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A matching of matrix elements and parton showers

J. André and T. Sjöstrand[1]

Department of Theoretical Physics,
Lund University, Lund, Sweden

Abstract

We propose a simple scheme to start a parton-shower evolution description from a given jet configuration in e^+e^- annihilation events. This allows a convenient combination of the full angular information content of matrix elements with the detailed sub-jet structure of parton showers, and should give a realistic overall description of event properties. Explicit studies with this hybrid approach are presented for the four-jet case, as a simple testing ground of the ideas.

[1]torbjorn@thep.lu.se

In modern high-energy particle physics, the ability to perform precision measurements or tests of fundamental concepts often requires an accurate modelling of multiparticle topologies. These events have their origin in the perturbative production of sets of partons — quarks and gluons — that subsequently hadronize nonperturbatively to produce the observable final state. The first step therefore is to obtain an accurate description of the perturbative production of multiparton topologies. Basically, there exists two approaches to this problem.

The standard method is the *matrix element* one. In this approach the relevant Feynman graphs are calculated, order by order in α_s, until the required accuracy is ensured. Taking the convenient example of the lowest-order process $e^+e^- \to \gamma/Z^0 \to q\overline{q}$, the first-order real production process is $e^+e^- \to \gamma/Z^0 \to q\overline{q}g$. The infrared and collinear gluon-emission divergences of the latter process are cancelled by the first-order virtual corrections to the lowest-order graph, to give a finite total cross section. In second order, two new processes appear, $e^+e^- \to \gamma/Z^0 \to q\overline{q}gg$ and $e^+e^- \to \gamma/Z^0 \to q\overline{q}q\overline{q}$, together with virtual corrections to the lower-order processes.

In principle, the matrix-element approach is the correct one, that gives the full information content of perturbative QCD. There are caveats, however. One is that the calculations become increasingly complex in higher orders. This in particular affects the virtual corrections, where subtle cancellations of singularities have to be performed between graphs corresponding to different number of phase-space dimensions. Therefore a full calculation of all virtual corrections only exists to second order in α_s [1]. Another caveat is that the matrix-element method is unreliable when applied to the exclusive production of two nearby partons. In an order-by-order calculation of such a fixed parton configuration, the virtual corrections are (to leading-log accuracy) expected to appear as an alternating series that sums up to give an exponential damping — a Sudakov form factor [2] . The closer a gluon is to another parton, the larger are the terms of this series, and the more terms are necessary to obtain a convergent answer.

The alternative approach is the *parton shower* one. Here the amplitude-based language of matrix elements is replaced by a simplified probabilistic one. The basic process $e^+e^- \to \gamma/Z^0 \to q\overline{q}$ is supplemented by standard QCD evolution-equation rules for subsequent branchings $q \to qg$, $g \to gg$ and $g \to q\overline{q}$. By repeated application of these simpler $1 \to 2$ branchings arbitrarily complicated multiparton topologies are generated. The shower approach obeys detailed balance — a branching transforms an n-parton configuration to an $n+1$ one — which implies that a Sudakov form factor enters in a natural fashion. While the traditional parton-shower approach formally is of leading-log accuracy only, in reality many next-to-leading effects are included: an event generator takes full account of energy–momentum conservation and recoil effects, it can be made to include angular ordering (coherence) of emissions [3], the α_s scale can be optimized based on knowledge of higher-order branching kernels [4], and azimuthal angles in branchings can be chosen non-isotropically to include both spin and coherence effects [5].

However sophisticated, the parton-shower approach still cannot be expected to cover the full information content available in the matrix-element expressions. This should be especially notable for situations when partons are well separated: here exact kinematics is important and several graphs can be expected to contribute comparably much, so that interference terms are significant.

