LU TP 97/31
hep-ph/9710555
October 1997

Report of the Working Group on Goldstone Boson Production and Decay

J. Bijnens[1], G. Colangelo[2], G. D'Ambrosio[3], F. Gabbiani[4], G. Isidori[2], J. Kambor[5], A. Kupsc[6], M. Moinester[7], B. Nefkens[8], J. Oller[9], E. Pallante[10], J.R. Peláez[11], B. Renk[12] and P. Talavera[13]

[1] Convener, Department of Theoretical Physics 2, University of Lund, Sölvegatan 14A, S22362 Lund, Sweden
[2] INFN, Laboratori Nazionali di Frascati, P.O. Box 13, I-00044 Frascati, Italy
[3] INFN-Sezione di Napoli and Dipartimento di Scienze Fisiche, Università di Napoli, I-80125 Naples, Italy
[4] Dept. of Physics, Duke University, Durham, NC 27708, USA
[5] Theoretische Physik, Universität Zürich, CH 8001 Zürich, Switzerland
[6] Department of Radiation Sciences, University of Uppsala, Box 535, S-75121 Uppsala, Sweden; representing WASA/PROMICE Collaboration
[7] School of Physics and Astronomy, Univ. of Tel Aviv, Tel Aviv 69978, Israel
[8] Dept. of Physics, University of California, Los Angeles, CA 90024, USA
[9] Dept. de Física Teórica and I.F.I.C. Universidad de Valencia - C.S.I.C. 46100 Burjassot (Valencia) - Spain
[10] Inst. f. Theoretische Physisk, Universität Bern Sidlerstr. 5, CH-3012 Bern, Switzerland
[11] Dept. de Física Teórica Universidad Complutense. 28040 Madrid. Spain
[12] Inst. f. Physik, Universität Mainz, D-55099 Mainz, Germany
[13] Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, E-08034 Barcelona, Spain

Abstract. This is the summary of the working group on Goldstone Boson Production and Decay of the Chiral Dynamics Workshop in Mainz, September 1-5, 1997. For the production aspects we discuss π^0 and η production in nucleon-nucleon collisions and the behaviour of hadrons in various sum rules. For the decays we present a discussion on various K, η and π decay channels. Other aspects discussed are a new treatment of meson-meson scattering, the light-by-light contribution to the muon anomalous magnetic moment and progress in various aspects of the p^6 generating functional in the mesonic sector.

1 Introduction

Chiral symmetry plays a prominent role in low energy hadronic physics. The workshop was basically fully devoted to theoretical and experimental progress in this area. In our working group we discussed some topics not covered by the other three working groups (Bernstein and Kaiser (1997), Meißner and Sevior (1997), Drechsel (1997)) and in the various plenary talks. The subject of chiral symmetry and the modern way to extract its consequences, Chiral Perturbation Theory (CHPT), was introduced by Jürg Gasser (Gasser (1997)). In this working group we did not discuss the extension to a small quark condensate (Stern (1997) and references therein). The plan of this contribution follows closely the summary as presented by the convener J. Bijnens. In particular some of the contributions to this working group are discussed in his plenary talk (Bijnens (1997)) and will not be repeated here.

On the first day we discussed the experiments done by WASA/PROMICE at Celsius in Uppsala (A. Kupsz) on meson productions and their future η decay program, as well as new η decay results and future programs (B. Nefkens) and the relevant CHPT aspects including the anomaly (J. Bijnens). The subject of anomalies was also covered experimentally (M. Moinester). The present status of theory for the anomalous magnetic moment of the muon and its relevance for beyond the standard model effects were covered. The main remaining uncertainty in the prediction remains the low-energy hadronic cross-section (E. Pallante). The subject of chiral sum rules in the VV and AA sector has recently had a boost by the two-loop calculations in CHPT (J. Kambor). Further progress at this order in CHPT was evident in the entire workshop. The topics covered here were $\pi \to e\nu\gamma$ (P. Talavera) and various subtleties in the general p^6 calculations as well as first results on the infinity structure at p^6 (G. Colangelo).

