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STRUCTURES IN MULTIPLICITY DISTRIBUTIONS AND OSCILLATIONS OF MOMENTS

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ABSTRACT

The possibility to relate multiplicity distributions and their moments, as measured in the hadronic final state in e^+e^- annihilation, to features of the initial partonic state is analyzed from a theoretical and phenomenological point of view. Recent developments on the subject are discussed.

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1 Introduction

One of the long standing problems in multiparticle dynamics is the description of multiplicity distributions and correlation functions within a common formalism which can emphasize the dynamical processes underlying multiparticle production. In this talk, I will illustrate one step in this direction, by discussing how the sign oscillations of the ratio of factorial cumulant moments to factorial moments of the Multiplicity Distribution (MD) can be related to the dynamics of the early stage of partonic evolution in e^+e^- annihilation.

In Section 2 the relevant observables are defined and illustrated with examples; in Section 3 and 4 theoretical calculations and experimental results are reviewed; in Section 5 the shoulder structure in the MD and the oscillations of moments are related to hard gluon radiation; finally in Section 6 the effect of flavour quantum numbers on the moments of the MD in 2-jet events is discussed; conclusions are drawn at the end.

2 Definitions and examples: the effect of truncation

In this section the definitions of some differential and integral observables and their relationships will be recalled; see reference [1] for further details. The definitions, summarized in Table 1, concern n-particle distributions: the *exclusive* ones, $P_n(y_1, \dots, y_n)$ where *all* n particles produced at rapidities $y_1, \dots, y_n$ are observed, and the *inclusive* ones, $Q_n(y_1, \dots, y_n)$ where *at least* n particles are observed. These quantities are of course related:

$$P_n(y_1, \dots, y_n) = Q_n(y_1, \dots, y_n) + \sum_{m=1}^{\infty} \frac{(-1)^m}{m!} \int dy'_1 \dots dy'_m Q_{n+m}(y_1, \dots, y_n, y'_1, \dots, y'_m) \quad (1)$$

From the inclusive distributions, via cluster expansion, one gets the *correlation functions* $C_n(y_1, \dots, y_n)$, thus subtracting from Q_n the statistical, uninteresting correlations due to combinations of lower order distributions $Q_{n-1} \dots Q_1$; however there remain correlations related to fluctuations in the number of particles n:[2] it is therefore only in an approximate sense that one can say that when C_2 is positive particles like to cluster together (which one relates to the dynamics of the process) and when C_2 is negative particles like to stay away from each other (which one relates to the effect of conservation laws).

By integrating the differential observables just discussed one obtains the integral observables which will be the subject of the rest of this talk (see Table 1): the *multiplicity distribution*, the *factorial moments* and the *factorial*

Table 1: Definitions of relevant differential (on the left) and integral (on the right) quantities; σ is the inclusive cross section, σ_{inel} the inelastic cross section, y_i is the rapidity of the i-th particle. The integrals in the right-hand column are over the full phase space.

Exclusive distribution $P_n(y_1, \dots, y_n)$	Multiplicity distribution $P_n = \frac{1}{n!}\int dy_1 \dots dy_n P_n(y_1, \dots, y_n)$
Inclusive distribution $Q_n(y_1, \dots, y_n) = \frac{1}{\sigma_{\text{inel}}}\frac{d^n\sigma}{dy_1 \dots dy_n}$	Factorial moments $F_n = \int dy_1 \dots dy_n Q_n(y_1, \dots, y_n)$
Correlation function, *e.g.* $C_2(y_1, y_2) = Q_2(y_1, y_2) - Q_1(y_1)Q_1(y_2)$	Factorial cumulant moments $K_n = \int dy_1 \dots dy_n C_n(y_1, \dots, y_n)$

cumulant moments. They are linked by the following relationships, analogous to Eq. 1, which allow to obtain the moments directly from the MD:

$$F_n = \sum_{r=n}^{\infty} r(r-1)\cdots(r-n+1)P_r \tag{2}$$

$$K_n = F_n - \sum_{r=1}^{n-1} \binom{n-1}{r} K_{n-r}F_r \tag{3}$$

It is apparent from the definitions that F_n and K_n receive contributions from events with at least n particles, which means that moments of high order are very sensitive to the tail of the MD. In particular, as will be shown in the following, it is interesting in this respect to study the behaviour of the ratio

$$H_q \equiv \frac{K_q}{F_q} \tag{4}$$

as a function of the order q: it is qualitatively different for different distributions and turns out to be suitable for analytical calculations (see Section 3). In order to illustrate these properties, Figure 1 (left column, dashed lines) shows the ratio H_q for some of the most common discrete distributions. It is worth pointing out that for the Poisson distribution, which one usually associates with a lack of dynamical correlations, the ratio H_q is zero for $q > 1$; for the geometric and for the negative binomial distributions, which one usually associates with dynamical effects because they give rise to positive correlations, the ratio H_q is always positive, but decreases toward zero; for the binomial