The picture then is one where matrix elements are likely to provide the better description of the main character of events, i.e. the topology of well separated jets, while parton showers should be better at describing the internal structure of these jets. A marriage of the two approaches seems rather natural, but is actually not so simple. Only for first-order e^+e^- events is this routinely done. One approach [6] is to generate a normal parton shower starting from a $q\overline{q}$ topology, and reweight (by a rejection technique) the first branchings of the q and the $\overline{q}$ so that the first-order $q\overline{q}g$ three-jet matrix elements are reproduced. An improvement is to include weights in all branches [7]. A clever choice of cascading description can automatically lead to a good agreement also with four-jet matrix elements [8]. Another technique is to use matrix elements to generate a varying number of initial partons that then are allowed to shower further [9]. This strategy has also been applied to deep inelastic scattering [10]. The number of event classes to consider can be reduced if cut-offs are chosen appropriately [11].

In this letter we propose another strategy for combining matrix elements with parton showers. It has the advantage that it can be applied to arbitrarily complicated partonic states, but the disadvantage that it does not tell how to mix different event topologies consistently. Its main application therefore is to events where the bulk properties are given by matrix elements, i.e. where the main partons are well separated, and the task is to provide a realistic representation of the structure of the resulting jets. One such example is four-jet studies at LEP 1 to test the coupling structure of QCD, another four-jet studies at LEP 2 to control backgrounds to W^+W^- events.

The basic idea is to cast the output of matrix element generators in the form of a parton-shower history, that then can be used as input for a complete parton shower. In the shower, that normally would be allowed to develop at random, some branchings are now fixed to their matrix-element values while the others are still allowed to evolve in the normal shower fashion. The preceding history of the event is also in these random branchings then reflected e.g. in terms of kinematical or dynamical (e.g. angular ordering) constraints.

The idea is best exemplified by e^+e^- four-jet events. The process receives contributions from seven graphs, see Fig. 1. The $q\overline{q}gg$ and $q\overline{q}q\overline{q}$ graphs easily separate, so we can concentate on the former processes. Then the matrix-element expression contains contributions from five graphs, and from interferences between them. The five graphs can also be read as five possible parton-shower histories for arriving at the same four-parton state, but here without the possibility of including interferences. The relative probability for each of these possible shower histories can be obtained from the rules of shower branchings. For example, the relative probability for the history shown in Fig. 2 is given by:

$$\mathcal{P} = \mathcal{P}_{1\to 34}\mathcal{P}_{4\to 56} = \frac{1}{m_1^2}\frac{4}{3}\frac{1+z_{34}^2}{1-z_{34}} \cdot \frac{1}{m_4^2}3\frac{(1-z_{56}(1-z_{56}))^2}{z_{56}(1-z_{56})} \tag{1}$$

where the probability for each branching contains the mass singularity, the colour factor and the momentum splitting kernel. The masses are given by

$$\begin{aligned} m_1^2 = p_1^2 &= (p_3+p_5+p_6)^2 \,, \\ m_4^2 = p_4^2 &= (p_5+p_6)^2 \,, \end{aligned} \tag{2}$$

and the z values by

$$\begin{aligned} z_{bc} = z_{a\to bc} &= \frac{m_a^2}{\lambda}\frac{E_b}{E_a} - \frac{m_a^2-\lambda+m_b^2-m_c^2}{2\lambda} \\ \text{with } \lambda &= \sqrt{(m_a^2-m_b^2-m_c^2)^2-4m_b^2m_c^2} \,. \end{aligned} \tag{3}$$

The form of the probability matches the expression used in the JETSET parton-shower algorithm [12]. Other programs use the opening angle [9] or the transverse momentum [8] as main evolution variable instead of mass. The z definition (again the one used in JETSET) reduces to energy fractions in the limit that the daughters are massless, and corresponds to unchanged decay angle in the rest frame of the mother when daughter masses are introduced.

Variants on the above probabilities are imaginable. For instance, in the spirit of the matrix-element approach we have assumed a common α_s for all graphs, which thus need not be shown, whereas the parton-shower language normally assumes α_s to be a function of the transverse momentum of each branching [4]. One could also include information on azimuthal anisotropies [5].