Beyond CHPT there is the discussion of the generalization of the Lippman-Schwinger approach and the Inverse Amplitude Method applied to meson-meson scattering (J. Peláez) and the attempts to calculate relevant higher order terms using various models, double VMD or not was a recurring theme in both η and K decays and the muon anomalous magnetic moment.

Experimental results in K decays and future prospects were reviewed (B. Renk) as well as the KLOE program at DAΦNE (G. Colangelo). Theoretical developments were discussed in the both the main decay modes (G. Isidori) as in various rare ones (G. D'Ambrosio, F. Gabbiani).

This report is organized as follows in Sect. 2 we discuss various η and π production experiments and future plans here. This also includes the anomalous process $\gamma\pi \to \pi\pi$. In Sect. 3 we discuss the parts mentioned above relevant for η and K decays. Here we also review the uncertainty on the decay $\pi^0 \to \gamma\gamma$. Sect. 4 contains the discussion of the sum rules for $g-2$ and VV and AA as well as the meson-meson scattering by the IAM. The last section, Sect. 5, covers the progress in the p^6 generating functional.

The topics discussed in this working group but covered in the plenary talk by J. Bijnens (Bijnens (1997)) are: radiative π^+ decay and the light-by-light contribution to the muon anomalous magnetic moment.

2 Production I

2.1 Hadronic Production of π and η

In Uppsala, an experimental programme for near threshold η and π meson production in proton-nucleon interactions is carried out. The experiments are done at the CELSIUS accelerator storage ring using an internal cluster gas-jet target and the WASA/PROMICE detector capable of measuring forward going charged particles and decays photons from the neutral mesons (Calén et al. (1996)). The photon detection allows for measurements very close to threshold when the outgoing protons or deuterons escape undetected in the beam pipe. At higher energies, a kinematically complete reconstruction of the events is possible.

An example is the near-threshold measurement of the reaction $pp \to pp\pi^\circ$ that has been measured at seven different excess energies, from 0.5 MeV to 14 MeV in the centre-of-mass system (Bondar et al. (1995)). The experiment gives added weight to an interpretation that the reaction near threshold is sensitive to the short range components of the nucleon-nucleon interaction. The understanding of the cross section magnitude remains a challenge for CHPT (van Kolck (1997)).

Production of η mesons in proton-nucleon collisions has been studied using both hydrogen and deuterium as targets. By exploiting the neutron Fermi momentum in the deuteron, the excitation functions for the reactions $pn \to pn\eta$ and $pn \to d\eta$ have been measured for CM excess energies from 15 MeV to 115 MeV (Fig. 1). The measured energy dependence of the cross section for the $pn \to d\eta$ reaction is the first firm evidence of the importance of the $S_{11}(1535)$ nucleon resonance in the reaction mechanism (Calén et al. (1997a)). The cross section for the reaction $pn \to pn\eta$ was found to be about six times larger than that of the reaction $pp \to pp\eta$.

Recently, the measurements of the $pn \to d\eta$ were extended down to the threshold by using the CELSIUS bending magnets as a spectrometer to identify deuterons at 0 degree. The cross section shows an enhancement at the threshold (Calén et al. (1997b)) which might be signal of a quasi-bound $d\eta$ state suggested by Ueda (1991).

Windowless thin internal targets offer unique possibilities to detect spectator protons. The way for systematical studies of $p+n$ reactions at CELSIUS is provided by placing semiconductor detectors inside the scattering chamber. An experiment which develops this technique has started recently.