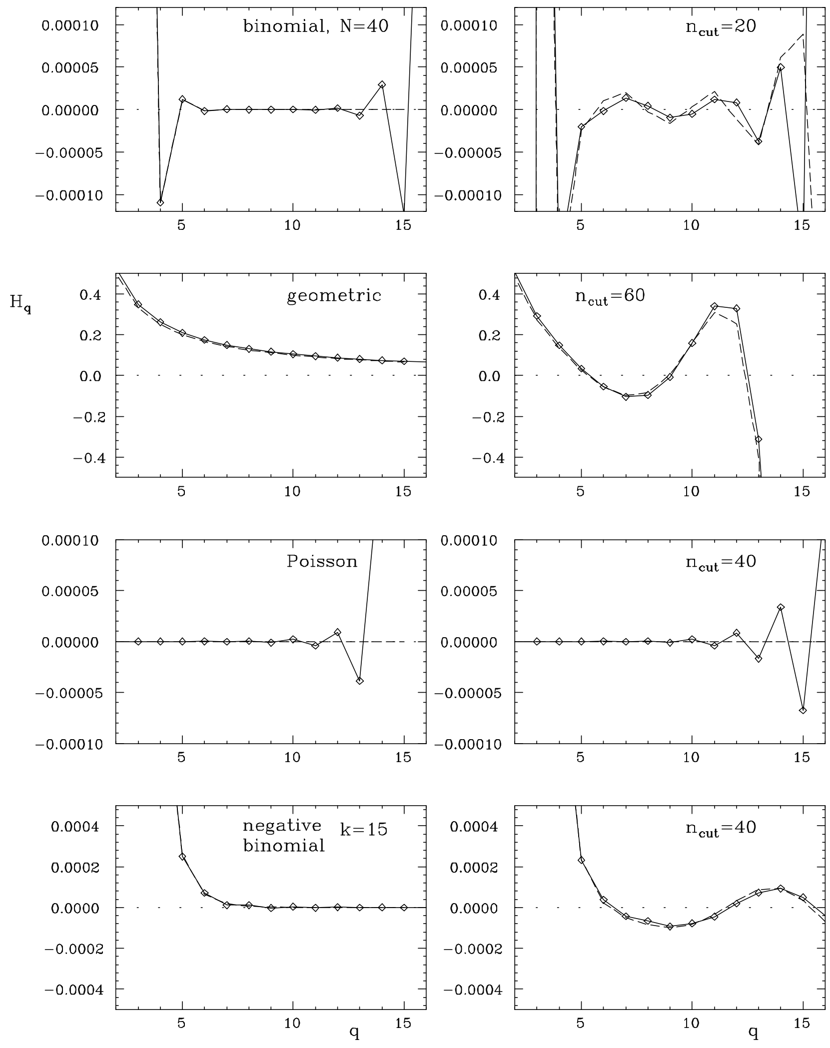

Figure 1: The ratio H_q for the most common discrete distributions. All the distributions are chosen to have the same average number of particles (namely, $\bar{n} = 10$); other parameters, if any, are listed in the figure. The plots in the left column correspond to the full distributions (dashed lines), and to the even-only distributions (solid lines joining the diamonds). The plots in the right column correspond to the same distribution after truncation has been taken into account as in Eq. 5; the effect of the truncation changes with the highest multiplicity $n_{\mathbf{cut}}$:[3] the values chosen here are such that the discarded part is always less than 1% of the cumulative distribution function.

distribution, which one usually associates with conservation laws because of negative correlations, the ratio H_q changes sign according to the parity of q. Before applying these considerations to the data in full phase space, one should remember that only even multiplicities are allowed for charged particles. In order to show how the suppression of the odd multiplicities affects the moments, the left column in Figure 1 presents also the ratio H_q vs the order q subject to the condition that $P_n = 0$ if n is odd (solid lines and points): it is seen that there is a small distortion with respect to the values of the full distributions.