The relative probability $\mathcal{P}$ for each of the five possible parton-shower histories can be used to select one of the possibilities at random. (A less appealing alternative would be a "winner takes all" strategy, i.e. selecting the configuration with the largest $\mathcal{P}$.) The selection fixes the values of the m, z and φ at two vertices. The azimuthal angle φ is defined by the daughter parton orientation around the mother direction. When the conventional parton-shower algorithm is executed, these values are then forced on the otherwise random evolution. This forcing cannot be exact for the z values, since the final partons given by the matrix elements are on the mass shell, while the corresponding partons in the parton shower might be virtual and branch further. The shift between the wanted and the obtained z values are rather small, very seldom above 10^{-6}. More significant are the changes of the opening angle between two daughters: when daughters originally assumed massless are given a mass the angle between them tends to be reduced. This shift has a non-negligible tail even above 0.1 radians. The "narrowing" of jets by this mechanism is compensated by the broadening caused by the decay of the massive daughters, and thus overall effects are not so dramatic.

All other branchings of the parton shower are selected at random according to the standard evolution scheme. In Fig. 2, this means that partons 2, 3, 5 and 6 (and any daughters) have random masses and branchings. There is an upper limit on the non-forced masses from internal logic, however. For instance, for four-parton matrix elements, the singular regions are typically avoided with a cut $y > 0.01$, where y is the square of the minimal scaled invariant mass between any pair of partons. Larger y values could be used for some purposes, while smaller ones give so large four-jet rates that the need to include Sudakov form factors can no longer be neglected. The $y > 0.01$ cut roughly corresponds to $m > 9$ GeV at LEP 1 energies, so the hybrid approach must allow branchings at least below 9 GeV in order to account for the emission missing from the matrix-element part. Since no 5-parton emission is generated by the second-order matrix elements, one could also allow a threshold higher than 9 GeV in order to account for this potential emission. However, if any such mass is larger than one of the forced masses, the result would be a different history than the chosen one. Thus one plausible strategy is to choose as threshold the smallest of the two forced masses. Another choice is a fixed mass threshold at 13 GeV, a value that gives the same average multiplicity as the original parton shower. This may be viewed as a pragmatical compromise between the two extremes above.

The resulting charged multiplicity distributions are shown in Fig. 3. The JADE P0 scheme [13] has been used to find four-jet events with a separation $y > 0.02$, and results are compared with DELPHI data [14]. The hybrid is executed with three different mass thresholds: fixed at 13 GeV (ME+PS I), fixed at 9 GeV (ME+PS II) and minimum of the two forced masses (ME+PS III). In all cases the partons have been hadronized with the string fragmentation scheme [15]. The distributions of the parton shower (PS) and the three hybrids nearly coincide, and agree with the data. The matrix elements distribution (ME) is peaked at significantly lower multiplicities, as should be expected from the lesser activity. This difference could be reduced by a retuning of the fragmentation parameters for the matrix-element approach, but this retuning would then have to be done separately at each energy, while the parton-shower tuned parameters should be valid at all energies and for all jet multiplicities.

The subjet multiplicity is shown in Fig. 4. This is the average number of jets as a function of $y < 0.03$, given that there are exactly four jets at $y = 0.03$. Thus the subjet multiplicity gives a picture of the width of the four original jets. The ME+PS events have been generated using the fixed mass threshold at 13 GeV. The curve corresponding to the matrix elements has the strongest tendency towards 4 jets, as could be expected, whereas ME+PS agrees rather well with the convenventional parton shower.

Angular distributions in four-jet events have been studied in order to test the predictions of QCD [16]. We therefore already know that these distributions are well described by the matrix-element approach, while the JETSET parton shower is somewhat less accurate [17]. Without entering into the (not easily reproducible) details of the experimental procedure, we can thus directly compare the hybrid approach with the ME and PS options. Fig. 5 gives the distribution of one such angle for four-jets at $y = 0.03$. As expected, the ME+PS curve here agrees well with the pure ME one.

In summary, we have therefore obtained a simple scheme that allows a convenient combination of the best aspects of the matrix-element and parton-shower approaches to the description of multihadronic e^+e^- annihilation events. The structure of well separated jets agrees with the matrix-element prediction, while the richer substructure of jets follows the parton-shower pattern. Further details on the algorithm and tests of it can be found elsewhere[18]; the four-parton program itself is also available [19].