The WASA/PROMICE setup will be dismounted and a new central detector with close to 4π coverage for photons and charged particles will be

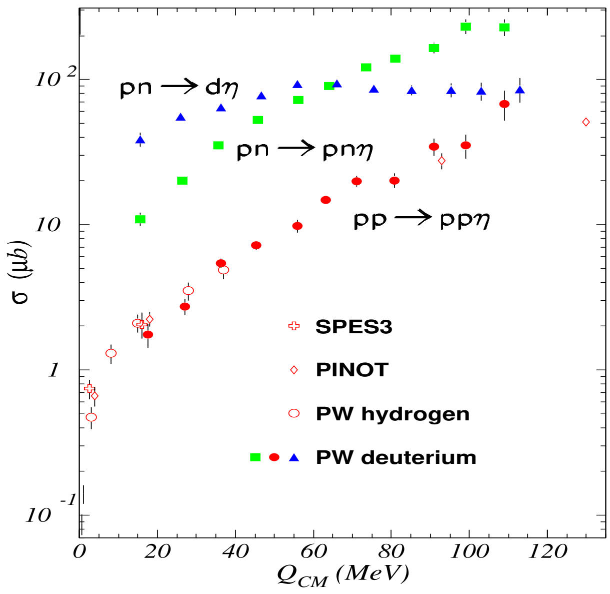

Fig. 1. Cross section for quasi-free η production in pp and pn interactions.

installed during 1998. The η decay studies will start and the meson production programme will be continued.

2.2 $\gamma\pi \to \pi\pi$

The value of the decay in the chiral limit and at zero for all the kinematical variables is predicted by the chiral anomaly. A precise measurement is thus a good test of this fundamental phenomenon. The prediction by the anomaly can be expressed in terms of a formfactor $F_{3\pi}$. In the chiral anomaly this is given by $F_{3\pi} = e/\left(4\pi^2 F_\pi^3\right) \approx 9.7\, GeV^{-3}$. This disagrees at the two sigma level with the experimental measurement of Antipov et al. (1987); $F_{3\pi} = 12.9 \pm 0.9 \pm 0.5$. The chiral loop corrections have been estimated by Bijnens, Bramon and Cornet (1990) and are 6 to 12 % over the phase space relevant for this experiment. The loop parts are fairly small and the major part of the upwards corrections comes from the tree level contributions at order p^6 estimated via Vector Meson Dominance. Higher order corrections have been estimated by Holstein (1996) by unitarizing the CHPT calculation of Bijnens, Bramon and Cornet (1990). An alternative way to estimate this is to use models of the meson structure. The results are similar to the VMD calculations and the unitarized calculation of Holstein (1996). An example of a recent calculation in this approach is (Alkofer and Roberts (1996)).

One recurring theme here is that the extrapolation used by Antipov et al. (1987) is not too reliable, there are large corrections from the measured region to the $s = t = u = 0$ point. We are therefore lucky to have several new experiments that will test this in the future. There is SELEX at Fermilab and COMPASS at CERN as discussed in the plenary talk by Moinester (1997). These are Primakoff type experiments using a pion beam scattering of the electromagnetic field of a nucleus. The opposite method, scattering a photon beam of a hadron target and extrapolating the result to the pion pole will be taken by the CLAS experiment at TJNAF (Miskimen, Wang and Yegneswaran (1994)).

3 Decays

3.1 η

The progress in η decays in the last years has been fairly slow. A few new results have been presented by the Crystal Barrel (Amsler (1997)) about $\eta \to \pi\pi\pi$, these are given in Bijnens (1997). The other new results are a preliminary limit on the branching ratio of $\eta \to \pi^0\pi^0$ of 7×10^{-4} from VEPP 2M in Novosibirsk (VEPP 2M). There is also a new limit on $\eta \to e^+e^-$ from CLEO at CESR, (T. Browder et al. (1997)).

A list of accelerators and associated experiments that perform experiments or will do so in the near future is in Table 1.

Table 1. Accelerators and experiments that will study the η meson.

