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COMMON ORIGIN OF THE SHOULDER STRUCTURE AND OF THE OSCILLATIONS OF MOMENTS IN MULTIPLICITY DISTRIBUTIONS IN e^+e^- ANNIHILATIONS

A. Giovannini [a]
Dip. di Fisica Teorica and I.N.F.N. - Sezione di Torino,
via P. Giuria 1, I-10125 Torino, Italy

S. Lupia [b]
Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)
Föhringer Ring 6, D-80805 München, Germany

R. Ugoccioni [c]
Department of Theoretical Physics, University of Lund,
Sölvegatan 14A, S-22362 Lund, Sweden

ABSTRACT

We show, via a simple parametrization of the multiplicity distribution of charged particles in e^+e^- annihilation at the Z^0 peak in terms of the weighted superposition of two negative binomial distributions, that both the shoulder structure in the intermediate multiplicity range and the oscillation in sign of the ratio of factorial cumulants over factorial moments of increasing order are related to hard gluon radiation.

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[a]E-mail: giovannini@to.infn.it
[b]E-mail: lupia@mppmu.mpg.de
[c]E-mail: roberto@thep.lu.se

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Föhringer Ring 6, D-80805 München, Germany

R. Ugoccioni
Department of Theoretical Physics, University of Lund,
Sölvegatan 14A, S-22362 Lund, Sweden

We show, via a simple parametrization of the multiplicity distribution of charged particles in e^+e^- annihilation at the Z^0 peak in terms of the weighted superposition of two negative binomial distributions, that both the shoulder structure in the intermediate multiplicity range and the oscillation in sign of the ratio of factorial cumulants over factorial moments of increasing order are related to hard gluon radiation.

1 Common origin of shoulder structure and moments oscillations

One of the main still open problems in multiparticle dynamics is that of attaining an integrated description of final particle Multiplicity Distributions (MD's) and of the corresponding correlation functions properties. One expects that features detected in terms of one of the two observables have a sound physical counterpart in terms of the other. It is of course a quite difficult task to explain facts occurring in the two domains by means of the same physical cause and accordingly to show that they have a common origin. We discuss in the following a successful example of this search in e^+e^- annihilation. Here two interesting features are observed at the Z^0 energy: the MD shows a shoulder in the intermediate multiplicity range [1,2,3,4] and the ratio of factorial moments to factorial cumulant moments changes sign as a function of its order.[4,5] The two observables are strictly linked, as factorial moments, F_q, and factorial cumulant moments, K_q, can be obtained in general from the MD, P_n, through the relations:

$$F_q = \sum_{n=q}^{\infty} n(n-1)\dots(n-q+1)P_n \qquad (1)$$

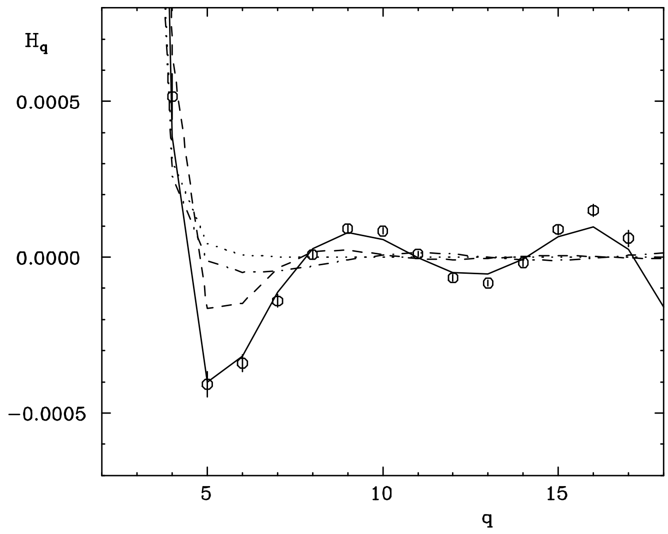

Figure 1: The ratio of factorial cumulant moments over factorial moments, H_q as a function of q; experimental data (diamonds) from the SLD Collaboration [4] are compared with the predictions of several parameterizations, with parameters fitted to the data on MD's: a full NBD (dotted line); a truncated NBD (dot-dashed line); sum of two full NBD's as per Eq. 4 (dashed line); sum of two truncated NBD's as per Eq. 5 (solid line).