As already mentioned, the ratio H_q probes the tail of the MD. Unfortunately, the tail is the most difficult part to measure experimentally, because only a finite number of events can be collected. This results in a MD which is truncated at some point:

$$\tilde{P}_n \propto \begin{cases} P_n & \text{if } n \le n_{\rm cut} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where the proportionality factor ensures proper normalization. The moments of high order are affected: the factorial moments are smaller than in the full distribution, and one finds that the factorial cumulant moments oscillate in sign as the order increases.[3] An example of this effect can be seen again in Figure 1; its importance for experimental MD's will be discussed in Section 4.

3 The ratio H_q in perturbative QCD

The first suggestion that the ratio H_q could be useful in the study of MD's came from analytical calculations in perturbative QCD,[4,5] where it was shown that one expects oscillations of the ratio H_q as a function of the order q. In the following I will summarize the derivation in the framework of pure gluodynamics,[4] starting from the evolution equations for the generating function $\mathcal{G}(\mathcal{Y};z) = \sum_{n=0}^{\infty}(1+z)^n P_n(\mathcal{Y})$:

$$\frac{d\mathcal{G}(\mathcal{Y};z)}{d\mathcal{Y}} = \int_0^1 dx \left(\frac{1}{x} - (1-x)[2 - x(1-x)]\right) \left(\frac{2\mathcal{N}_c \alpha_s(\mathcal{Y})}{\pi}\right) \times \\ \left[\mathcal{G}(\mathcal{Y} + \ln x; z)\mathcal{G}(\mathcal{Y} + \ln(1-x); z) - \mathcal{G}(\mathcal{Y};z)\right] \tag{6}$$

where the first term in parentheses is the DGLAP kernel for the process $g \to gg$ with the emitted gluon carrying a fraction x of the parent momentum, $\mathcal{Y} = \ln(k_\perp/Q_0)$ is the evolution variable ($k_\perp$ the jet's transverse momentum and Q_0 the cutoff, which limit the integration interval via $x(1-x)k_\perp > Q_0$), $\alpha_s(\mathcal{Y})$ is the running coupling constant and $\mathcal{N}_c$ is the number of colours. Notice that the terms in the square brackets take recoil into account, thus going beyond the pure Double Log Approximation (DLA). In order to solve Eq. 6 for the moments of the distribution one expands it in powers of $\mathcal{Y}$, remembering that, when calculating correlations of order q, one should use $q\gamma$ as expansion parameter (because the NLO contribution to such correlations is proportional to $q\gamma$, rather than γ; γ is here the anomalous dimension). Assuming asymptotic KNO scaling, so that only the average number of partons depends on $\mathcal{Y}$, while the normalized moments of higher order are constant as $\mathcal{Y} \to \infty$, one calculates now the derivatives in $z = 0$ of the generating function $\mathcal{G}$, obtaining the factorial moments; the factorial cumulant moments are then computed via Eq. 3. The normalization of these moments is not fixed but can be eliminated by taking their ratio. One finds for the ratio H_q a negative minimum at $q \approx 5$ and sign oscillations at larger q. This result has been qualitatively improved by including an estimate of the contribution of the vertices $q \to qg$ and $g \to q\bar{q}$;[6] further confirmation comes from the exact solution of the full evolution equations including both quarks and gluons in the case of fixed coupling.[7] It could be noted that the oscillations found in this last case happen around the (monotonically decreasing) value of H_q of a negative binomial distribution (NBD) with $k \approx 5$. On the other hand, it is interesting to point out that in DLA the oscillations disappear and the ratio H_q is very close to that of a NBD with $k \approx 3$.

However, before doing comparison of partonic results with experimental data, one should investigate what the possible role of hadronization is. Here I will just recall[8] that if the simplest possible Generalization to Local Parton Hadron Duality[9] (GLPHD), which requires the proportionality of all inclusive distributions at parton (p) and hadron (h) level via a single parameter ρ:

$$Q_n^{\mathrm{h}}(y_1, \dots, y_n) = \rho^n Q_n^{\mathrm{p}}(y_1, \dots, y_n), \tag{7}$$

is used, it is easy to show, by direct substitution into the formulae of Table 1, that one obtains:

$$H_n^{\mathrm{h}} = \frac{\rho^n K_n^{\mathrm{p}}}{\rho^n F_n^{\mathrm{p}}} = H_n^{\mathrm{p}} \tag{8}$$

Thus, if the GLPHD hadronization prescription is used, the parton level result on the ratio H_q can be directly applied to the hadrons: in this case, it is seen that the QCD prediction for e^+e^- annihilation[6] fails quantitatively.