Accelerator	Experiment	Comments
AGS	Crystal Ball	neutral decays, to $\approx 10^{-6}$
CELSIUS	WASA	first 10^{-7} later to 10^{-9}–10^{-10}
DAΦNE	KLOE	to 10^{-7}–10^{-8}
GRAAL/ESRF	Lagrange	production and neutral decays
VEPP2M. Novosibirsk	CMD-2	
MAMI	TAPS	production
ELSA	Crystal Barrel and TAPS	production
CEBAF	CLAS	production
CESR	CLEO	
Serpukhov	GAMS	
...		

One problem that still needs clarification is $\Gamma(\eta \to \gamma\gamma)$. The old Primakoff experiment has a significantly lower value for the width than the more recent experiments in e^+e^-–colliders. This width is used to normalize all the other η widths and is therefore rather important.

Other η decays are $\eta \to 3\pi$ and $\eta \to \pi^0\gamma\gamma$ as discussed in Bijnens (1997). In the anomalous decays, $\eta \to \gamma^*\gamma^*$, only the two real photons and the single Dalitz decays $\eta \to \gamma e^+e^-, \gamma\mu^+\mu^-$ have been measured with a single measurement of the formfactor in the latter. As discussed below for the Kaon decays and in Bijnens (1997) for the muon anomalous magnetic moment, it is fairly important to know whether double Vector Meson Dominance holds, or whether something else needs to be used for the case when the two photons are off-shell. Measurements of the double Dalitz decays and the formfactors in there as well as double tagged production experiments of π^0, η and η' are therefore very welcome. The decay $\eta \to \pi^+\pi^-\gamma$ has a width of $58 \pm 6\ eV$ while the one-loop CHPT result is about $47\ eV$ (Bijnens, Bramon and Cornet (1990)). Improved measurements of the form-factors and the decay with the photon off-shell would provide information about precisely which type of contributions are the reason for this discrepancy.

On the theoretical front there have been several studies of how to include $\eta - \eta'$ mixing in CHPT. In Gasser and Leutwyler (1985) the massive degree was integrated out and treated perturbatively, more recent studies try to include it as a propagating degree of freedom. The combined $1/N_c$ and chiral expansion then provides a proper framework to discuss these questions, (Leutwyler (1996), Herrera-Siklody et al. (1997), Georgi (1994), Peris and de Rafael (1995), Peris (1994)).

3.2 K

Experiments : Rare decays and CP-violation In the past year several new Kaon Decay modes have been observed. The most interesting one is probably $K^+ \to \pi^+\nu\bar{\nu}$ (Adler et al. (1997c)) that in the long run will provide an accurate measurement of V_{td}. In the field of tests of CHPT we have seen measurements of $K^+ \to \pi^+\gamma\gamma$ (Kitching et al. (1997)) and $K^+ \to \pi^+\mu^+\mu^-$ (Adler et al. (1997b)). Moreover, we have about 500 candidates for $K_L \to \pi^+\pi^-e^+e^-$ (O'Dell (1997)) that can be useful for CP-violation studies.

In the near future we expect a significant improvement in the measurements of ε'/ε. This constitutes our major CP-violation effort in the K system at present. We also expect improvements in the modes $K_L \to \gamma\gamma, \gamma\gamma^*, \gamma^*\gamma^*$ very interesting both in the framework of CHPT and to estimate the 'long-distance contamination' of more rare modes, as discussed below.

In the longer term we expect to observe direct CP violation in $K_L \to \pi^0e^+e^-$ and $K_L \to \pi^0\nu\bar{\nu}$. In these channels, as well as in $K^+ \to \pi^+\nu\bar{\nu}$, the progress in rare kaon decays is complementary and competitive with the one in the B-system.