and

$$K_q = F_q - \sum_{i=1}^{q-1} \binom{q-1}{i} K_{q-i} F_i. \tag{2}$$

The ratio of the two mentioned quantities:

$$H_q = K_q / F_q \tag{3}$$

is well suited to theoretical and experimental studies.[6]

QCD calculations are available for both the MD [7] and the ratio H_q.[6] Both fail to reproduce the data after a simple application of Local Parton Hadron Duality [8] as hadronization prescription (or its generalized version,[9] to be used with moments of higher order [10]). The most recent calculation of the MD has been done in MLLA gluodynamics:[7] KNO scaling is present only asymptotically, and the predicted shape at current energies is much narrower than in previous calculations,[11] but still the head and the tail of the distribution cannot be correctly reproduced. The ratio H_q has also been calculated beyond

DLA,[6] and has been predicted to show a negative minimum around $q = 5$ and then to oscillate in sign as a function of the order q; this agrees qualitatively with the behaviour seen in the data.[5]

Many phenomenological parameterizations have been used to describe data on MD's, the most common (and simplest) being the Negative Binomial Distribution (NBD) [12] and the Log-Normal Distribution (LND).[13] Both work well at lower energies, but fail to reproduce the shoulder structure at $\sqrt{s} = 91$ GeV. In particular this failure can be seen if the residuals (difference between the data and the parametrization divided by the error on the data) are examined: a clear structure is seen.[4] The SLD Collaboration has also shown that these parameterizations fail in the sector of the H_q moments,[4] even after taking into account the effect of finite statistics.[14] This is particularly evident in Figure 1, if one looks at the dotted (full NBD) and dot-dashed (truncated NBD) lines.

The shoulder structure was first explained by the Delphi Collaboration: [1] it results from the superposition of samples of events with a fixed number of jets. In each sample taken separately there is no shoulder but, since they have a different average multiplicity, a shoulder appears in the full sample. In addition it was found that the NBD describes well the MD in these samples of events with a fixed number of jets.[15] It should also be remembered that a shoulder has been seen in $p\bar{p}$ collisions at $\sqrt{s} = 900$ GeV by the UA5 Collaboration,[16] and it has been confirmed at the Tevatron.[17] A good fit to the UA5 data was performed with the weighted sum of two NBD's.[18]

Following the above observations, we propose a parametrization of the MD which is the weighted sum of two components, one to be associated with 2-jet events and one to be associated with events with 3 or more jets. The weight in this superposition is then the fraction of 2-jet events, which is experimentally determined, it is not a fitted parameter. This decomposition depends of course on the definition of jet; in particular it depends on the particular jet-finding algorithm, and on the value of the parameter $y_{\rm min}$ which controls the algorithm. The Delphi Collaboration has used the JADE algorithm and published values for the 2-jet fraction and for the MD's at $y_{\rm min} = 0.02$, 0.04, 0.06, 0.08. We have performed the fit for each of these values.

Concerning the experimental data, it should be noticed that the experimental errors on each point of the published MD's are correlated with adjacent bins: in our fit we cannot take this correlation into account, therefore it is not possible to compare directly the values of the χ^2 we obtain with those obtained by the experimental collaborations. Furthermore, the extraction of H_q from the published P_n also suffers from a similar problem, and the errors we show in the figures are estimates obtained by a statistical method [19] (except for the data of the SLD Collaboration [4]).