The method has here been illustrated for four-jet events, but could easily be extended to any other fixed multiplicity. A possible weakness of the approach, however, is that it does not automatically tell how to mix different jet multiplicities in the proper proportions. One could use the matrix-element mixture as a starting point, and generate three- and two-jet events with a veto against too large masses, similarly to what was discussed above. This would not be unreasonable, but the Sudakov form factors of the pure parton-shower approach are then missing and therefore the transition between jet multiplicities is not as well controlled.

The same kind of approach could also be applied to other processes, such as three-jet production in hadron collisions, which could be viewed as a basic $2 \to 2$ scattering combined with one forced parton-shower branching. This branching could here occur either in the initial or in the final state, however, so there are more possibilities to keep track of. Furthermore, the cross section of the $2 \to 2$ scattering here needs to be included in the relative probability to select a given configuration, since this scattering varies between the possible event histories.

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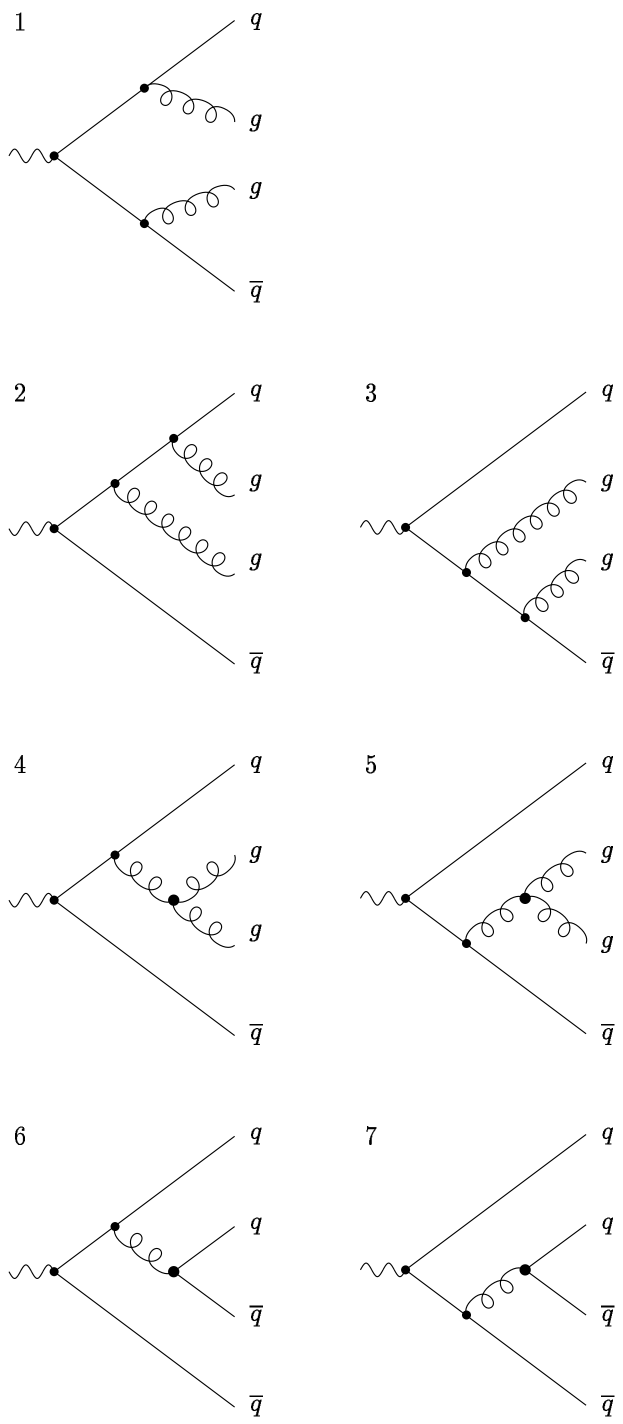

