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MULTIPLICITY DISTRIBUTIONS AND FLUCTUATIONS IN THE DISCRETE QCD MODEL

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ABSTRACT

Results are presented from calculations carried out in the discrete QCD model, concerning multiplicity distributions in rapidity intervals and fluctuations.

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1 Introduction

One of the long standing problems in multiparticle dynamics is an integrated description of multiplicity distributions and correlations. In particular, from the point of view of perturbative QCD, these observables are not satisfactorily described: this is particularly evident in restricted regions of phase space, where the most interesting experimental results are.[1] The discrete approximation to QCD [2] allows to perform exclusive and inclusive calculations at parton level; in this talk, a few results are described concerning Multiplicity Distributions (MD's) and fluctuations.

2 A short description of discrete QCD

Here I will present only the fundamental ideas behind this model; for a detailed discussion, follow the pointers in the bibliography.[2] Consider an event in which a quark-antiquark pair is produced: the cross section for the emission of gluons by this pair is, in the Double Log Approximation (DLA), given by the "colour dipole radiation" formula:[3]

$$dP \approx \frac{C\alpha_s}{\pi} \frac{dk_\perp^2}{k_\perp^2} dy \tag{1}$$

where C is a constant depending on the colour charges of the partons at the end points of the dipole, and y and $k_\perp$ are the rapidity and transverse momentum of the emitted gluon, respectively. Notice that the azimuthal direction is here integrated over, but it is of course possible to take it fully into account. The phase space limits for the gluon emission are given approximately by

$$|y| < \log(W/k_\perp) \tag{2}$$

where W is the dipole mass (which in the case of the original $q\bar{q}$ dipole equals the c.m. energy); the rapidity range available for the first gluon emission is

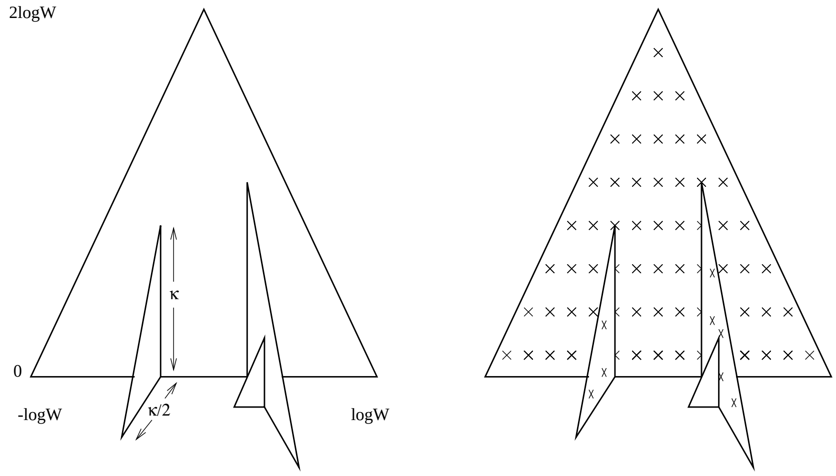

Figure 1: **left)** Phase space in the Dipole Cascade Model. The dimensions shown are y and $\kappa = \log k_\perp^2$, respectively rapidity and log of transverse momentum, taken in each dipole's c.m. frame. **right)** Phase space in the discrete QCD model. Emission of "effective gluons" can only take place at the points marked with a cross.

thus $\Delta y = \log(W^2/k_\perp^2)$. After the first emission, the three partons form two independent dipoles: they will in turn emit gluons according to Eq. 1 (in the c.m. frame of each dipole) and the cascade develops.[4] This is illustrated in Figure 1 (left part) where it is also shown that, because the gluons carry colour charges, the phase space actually increases with each emission: a new double faced triangular region is added for each new gluon emitted; notice that according to Eq. 2 the baseline of each face added is half the height of the same face. The Modified Leading Log Approximation (MLLA) changes this picture in that it provides a region of phase space, close to the end points of the dipole, where gluon emission is suppressed.[5] Thus the effective rapidity range becomes smaller: for example, for a gg dipole one has for large W:

$$\Delta y = \log \frac{W^2}{k_\perp^2} - \delta y_g \tag{3}$$

It can be argued,[2,6] from the form of the beta term in the Callan-Symanzik equation, that δy_g equals 11/6. Dipole splitting with ordering in $k_\perp$ can also be shown to be equivalent to the angular ordering prescription in standard parton cascades.[2,5]