As for the particular form of the MD in the two components of our fit, we have chosen the NBD, because it has successfully been fitted to the data for the samples of events with fixed number of jets. In practice we perform a fit to the MD's with a four parameter formula:

$$P_n \propto \begin{cases} \alpha P_n^{\rm NBD}(\bar{n}_1, k_1) + (1-\alpha) P_n^{\rm NBD}(\bar{n}_2, k_2) & \text{if } n \text{ is even} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Here $P_n^{\rm NBD}(\bar{n}, k)$ is the standard NBD of parameters $\bar{n}$ and k; notice that we have taken into account the charge conservation law, which requires the final charged particle multiplicity to be even. The proportionality factor is fixed by requiring the proper normalization for P_n.

Results of our fit to the data of four experiments are shown in Table 1: we find χ^2 per degree of freedom equal to or smaller than 1, and values of the parameters consistent between different experiments; they are also consistent with those obtained by the Delphi Collaboration in fitting their 2-jet and 3-jet data separately with a NBD. These findings are visually summarized for $\alpha = 0.767$ in Figure 2, where the residuals do not show structures.

In Figure 3 we compare the experimental data on H_q's with the values obtained from a formula that takes the truncation effect into account, too:

$$\tilde{P}_n \propto \begin{cases} P_n & \text{if } (n_{\rm min} \le n \le n_{\rm max}) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where $n_{\rm min}$ and $n_{\rm max}$ are the minimum and maximum observed multiplicity, and a proportionality factor ensures proper normalization. We obtain a very good agreement with the data. Notice that it is not possible to reproduce the behaviour of the ratio H_q without taking into account the limits of the range of the available data. This can be seen in Figure 1, by comparing the dashed (full formula) and the solid (truncated formula) lines.

In conclusion, the observed behavior of H_q's results from the convolution of two different effects, a statistical one, i.e., the truncation of the tail due to the finite statistics of data samples, and a physical one, i.e., the superposition of two components. The two components can be related to 2- and 3-jet events, i.e., to the emission of hard gluon radiation in the early stages of the perturbative evolution.

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Table 1: Parameters and χ^2 per Degree of Freedom (df) of the fit to experimental data from ALEPH,[3] DELPHI,[1] OPAL[2] and SLD[4] Collaborations with the weighted sum of two NBD's. Results are shown for different values of α corresponding to the fraction of 2-jet events, f, experimentally measured by DELPHI Collaboration[15] at different values of the jet-finder parameter $y_{\min}$. NBD parameters extracted by the DELPHI Collaboration by fitting MD's of samples of events with 2- and 3-jets at different values of $y_{\min}$ are also shown for comparison in the last column.

	ALEPH	DELPHI	OPAL	SLD	DELPHI	
	$\alpha = 0.463$			$y_{\min} = 0.02$	f=0.463	
$\bar{n}_1$	17.7±1.1	18.2±0.2	18.4±0.2	18.4±0.2	$\bar{n}_{2-jet}$	18.5±0.1
k_1	111±168	90±20	71±11	47±4	k_{2-jet}	57± 4
$\bar{n}_2$	23.6±0.8	23.9±0.2	24.0±0.2	23.0±0.2	$\bar{n}_{3-jet}$	22.9±0.1
k_2	32±15	31±3	28±2	29±2	k_{3-jet}	44±2
χ^2/df	3.56/22	8.95/21	3.32/21	17.6/21		
	$\alpha = 0.659$			$y_{\min} = 0.04$	f=0.659	
$\bar{n}_1$	18.5±0.7	18.9±0.2	19.0±0.1	18.9±0.1	$\bar{n}_{2-jet}$	19.4±0.1
k_1	66±46	63±8	54±5	42±3	k_{2-jet}	44±2
$\bar{n}_2$	25.5±1.0	25.8±0.3	25.9±0.2	24.7±0.2	$\bar{n}_{3-jet}$	24.8±0.1
k_2	47±33	44±5	40±5	37±3	k_{3-jet}	42±2
χ^2/df	3.72/22	10.1/21	4.40/21	16.3/21		
	$\alpha = 0.767$			$y_{\min} = 0.06$	f=0.767	
$\bar{n}_1$	19.1±0.5	19.4±0.2	19.5±0.07	19.3±0.09	$\bar{n}_{2-jet}$	20.0±0.1
k_1	53±24	52±6	46±3	39±2	k_{2-jet}	38±1
$\bar{n}_2$	27.0±1.1	27.3±0.3	27.5±0.2	26.0±0.2	$\bar{n}_{3-jet}$	26.0±0.1
k_2	65±62	61±10	55±8	47±5	k_{3-jet}	45±2
χ^2/df	3.86/22	11.7/21	6.30/21	15.6/21		
	$\alpha = 0.834$			$y_{\min} = 0.08$	f=0.834	
$\bar{n}_1$	19.5±0.4	19.8±0.1	19.9±0.6	19.6±0.1	$\bar{n}_{2-jet}$	20.4±0.1
k_1	45±15	46±3	40±2	37±2	k_{2-jet}	34±1
$\bar{n}_2$	28.2±1.2	28.6±0.3	28.8±0.2	27.1±0.3	$\bar{n}_{3-jet}$	26.8±0.1
k_2	92±121	85±18	76±15	59±7	k_{3-jet}	49±1
χ^2/df	3.99/22	13.9/21	8.81/21	15.2/21		