Figure 1: 4-parton histories to second order in α_s

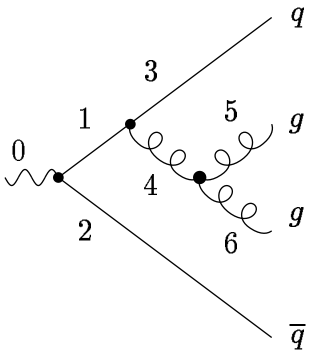

Figure 2: One 4-parton history

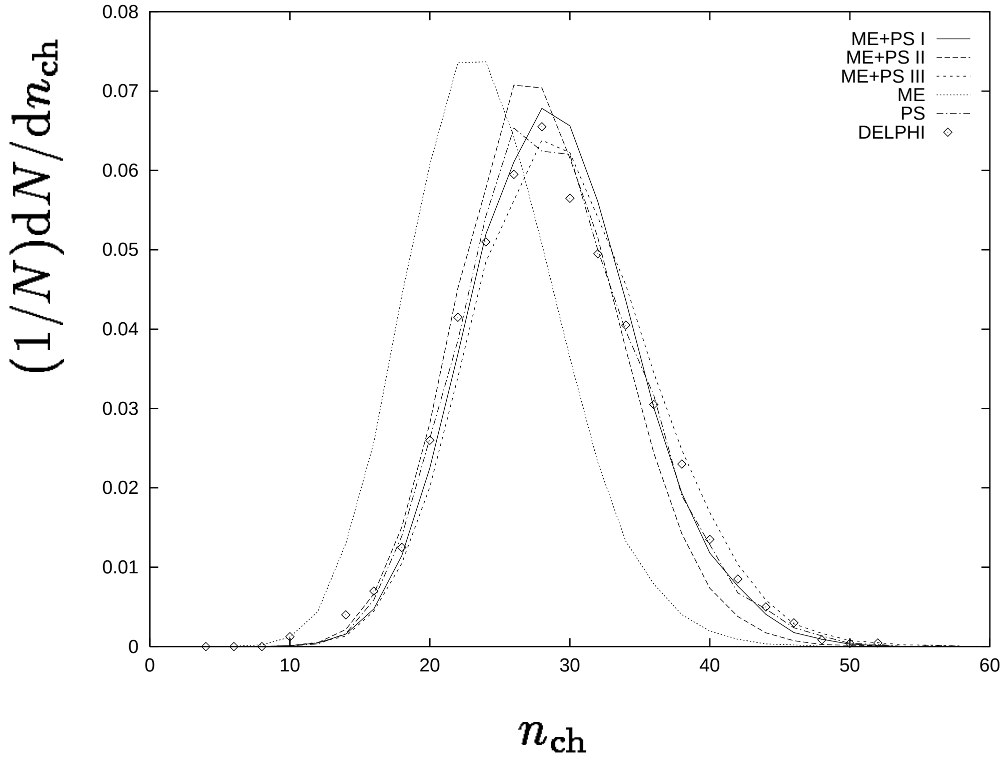

Figure 3: Distributions of charged particle multiplicity