Experiments : K_{l4} Here there is the BNL experiment as presented by J. Lowe at this meeting (Lowe (1997)) and the progress to be expected from the KLOE experiment at DAΦNE. Both of them aim at about 300000 events in a much better acceptance environment than the previous experiment (Rosselet et al. (1977)). The 4 form factors should all be measurable. The F and G form factor provides tests of $\pi\pi$ scattering (Ecker (1997)) but their absolute value is important in determining the values of the input parameters for CHPT. KLOE should at the same time be able to measure the decays $K^+ \to \pi^0\pi^0 e^+\nu$ and $K^0 \to \pi^0\pi^- e^+\nu$ as well. These latter only depend on one form-factor to a much larger extent thus allowing for an independent cross-check of the decay with two charged pions. At the level of statistics expected this will also provide an important check of the isospin breaking corrections that might be needed at the expected level of prediction. Once F and G are well measured we can turn to the anomalous formfactor H where the pattern of $SU(3)$ breaking corrections is expected to be quite different from the one in F and G. And, last but not least, once F and G are accurately determined in the electronic decay modes, we can then use these to determine R in the muonic decay mode. That way we can determine the otherwise elusive parameter L_4 of the CHPT Lagrangian and provide additional tests of the $1/N_c$ approximations.

Theory: $K \to \pi\pi, \pi\pi\pi$ The starting tool for a theoretical analysis of non–leptonic kaon decays is the chiral realization of the effective four–quark Hamiltonian for $|\Delta S| = 1$ transitions. At the leading order $(O(p^2e^0) + O(p^0e^2))$ there are only three independent operators (and thus three unknown couplings) transforming under $SU(3)_L \times SU(3)_R$ as the effective four-quark Hamiltonian, i.e. as $(8_L, 1_R)$, $(27_L, 1_R)$ and $(8_L, 8_R)$. As shown by Kambor et al. (1989), the number of independent couplings increases substantially at next–to–leading order: limiting to the presumably dominant $(8_L, 1_R)$ sector there are already 37 independent operators (Ecker et al. (1993)). Nonetheless, the situation is not dramatic since there exist various classes of processes where only few couplings appear and the predictive power of the theory is not destroyed (see e.g. de Rafael (1994), Ecker (1995), D'Ambrosio et al. (1995), D'Ambrosio and Isidori (1996), and references therein).

$K \to \pi\pi$ amplitudes are typically used as input to determine the low–energy constants of the weak chiral Lagrangian. Even though some recent progress has been made in understanding the origin of the $\Delta I = 1/2$ rule (see e.g. Antonelli et al. (1996) and references therein) a reliable prediction of $K \to \pi\pi$ amplitudes from first principles is still far.

$K \to 3\pi$ decays is one of the classes where the chiral Lagrangian approach leads to interesting predictions even at next–to–leading order. In particular, the quadratic slopes in the Dalitz variables are unambiguously predicted in terms of widths and linear slopes (Kambor et al. (1991), Kambor et al. (1992)). The predictions of $\Delta I = 1/2$ quadratic slopes are in good agreement with present data. On the other hand, the situation in the $\Delta I = 3/2$ sector is still unclear because of large experimental error. The situation will certainly improve in the next years with the advent of new high precision data. However, it must be stressed that also a complete theoretical analysis of the isospin–breaking terms is needed in order to make a precise test in the $\Delta I = 3/2$ sector.

Another interesting aspect of $K \to 3\pi$ decays is the interference measurement of $K_{L,S} \to \pi^+\pi^-\pi^0$, that could lead to determine both the suppressed $K_S \to \pi^+\pi^-\pi^0$ amplitude and the 3π rescattering phases (see e.g. D'Ambrosio et al. (1994) and references therein). Here the chiral predictions are again quite firm, thus interesting consistency tests can be made. Moreover, a determination of 3π rescattering phases is important to estimate the magnitude of direct CP violation in $K \to 3\pi$. The interference effect in $K_{L,S} \to \pi^+\pi^-\pi^0$ has been recently observed by Adler et al. (1997a) and Zou et al. (1996), showing a clear signal for a non–vanishing $K_S \to \pi^+\pi^-\pi^0$ amplitude and a preliminary evidence for the 3π phases. Substantial improvements are expected at DAΦNE.

