the following 3-parameter MD (plus the correction for the even-odd effect in eq. (13)):

$$P_n(\bar{n}_l, \bar{n}_b, k) = \alpha P_n^{\mathrm{NB}}(\bar{n}_b, k) + (1-\alpha) P_n^{\mathrm{NB}}(\bar{n}_l, k) \tag{20}$$

The parameters of the fits and the corresponding χ^2/NDF are given in Table 2 (third group) for different values of the resolution parameter $y_{\min}$. A really accurate description of experimental data is achieved. Notice that the best-fit value for the difference between the average multiplicities in the two samples, δ_{bl}, also given in Table 2, is quite large. This difference grows with increasing $y_{\min}$, i.e., with increasing contamination of 3-jet events.

Figure (3) compares the predictions of eq. (20) with the experimental MD's for two-jet events at different values of the resolution parameter $y_{\min}$. The residuals are also shown in units of standard deviations. One concludes that the proposed parametrization can reproduce the experimental data on MD's very well; no structure is visible in the residuals.

As already discussed, the ratio H_q gives a more stringent test of theoretical parametrizations; it is then interesting to study the predictions of eq. (20) for this ratio. In this case, one can obtain a closed expression for the factorial moments in terms of the parameters δ_{bl}, $\bar{n}_l$ and k. Let us notice indeed that, since the two components are given by a NBD with the same k, they have the same normalized factorial moments, which for a NBD are given by:

$$F_q^{(l)} = F_q^{(b)} = \prod_{i=1}^{q-1} (1 + \frac{i}{k}) \tag{21}$$

From eq. (20), one obtains a similar relation for the generating function:

$$G(z) = \alpha G^{(b)}(z) + (1-\alpha) G^{(l)}(z) \tag{22}$$

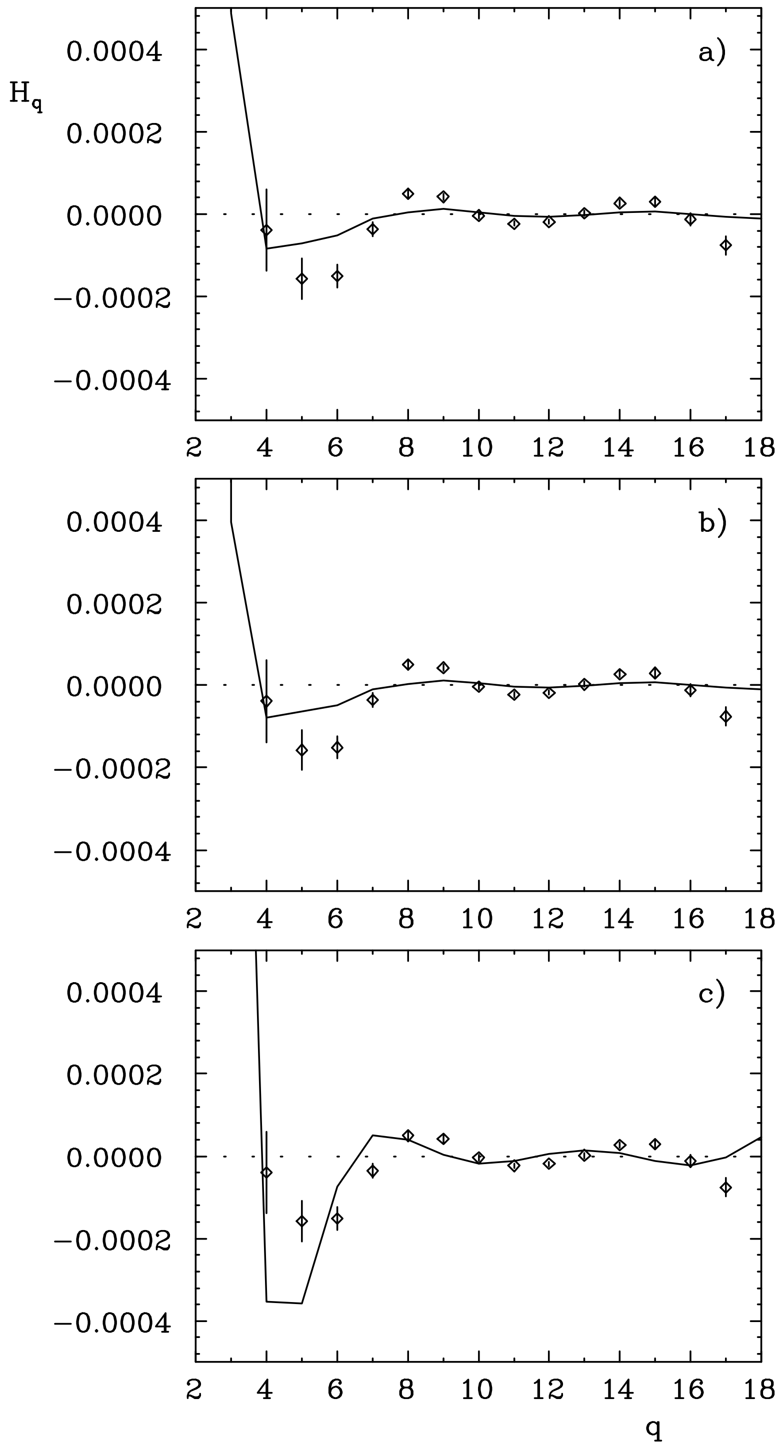

Figure 2: The ratio of factorial cumulant over factorial moments, H_q, as a function of q. **a)**: Experimental data (diamonds) for 2-jet events with $y_{\min} = 0.02$ are compared with the fit with a single NBD as performed by DELPHI Collaboration (solid lines). **b)**: Same as in **a)**, but the solid line here shows the result of fitting eq. (11), with parameters given in Table 2. **c)**: Same as in **a)**, but the solid line here shows the result of fitting eq. (14), with parameters given in Table 2. In all three cases the even-odd and the truncation effects have been taken into account (see eq. (19)).