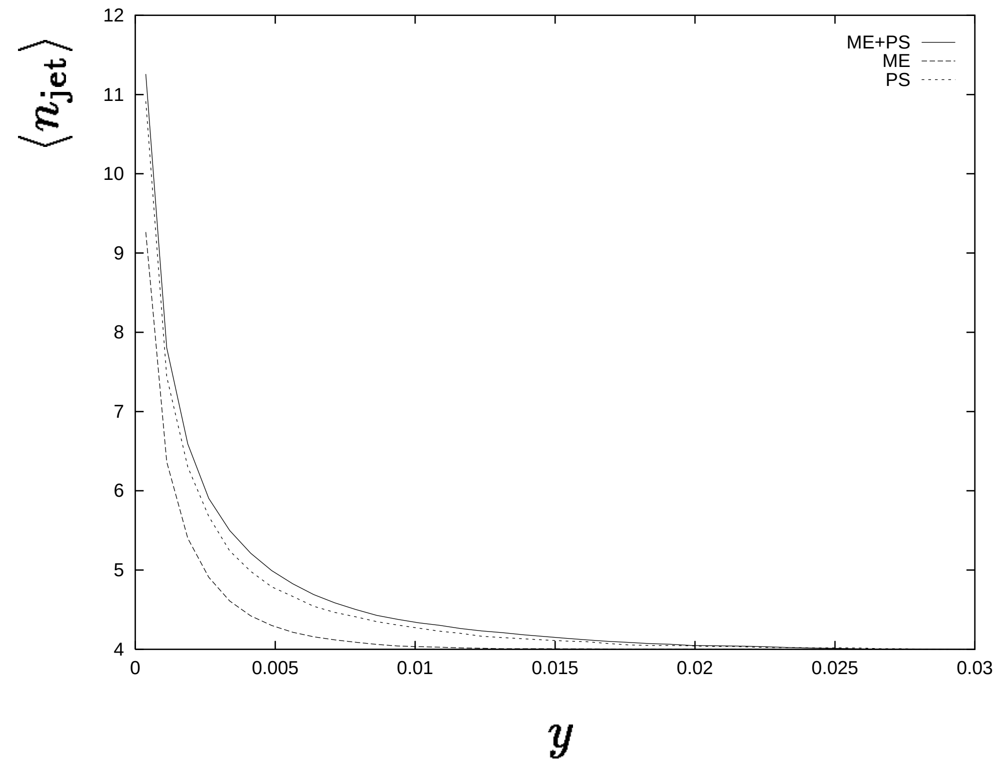

Figure 4: Subjet multiplicities

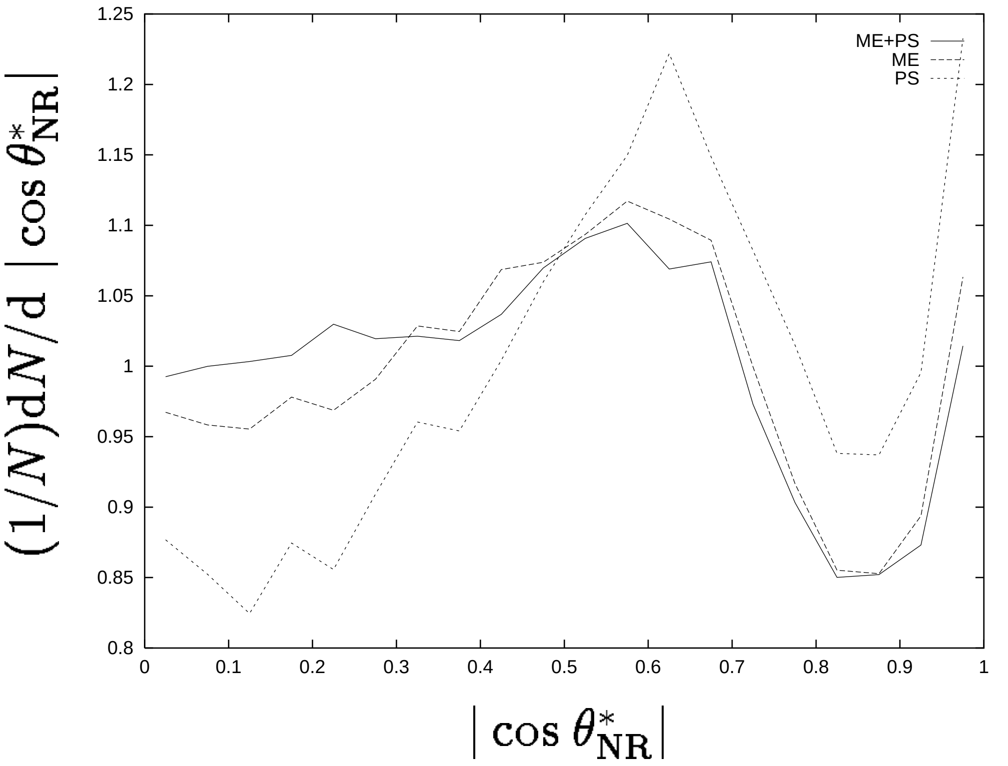

Figure 5: $|\cos\theta^*_{\mathrm{NR}}|$ distributions