4 The ratio H_q in experiments

The theoretical work described in the previous Section triggered the analysis (a posteriori) of available data on MD's in order to extract the features of the

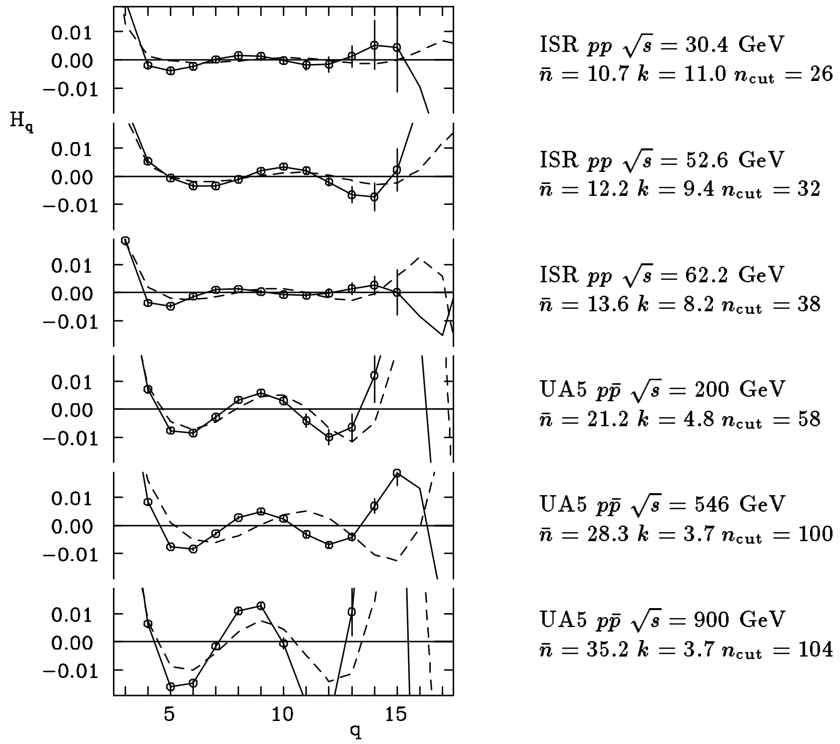

Figure 2: The ratio H_q vs the order q for a collection of experiments in hadron-hadron collisions.[12] The solid line joins data points and is drawn to guide the eye only. The dashed line shows the prediction of the NBD which fits well the MD's, after the truncation effect has been taken into account.

ratio H_q.[10] It should be immediately said that this is not an easy task: in order to measure the MD in a modern experiment an 'unfolding' procedure has to be used;[11] the resulting published MD has correlations between adjacent bins which are not taken into account when moments are extracted from it without knowledge of the correlation matrix. This makes the calculation of the errors on the moments thus obtained very difficult: it should be considered as an order of magnitude estimate rather than a precise determination. Keeping this in mind, one can proceed to review some experimental results. The main point here is that oscillations of very different amplitudes are seen in the data in all reactions, in most cases (but not all! see below) compatible with being due to the truncation of the MD. As an illustration, in Figure 2 some results relative to hadron-hadron collisions are shown: large oscillations are found. The MD's from which these moments are extracted are well described (with the exception of UA5 data at $\sqrt{s} = 900$ GeV) by a NBD. Also shown in the same figure is the ratio H_q predicted by the fitted NBD after taking into account the effect of truncation (as in Eq. 5): it is seen that the oscillations thus obtained are of the same order of magnitude as the data, so that no further dynamical effects, beyond those described by a single NBD, are apparent (see also the analysis in [13]).

The case is different in e^+e^- annihilation data at the Z^0 peak, as exemplified in Figure 3, where data from the SLD Collaboration [14] are compared with the predictions of a NBD (dotted line) and a truncated NBD (dot-dashed line). It should be mentioned that in this plot the errors on the data are the statistical errors as published by the SLD Collaboration, which take into full account all correlations from the unfolding matrix. Clearly the truncated distribution cannot describe the data: it will be shown in the next section how one can relate these oscillations to hard gluon radiation.

5 Common origin of the shoulder structure and of the oscillations

A very interesting feature in the data on charged particles MD's at the Z^0 peak is the shoulder structure which is clearly visible in the intermediate multiplicity range.[15,16,17] This peculiar behaviour appears evident if one looks at the residuals (difference between the data and fitted values, in units of standard deviations) with respect to a NBD (or even a Lognormal Distribution):[14] the curve starts below the data, goes above and then below again. A pattern is seen, instead of a random sign and size of residuals.