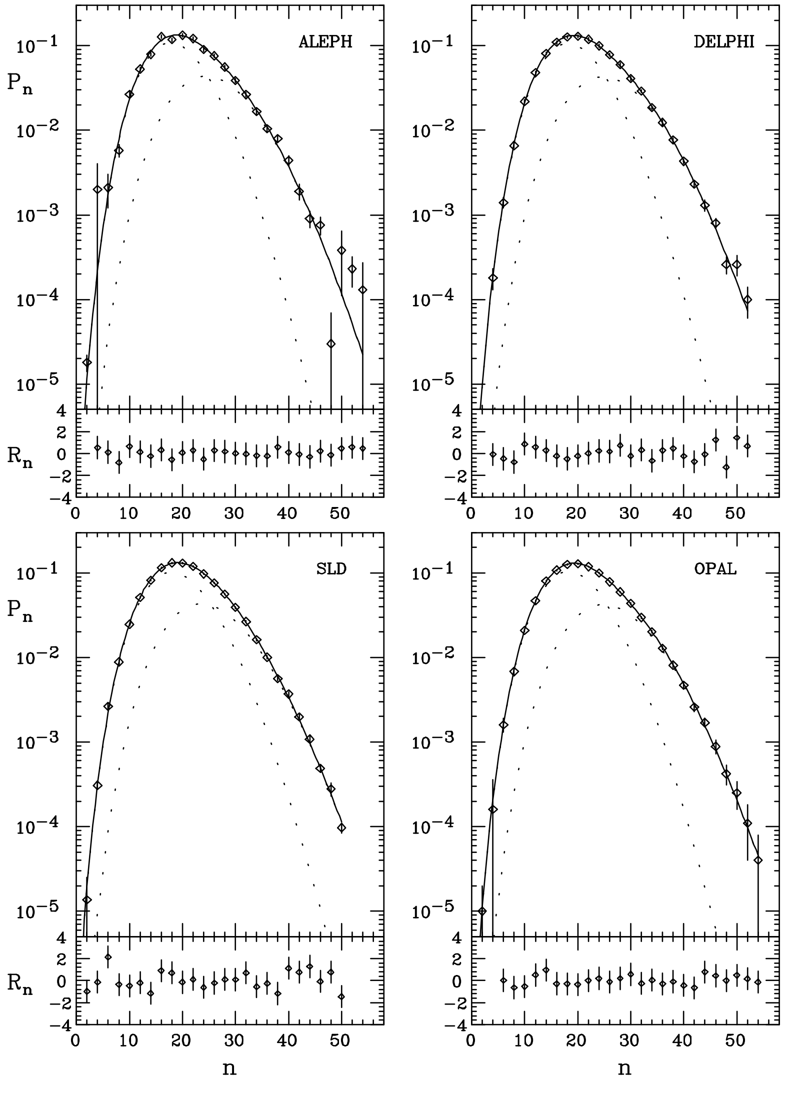

Figure 2: Charged particles MD's in full phase space, P_n, at the Z_0 peak from ALEPH,[3] DELPHI,[1] SLD [4] and OPAL [2] Collaborations are compared with Eq. 4 with $\alpha = 0.767$ (see Table 1 for the values of the corresponding parameters) (solid lines); dotted lines indicate the two separate NBD contributions. The lower part of each plot shows the residuals, R_n, i.e., the difference between data and theoretical predictions, in units of standard deviations.