Strictly related to $K \to 3\pi$ are also $K \to 3\pi\gamma$ transitions. These have been completely analyzed at next–to–leading order by D'Ambrosio et al. (1996) and could offer interesting tests of the theory. Unfortunately the present experimental information about these transitions is very poor, but again substantial improvements are expected in the near future.

Theory: Rare decays Radiative non–leptonic kaon decays may play a crucial role in our understanding of fundamental questions like the validity of the Standard Model, the origin of CP violation and the realization of chiral symmetry in the framework of non–leptonic weak interactions (see D'Ambrosio et al. (1995), D'Ambrosio and Isidori (1996), and references therein). For instance the measurement of $K_L \to \pi^0\nu\nu$ and $K^\pm \to \pi^\pm\nu\nu$ rates should allow a determination of the CKM matrix elements competitive and complementary to the one attainable from B decays (Littenberg (1989), Buchalla and Buras (1996)).

In the following we will discuss some recent theoretical progress in various channels.

$\underline{K \to \pi\gamma\gamma \text{ and } K_L \to \gamma\gamma^*}$. The amplitude for $K_L(p) \to \pi^0\gamma(q_1)\gamma(q_2)$ can be generally decomposed in terms of two independent Lorentz and gauge invariant amplitudes: $A(z,y)$ and $B(z,y)$, where $y = p\cdot(q_1 - q_2)/m_K^2$ and $z = (q_1+q_2)^2/m_K^2$. Then the double differential rate is given by

$$\frac{\partial^2\Gamma}{\partial y\,\partial z} = \frac{m_K}{2^9\pi^3}\left[z^2\,|\,A + B\,|^2 + \left(y^2 - \frac{\lambda(1,r_\pi^2,z)}{4}\right)^2 |\,B\,|^2\right] , \tag{1}$$

where $\lambda(a,b,c)$ is the usual kinematical function and $r_\pi = m_\pi/m_K$. Thus in the region of small z (collinear photons) the B amplitude is dominant and can be determined separately from the A amplitude. This feature is important in order to evaluate the CP–conserving contribution $K_L \to \pi^0\gamma\gamma \to \pi^0 e^+e^-$.

Both on–shell and off–shell two–photon intermediate states generate, through the A amplitude, a contribution to $K_L \to \pi^0 e^+ e^-$ that is helicity suppressed (Ecker et al. (1988)). Instead the B–type amplitude, though appearing only at $\mathcal{O}(p^6)$, generates a relevant unsuppressed contribution to $K_L \to \pi^0 e^+ e^-$ through the on–shell photons, due to the different helicity structure.