The DELPHI Collaboration has shown[18] that the shoulder structure in the MD in e^+e^- annihilation can be explained by the superposition of the MD's coming from events with 2, 3 and 4 jets, as identified by a suitable jet-finding algorithm, and that the MD's in these classes of events are well reproduced by a single NBD for a range of values of the jet-finder parameter, $y_{\rm min}$. It is thus suggested that the shoulder is associated with the radiation of hard gluons resulting in the appearance of one of more extra jets in the hadronic final state. One should also recall that a shoulder structure similar to the one observed in e^+e^- annihilation has been observed in $p\bar{p}$ collisions at high energies [19,20] and was shown to be well described by a 5-parameter parametrization in terms of the weighted superposition of two NBD's.[21]

Following these observations, a parametrization of the MD which is the weighted sum of two components, one to be associated with 2-jet events and one to be associated with events with 3 or more jets, was proposed.[22] The weight

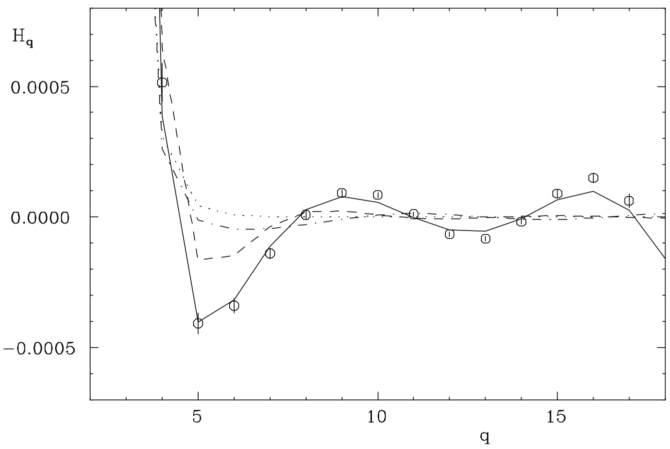

Figure 3: The ratio of factorial cumulant moments over factorial moments, H_q as a function of q; experimental data (diamonds) from the SLD Collaboration[14] are compared with the predictions of several parameterizations, with parameters fitted to the data on MD's: a full NBD (dotted line); a truncated NBD (dot-dashed line); sum of two full NBD's as per Eq. 9 (dashed line); sum of two truncated NBD's as per Eq. 10 (solid line).

in this superposition is then the fraction of 2-jet events, which is experimentally determined, not a fitted parameter. This decomposition depends of course on the definition of jet, and in particular on the value of the parameter which controls the jet-finding algorithm. The DELPHI Collaboration[18] has used the JADE algorithm and published values for the 2-jet fraction and for the MD's at $y_{\min} = 0.02$, 0.04, 0.06, 0.08.

As for the particular form of the MD in the two components, the NBD was chosen because it has successfully been fitted to the data for the samples of events with fixed number of jets.[18] In practice a fit was performed to the MD's with a four parameter formula:

$$P_n \propto \begin{cases} \alpha P_n^{\rm NBD}(\bar{n}_1, k_1) + (1-\alpha) P_n^{\rm NBD}(\bar{n}_2, k_2) & \text{if } n \text{ is even} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Here $P_n^{\rm NBD}(\bar{n}, k)$ is the standard NBD of parameters $\bar{n}$ and k; notice that the charge conservation law is taken into account. The proportionality factor is fixed by requiring the proper normalization for P_n.

Results of the fit to the data of four experiments can be summarized as follows: the χ^2 per degree of freedom are equal to or smaller than 1, and values of the parameters are consistent between different experiments; they are also consistent with those obtained by the Delphi Collaboration in fitting their 2-jet and 3-jet data separately with a NBD.

Once the fits to the MD's have been done, one can compare the experimental data on H_q's with the values obtained from the fitted MD's by using a formula that takes into account the truncation effect too:

$$\tilde{P}_n \propto \begin{cases} P_n & \text{if } (n_{\min} \leq n \leq n_{\max}) \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where $n_{\min}$ and $n_{\max}$ are the minimum and maximum observed multiplicity, and a proportionality factor ensures proper normalization. A very good agreement with the data is obtained, as exemplified in Figure 3 (solid line). Notice that it is not possible to reproduce the behaviour of the ratio H_q without taking into account the limits of the range of the available data: this can be seen again in Figure 3, where the dashed line corresponds to Eq. 9 and does not agree with the data.