In the discrete QCD model,[2] the idea expressed in Eq. 3 is carried to its extreme consequences: it is assumed that if two gluons are closer than δy_g they will effectively be re-absorbed into a single gluon. One needs then only consider "effective gluons": the "rapidity" dimension of phase space is thus partitioned in discrete steps of size δy_g. However, this is true also for every triangle "added" to the original phase space, which means that discrete steps must be taken also in the vertical dimension, i.e., in $\kappa \equiv \log k_\perp^2$: phase space is thus partitioned in rectangular cells of size $\delta y_g \times 2\delta y_g$. In a complementary way, one can say that emission of effective gluons takes place only at a discrete lattice of points, δy_g units apart horizontally and $2\delta y_g$ units apart vertically (see Figure 1). From the very structure of the cascade described above, it follows that each "column" of cells is independent of the others: such a column will be called a *tree* because of the possibility of emitting further triangles ("branches"). The probability of emitting a gluon with transverse momentum κ in such a cell can be obtained from Eq. 1 using the leading order expression for the running coupling constant α_s and the appropriate colour factor for the $g \to gg$ vertex:

$$dP = \frac{1}{\delta y_g} \frac{d\kappa}{\kappa} \delta y_g = \frac{d\kappa}{\kappa} \tag{4}$$

Normalizing with a Sudakov factor, the distribution in κ for an effective gluon at a given rapidity is uniform:

$$d\text{Prob} = \exp\left\{ -\int_\kappa^{\kappa_{\max}} dP \right\} dP = \frac{d\kappa}{\kappa_{\max}} \tag{5}$$

where $\kappa_{\max}$ corresponds to the maximum $k_\perp$ available at that rapidity. Each triangle emerging from the original one can then be described by the position and length of its baseline (both as integer multiples of δy_g): an events is a collection of independent trees of different height, the number of trees being given by $L = 2\log(W/\delta y_g)$. The maximum height H_j^L of each tree (numbering, e.g., the trees from left to right) depends only on the tree's position j and on the event "baseline length" L. A tree of height H corresponds to an emitted gluon of transverse momentum $\kappa = H \cdot 2\delta y_g$. A tree of height 0 corresponds to the emission of no gluons; notice that this introduces a natural cut-off for cascade evolution. Notice also that only a finite number of gluons can be emitted. These points are important in characterizing discrete QCD.

In order to calculate the multiplicity distribution, one can now define an *N-tree* as a tree with height between 0 and $N-1$ (in units of δy_g) where N is an integer. It is convenient to define also a *true N-tree* as a tree of height $N-1$. Because of Eq. 5, an N-tree is obtained by summing true H-trees ($H = 1, \ldots, N$) with equal probability $1/N$. This is easily expressed in terms of the generating functions for the MD's in an N-tree, $\mathcal{P}_N(z)$, and in a true N-tree, $\mathcal{Q}_N(z)$:

$$\mathcal{P}_N(z) = \frac{1}{N} \sum_{H=1}^{N} \mathcal{Q}_H(z) \tag{6}$$

It will also be recognized that a true $(N+1)$-tree can be obtained from a true N-tree by attaching two N-trees to it (one on each side)

$$\mathcal{Q}_{N+1}(z) = \mathcal{Q}_N(z)\,[\mathcal{P}_N(z)]^2 \tag{7}$$

Using the obvious initial conditions:

$$\mathcal{P}_1(z) = \mathcal{Q}_1(z) = 1; \qquad \mathcal{Q}_2(z) = z; \qquad \mathcal{P}_2(z) = \frac{1+z}{2} \tag{8}$$

Eq.s 6 and 7 can be solved,[2] thus giving the MD in a tree.

3 Calculations in rapidity intervals

In order to approach the calculation in rapidity intervals, one should understand that the size of δy_g does not imply that there is no structure at distances smaller than 11/6. In fact, δy_g correspond to a separation in rapidity along the original $q\bar{q}$ axis (which one can identify to a good approximation with the thrust axis of the event) only for trees which 'grow' from the original baseline, while inside each branch, δy_g is a distance along the local 'jet' direction. Thus one can say that gluons belonging to the same tree are all localized in rapidity inside one unit of δy_g. That this is actually a good approximation has been checked with a Monte Carlo implementation of the discrete QCD model. In this framework one can then calculate observables integrated over intervals of rapidity whose width is a multiple of δy_g, i.e., which contain a whole number of trees. So one recognizes that the MD in any of these rapidity interval is given by $\mathcal{P}_{H_j^L}(z)$. Adjacent trees are independent, and so are adjacent rapidity intervals in this approximation; thus the MD generating function in an interval Δy is given by

$$\mathcal{P}_{\Delta y}(z) \;=\; \prod_{j \in \Delta y} \mathcal{P}_{H_j^L}(z) \tag{9}$$

where the product is over all trees inside the interval Δy.