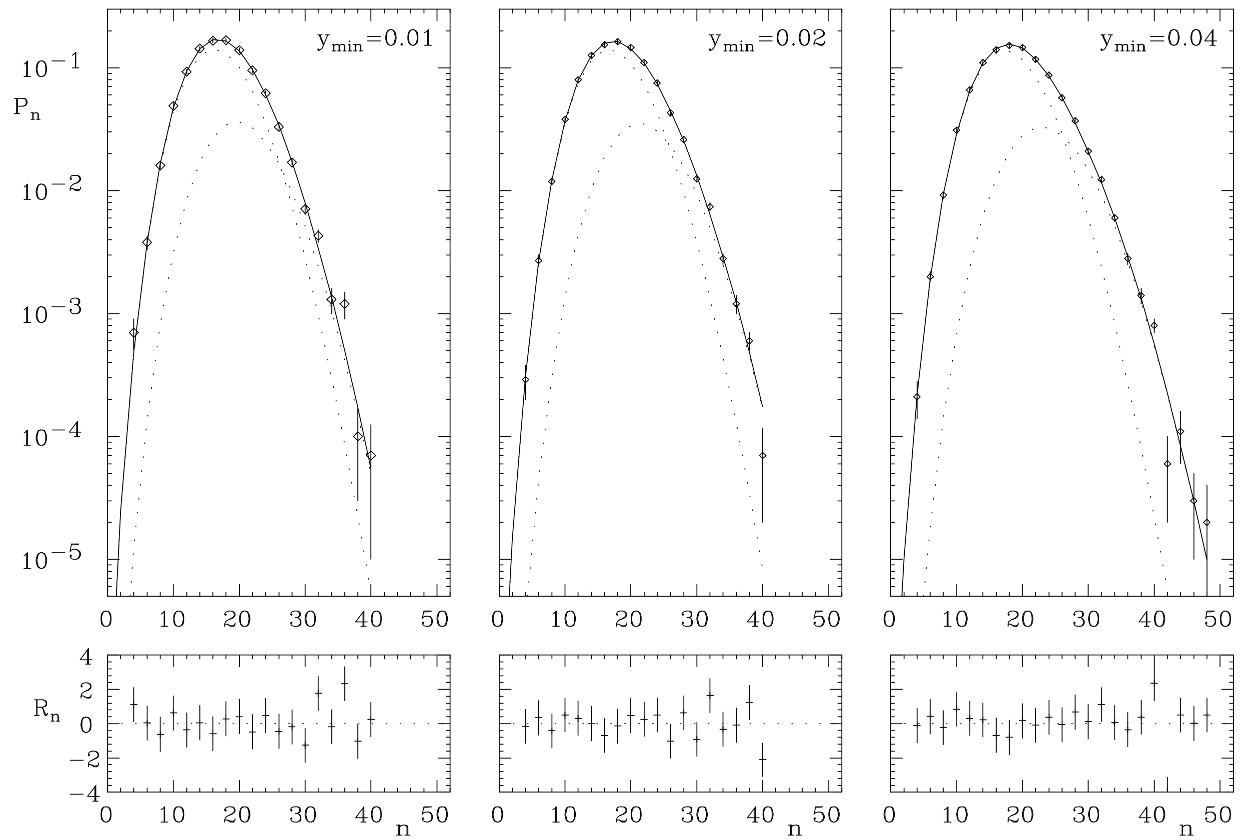

Figure 3: Charged particles' MD for two-jet events in full phase space, P_n, at the Z^0 peak from DELPHI[7] with different values of $y_{\min}$ are compared with a fit with the sum of 2 NBD's with the same parameter k as in eq. (20). The even-odd effect has been taken into account (see eq. (13)). The lower part of the figure shows the residuals, R_n, i.e., the difference between data and theoretical predictions, expressed in units of standard deviations.

By differentiating the previous equation, one then gets the following expression for the unnormalized factorial moments, $\tilde{F}_q$:

$$\bar{n} = \tilde{F}_1 = \bar{n}_l + \alpha\delta_{bl} \tag{23}$$

$$\begin{aligned} \tilde{F}_q &= \alpha\tilde{F}_q^{(b)} + (1-\alpha)\tilde{F}_q^{(l)} = \\ &= \left[\alpha(\bar{n}_l + \delta_{bl})^q + (1-\alpha)\bar{n}_l^q\right] \prod_{i=1}^{q-1}\left(1+\frac{i}{k}\right) \end{aligned} \tag{24}$$

Predictions of the ratio H_q as a function of the order q are obtained by inserting eq. (20) into eq. (19). These predictions with parameters fitted to reproduce the MD as given in Table 2 (third group) are compared in Figure 4 with the H_q's extracted from experimental data on MD's for 2-jet events[7] at different values of $y_{\rm min}$ according to the procedure described in [6]. The new parametrization gives an accurate description of the shape of MD's and is shown to describe well the ratio H_q too. Small deviations are still present for $y_{\rm min} = 0.01$, where 2-jet events are more collimated. They might be due to more subtle not yet understood effects. This consideration notwithstanding, the overall description of 2-jet MD's and H_q's appears quite satisfactory. This result gives therefore further support to the parametrization of the MD in quark-jets with fixed flavour in terms of a single NBD. It is also remarkable that the average number of particles only depends on flavour quantum numbers, whereas the NBD parameter k is flavour-independent.

As a further check, we also investigated the MD of 2-jet events with fixed flavour in the Monte Carlo program JETSET 7.4 PS[16]. For each flavour, we generated a sample of 60000 events and of 60000 2-jet events (selected using the JADE algorithm with $y_{\rm min} = 0.02$) by using the OPAL tuning[17] and we fitted the MD's in full phase space with a single NBD, eventually with a finite shift. In the all-events sample, the χ^2/NDF is really bad, thus indicating that a single NBD cannot describe the MD of events with fixed flavour. By requiring the 2-jet selection, the description improves strongly; the MD for light quarks are indeed well reproduced by a single NBD, while $b\bar{b}$ events are better described by a shifted NBD.

5. Conclusions

It has been shown that a single NBD cannot reproduce the observed behavior of the ratio H_q for events with a fixed number of jets in full phase in e^+e^- annihilation at the Z^0 peak. A simple phenomenological parametrization of the MD in terms of the weighted superposition of two NBD's has been shown to describe simultaneously both the MD's and the ratio H_q. The weight of the first component was taken equal to the fraction of $b\bar{b}$ events, i.e., the two components were identified with the b- and the light flavours contribution respectively. The simple NBD parametrization is thus reestablished at the level of 2-jet events with fixed quark flavour composition. It is interesting to note that this result is consistent with the results obtained in the context of a thermodynamical model of hadronization[18].