It can thus be concluded that the observed behavior of H_q's results from the convolution of two different effects, a statistical one, i.e., the truncation of the tail due to the finite statistics of data samples, and a physical one, i.e., the superposition of two components. The two components can be related to 2- and 3-jet events, i.e., to the emission of hard gluon radiation in the early stages of the perturbative evolution. Notice that as the energy increases, the number of components also grows so that, asymptotically, the oscillations should be washed out, in agreement with the DLA expectations.

6 Effects of flavour in 2-jet events

A simple check of the picture described in the previous section consists in looking at the behaviour of the ratio H_q in 2-jet events, using the MD given by the DELPHI Collaboration:[18] one does not expect oscillations. It is found[10] that oscillations are present, although their amplitude is one order of magnitude smaller than in the full sample. If one looks at the MD itself in 2-jet events, one discovers that the residuals, with respect to a NBD, show a structure similar to the one seen in the full sample; furthermore, the oscillations in the ratio H_q cannot be described by truncating one NBD.[25] The hint that there could be a substructure comes once again from DELPHI data:[23] they showed that the MD in a single hemisphere in a sample enriched in $e^+e^- \to b\bar{b}$ events is identical in shape to the MD in a sample without flavour selection, except for a shift of 1 unit:

$$P_n^{(b)} = P_{n-1}^{(all)} \tag{11}$$

This effect could be due to weak decays of B-hadrons.[24] This result hints at a possible substructure in terms of events with heavy quarks and events with light quarks, each sample contributing to the MD with one NBD. One can therefore try a parametrization of the form[25]

$$P_n(\bar{n}_l, \bar{n}_b, k) = \alpha_b P_n^{\rm NB}(\bar{n}_b, k) + (1 - \alpha_b) P_n^{\rm NB}(\bar{n}_l, k) \tag{12}$$

where now α_b is the fraction of $b\bar{b}$ events at the Z^0 peak, measured at LEP to be approximately 0.22. Notice that, as suggested by the DELPHI result, the parameter k is the same in the two NBD's: this is therefore a fit with three parameters. The parameters of the fits and the corresponding χ^2/NDF are given in Table 2 for different values of the jet resolution parameter $y_{\rm min}$; the fit has been performed taking the charge conservation law into account similarly to Eq. 9. A really accurate description of experimental data is achieved. Notice that the best-fit value for the difference between the average multiplicities in the two samples, δ_{bl}, also given in Table 2, is quite large. This difference grows with increasing $y_{\rm min}$, i.e., with increasing contamination of 3-jet events. By looking at the residuals one concludes that the proposed parametrization can reproduce the experimental data on MD's very well, as no structure is visible.[25] Furthermore this parametrization describes well also the ratio H_q.[25] It is also remarkable that only the average number of particles depends on flavour quantum numbers, whereas the NBD parameter k is flavour-independent.

One can thus conclude that the examination of the behaviour of the ratio H_q as a function of q has allowed to link the final hadronic level with the flavour composition of the event.

Table 2: Parameters and χ^2/NDF of the fit to experimental data on 2-jet events MD's from the DELPHI Collaboration[18] with the weighted superposition of two NBD's with the same parameter k (Eq. 12); the weight used is the fraction of $b\bar{b}$ events. Results are shown for different values of the jet-finder parameter $y_{\rm min}$.

	$y_{\rm min} = 0.01$	$y_{\rm min} = 0.02$	$y_{\rm min} = 0.04$
$\bar{n}_l$	16.81±0.21	17.22±0.15	17.98±0.15
$\bar{n}_b$	20.26±1.71	21.96±1.57	23.61±1.64
k	124±51	145±53	120±33
χ^2/NDF	17.4/16	12.6/16	27.5/20
δ_{bl}	3.44±0.83	4.6±0.5	5.6±0.5

7 Conclusions

The ratio of factorial cumulant moments to factorial moments, H_q, is a good observable for exploring substructures in hadronic final states. It is possible to calculate its behaviour in perturbative QCD and, after making allowance for the effect of finite statistics in the data, to extract dynamical information about the first stages of the perturbative evolution. In fact, hard gluon radiation explains the substructures observed in the full sample of events in e^+e^- annihilation at the Z^0 peak; flavour dependent properties explain the additional substructures observed in the sample of 2-jet events. It is also remarkable that in this analysis the most elementary substructures are well described by negative binomial distributions (down to 2-jet events of fixed flavour). A final point should be made, as the fits discussed in this talk have been performed on the published data, and therefore could not take into account the full correlations: the hope is that the interesting results thus obtained will spawn experimental work on the original data.

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