Figure 2, on the left, shows the resulting MD in different central rapidity intervals: the solid line shows a negative binomial distribution (NBD), phenomenologically widely used for describing the data.[1] The same figure shows on the right the MD for what I will call in the following "2-jet like" events,

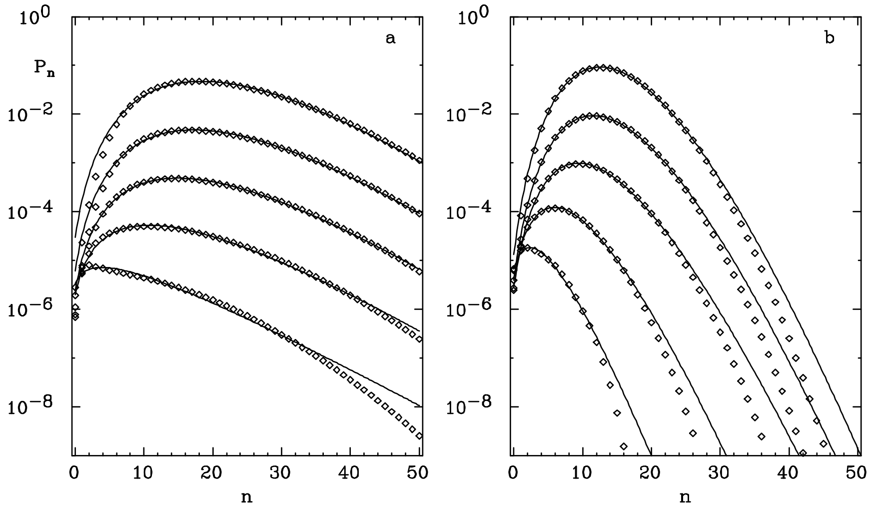

Figure 2: **a)** Multiplicity distributions in central intervals of rapidity of size (from bottom to top) 2, 4, 6, 8 and 10 (in units of δy_g) for standard events with baseline length $L = 12$. Each distribution is divided by 10 with respect to one above it in order to make the figure legible. **b)** Same as in a), but for "2-jet like" events.

obtained by cutting away the tip of the phase space triangle, i.e., requiring a cut-off on the *maximum* $k_\perp$ of an emitted gluon. Notice that in the standard events, for small intervals, the NBD behaves w.r.t. the points in a way similar to the way it behaves w.r.t to experimental data on hadronic MD, going first below the data, than above and then below again.[7] In the 2-jet like events, on the other hand, the NBD is closer to the points than in the standard events: one is reminded of the experimental result that a NBD is found to describe the hadronic MD in 2-jet events (defined via a jet-finder algorithm) better than in the full sample.[8]

In the literature, the NBD is often explained in terms of clan structure:[9,10] independent emission of clans, according to a Poisson distribution, followed by particle emission from each clan in a cascade way (logarithmic distribution). In the framework of discrete QCD, it is tempting to notice that each tree which comes from the baseline of the original triangle is independent from the others; a tree which produces no partons is not "observed", so the number of trees in a rapidity interval is variable. One can therefore call trees of height larger than 0 *non-empty trees*; by definition these contain at least one parton.