It is remarkable that the two NBD's associated to b- and light flavours contributions have the same parameter k; since k^{-1} is the second order normalized factorial cumulant,

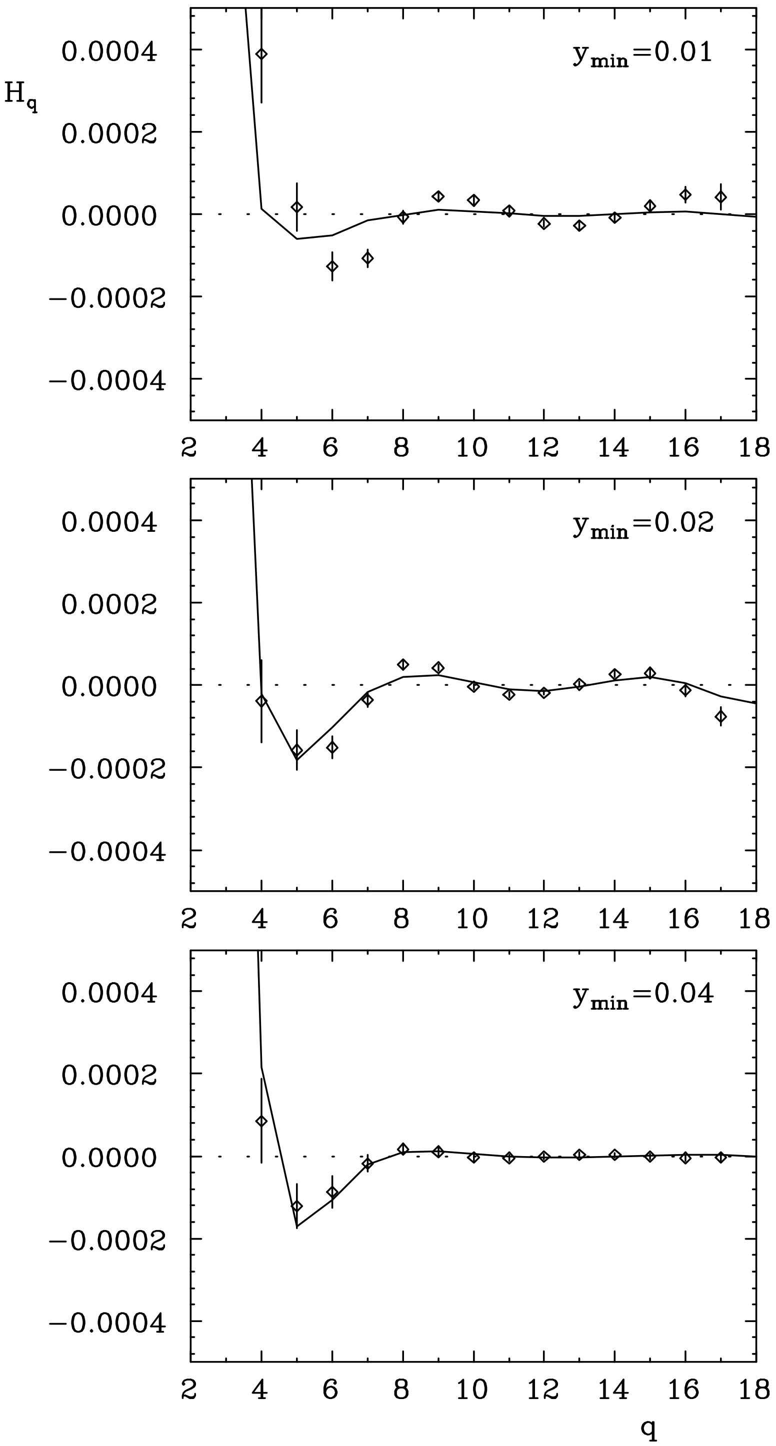

Figure 4: The ratio of factorial cumulant over factorial moments, H_q, as a function of q; experimental data (diamonds) for 2-jet events with different values of $y_{\min}$ are compared with equation (20) (solid lines). The parameters used are shown in Table 2. The even-odd and the truncation effects have been taken into account (see eq. (19)).

i.e., it is directly related to two-particle correlations, one concludes that two-particle correlations are flavour-independent in this approach. In addition, since both MD's are well described by a NBD, higher order correlations show a hierarchical structure[19], which is also flavour independent. This result can also be interpreted in the framework of clan structure analysis[20], where k^{-1} gets the meaning of an aggregation coefficient, being the ratio of the probability to have two particles in the same clan over the probability to have the two particles in two separate clans: in this language, one concludes that the aggregation among particles produced into clans in $b\bar{b}$ and light flavoured events turns out to be the same. The flavour quantum numbers affect then the average multiplicity of the corresponding jets only, but not the structure of particle correlations. It would be interesting to see, when appropriate samples of events will be available, whether this property established in full phase space continues to be valid in restricted regions of it.

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