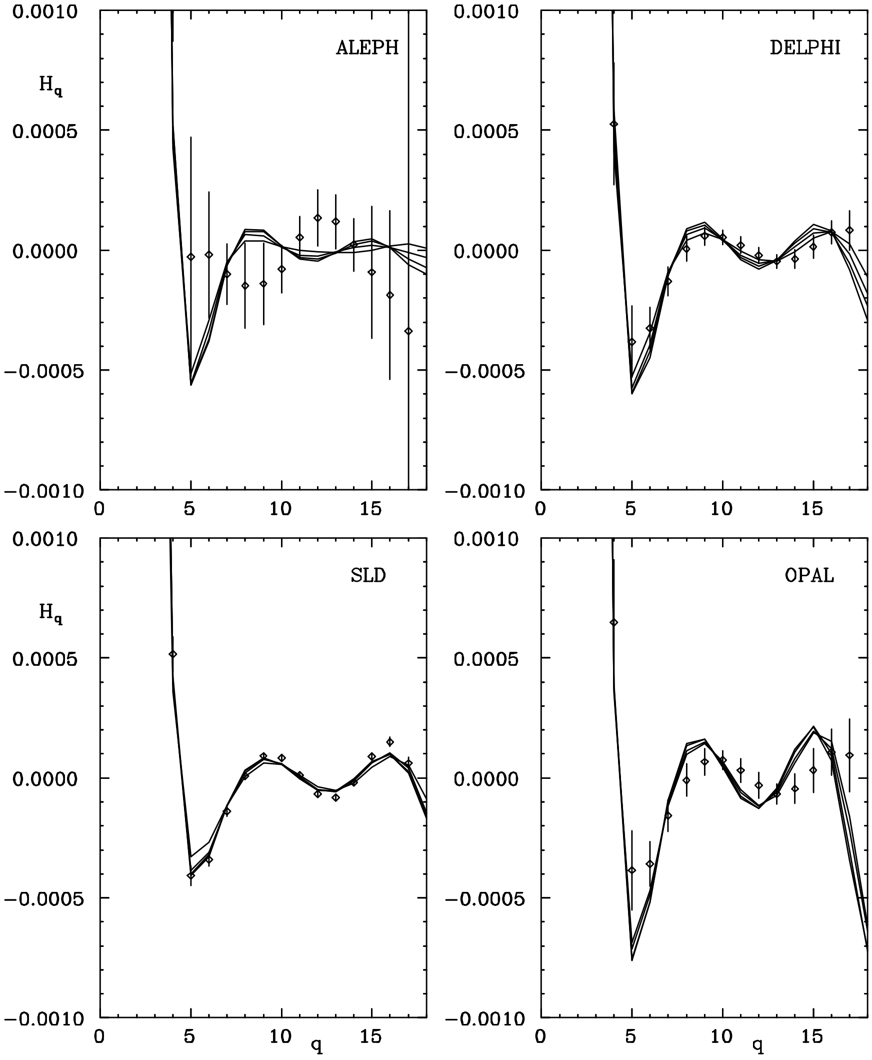

Figure 3: The ratio of factorial cumulant moments over factorial moments, H_q as a function of q; experimental data (diamonds) from ALEPH, DELPHI, SLD and OPAL Collaborations are compared with Eq. 5 for different values of α (see Table 1 for the values of the corresponding parameters). In the figure only statistical errors of SLD data[4] are shown.

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