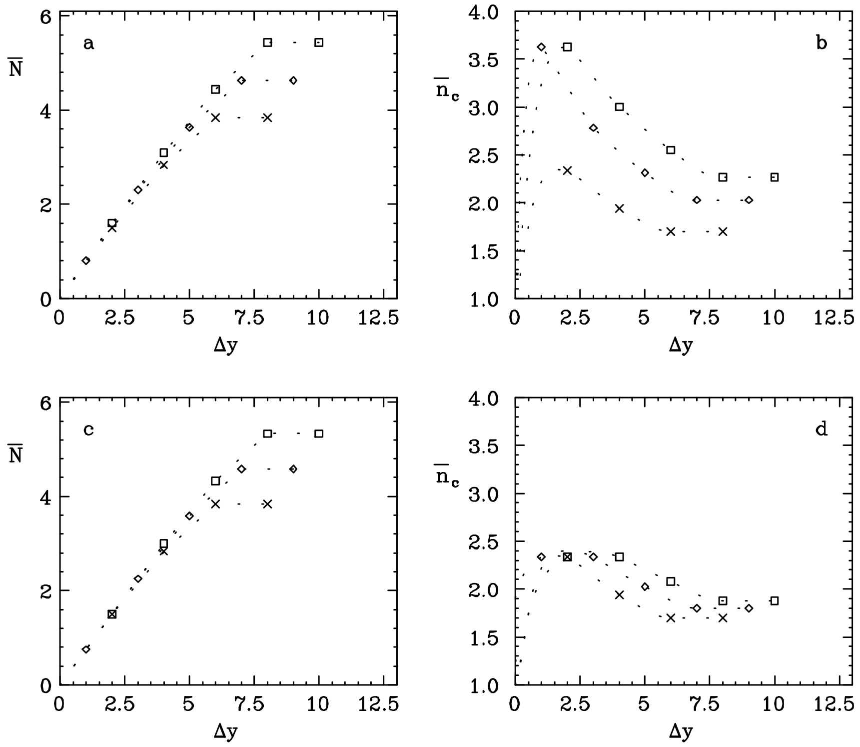

Figure 3: **a)** Average number of non-empty trees in rapidity intervals vs size of the interval (in units of δy_g), for three sizes of the event: $L = 8$ (crosses), $L = 9$ (diamonds) and $L = 10$ (squares). **b)** Average multiplicity in an average non-empty tree vs size of the rapidity interval for the said event sizes. **c)** As in a), but for 2-jet like events. **d)** As in b), but for 2-jet like events.

Consider now an event with L trees:

$$x_j \equiv \mathcal{P}_{H_j^L}(0) = \frac{1}{H_j^L} \tag{10}$$

is the probability that the j-th tree is empty. The MD in the number of non-empty trees n is given by the product of the probabilities that $L - n$ trees are empty and n are not:

$$P_n^{(L)} = \sum_{\text{perm}} (1 - x_1) \dots (1 - x_n) x_{n+1} \dots x_L \qquad n = 0, \dots, L \tag{11}$$

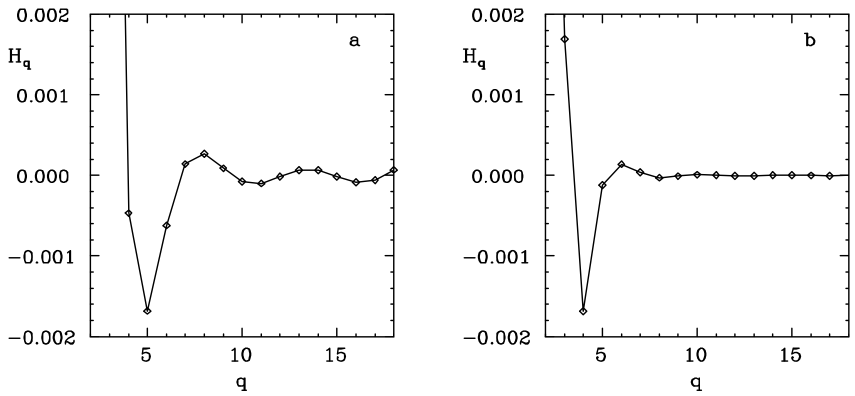

Figure 4: **a)** Ratio of factorial cumulant moments to factorial moments, H_q, vs the order of the moments, q, for standard events of baseline length $L = 10$. **b)** Same as in a) but for 2-jet like events.

where the sum is over all permutations of x's. It can be shown that $P_n^{(L)}$ satisfies the recurrence relation

$$P_n^{(L)} = \sum_{j=1}^{n} \frac{(-1)^{j+1}}{n} a_j P_{n-j}^{(L)} \tag{12}$$

where

$$a_r = \sum_{i=1}^{L} \left(\frac{1}{x_i} - 1\right)^r \qquad \text{and} \qquad P_0^{(L)} = \prod_{i=1}^{L} x_i \tag{13}$$

From these relations $P_n^{(L)}$ can be calculated; it is easy to show that one obtains a binomial distribution when x_j is independent of j, and a Poisson distribution when $a_r = \delta_{1,r}$: the former is incompatible with Eq. 6, but the latter is a good approximation when x_j is not very small. Figure 3 shows the result for the average number of non-empty trees $\bar{N}(\Delta y)$ and for the average multiplicity within an average non-empty tree $\bar{n}_c(\Delta y)$ (defined as the ratio of the average multiplicity to $\bar{N}(\Delta y)$) for standard events. The noticeable features for the average number of non-empty trees are an independence from the size L of the event in small intervals, and a linearity with the size of the interval at fixed L. Both these characteristics are seen in experimental data with the statistical definition of clans.[7,11]

Finally, in Figure 4 the results of a calculation in full phase space of the ratio H_q of factorial cumulant moments to factorial moments are shown: these where shown to be sensitive to the structures in the MD,[12,13] and in particular to the radiation of hard gluons in the early stages of the partonic evolution. The figure shows that oscillations in sign of the ratio H_q appear in the discrete QCD model, and when examined in the 2-jet like sample these oscillations diminish strongly in amplitude, in accordance with the behaviour of data.[12]

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