The leading finite $\mathcal{O}(p^4)$ amplitudes of $K_L \to \pi^0 \gamma\gamma$ were evaluated by Ecker et al. (1987a), Cappiello and D'Ambrosio (1988), and generate only the A–type amplitude in Eq. (1). The observed branching ratio for $K_L \to \pi^0 \gamma\gamma$ is $(1.7 \pm 0.3) \times 10^{-6}$ (Barnett et al. (1996)), about 3 times the $\mathcal{O}(p^4)$ prediction. However, the $\mathcal{O}(p^4)$ spectrum of the diphoton invariant mass nearly agrees with the experiment, in particular no events for small $m_{\gamma\gamma}$ are observed, implying a small B–type amplitude. Thus $\mathcal{O}(p^6)$ corrections have to be important. No complete calculation is available, but the supposedly larger contributions are known : $\mathcal{O}(p^6)$ unitarity corrections (Cappiello et al. (1993), Cohen et al. (1993), Kambor and Holstein (1994)) enhance the $\mathcal{O}(p^4)$ branching ratio by 30%, and generate a B–type amplitude. Ecker et al. (1990) parameterized the $\mathcal{O}(p^6)$ vector meson exchange contributions by an effective vector coupling a_V. There are two sources for a_V : i) strong vector resonance exchange with an external weak transition (a_V^{ext}), and ii) direct vector resonance exchange between a weak and a strong $VP\gamma$ vertices (a_V^{dir}). Then $a_V = a_V^{ext} + a_V^{dir}$. The first one is model independent and gives $a_V^{ext} \simeq 0.32$ (Ecker et al. (1990)), while the direct contribution depends strongly on the model for the weak $VP\gamma$ vertex. (Cohen et al. (1993)) noticed that one could, simultaneously, obtain the experimental spectrum and width of $K_L \to \pi^0 \gamma\gamma$ with $a_V \simeq -0.9$. The question of the relevant $\mathcal{O}(p^6)$ contributions relative to the leading $\mathcal{O}(p^4)$ result for $K^+ \to \pi^+ \gamma\gamma$, (Ecker et al. (1988)), can also be studied. Unitarity corrections, (D'Ambrosio and Portolés (1996)), generate a B–type amplitude and increase the rate by a 30–40% while, differently from $K_L \to \pi^0 \gamma\gamma$, vector meson exchange is negligible in $K^+ \to \pi^+ \gamma\gamma$. The detection by Kitching et al. (1997) of a few events, confirms the relevance of the unitarity corrections and a first measurement (though with large error) of the $\mathcal{O}(p^4)$ local contributions.

The decay $K_L \to \gamma(q_1, \epsilon_1)\gamma^*(q_2, \epsilon_2)$ is given by an amplitude $A_{\gamma\gamma^*}(q_2^2)$ that can be expressed as $A_{\gamma\gamma^*}(q_2^2) = A_{\gamma\gamma}^{exp} f(x)$, where $A_{\gamma\gamma}^{exp}$ is the experimental $A(K_L \to \gamma\gamma)$ amplitude and $x = q_2^2/m_K^2$. The form factor $f(x)$ is normalized to $f(0) = 1$ and the slope b is defined as $f(x) = 1 + bx$. Traditionally experiments (see Barnett et al. (1996)) do not measure directly the slope but they input the full form factor suggested by Bergström et al. (1983-1990). However, as discussed by D'Ambrosio and Portolés (1997), it is more appropriate to measure directly the slope, they estimate $b_{exp} = 0.81 \pm 0.18$.

D'Ambrosio and Portolés (1997) have analyzed the general effective weak coupling $VP\gamma$ contributing at $\mathcal{O}(p^6)$ to both $K \to \pi\gamma\gamma$ and $K_L \to \gamma\gamma^*$. The corresponding Lagrangian can be written in terms of the 5 possible relevant $P\gamma$ structures and contains thus 5 coupling constants, κ_i to be determined from phenomenology or theoretical models. The Factorization Model (FM), motivated by the $1/N_c$ expansion (see e.g. Pich and de Rafael (1991)), assumes that the dominant contribution to the four–quark operators of the $\Delta S = 1$ Hamiltonian has a *current*$\times$ *current* structure. This assumption is then implemented with a bosonisation of the left–handed quark currents. Applying the factorization procedure to the construction of the weak $VP\gamma$ vertex and integrating out the vector mesons afterwards, (D'Ambrosio and Portolés (1997)) have identified *new contributions* to the left–handed currents and therefore to the chiral structure of the weak amplitudes at $\mathcal{O}(p^6)$. Within this prescription, called Factorization Model in the Vector couplings (FMV), one can determine the κ_i and then predict both the a_V^{dir} parameter of $K_L \to \pi^0\gamma\gamma$ and the slope b_D of $K_L \to \gamma\gamma^*$. The results are $a_V^{dir}\,|_{FMV} \simeq -0.95$ and $b_D^{octet}\,|_{FMV} \simeq 0.51$, leading to (see Ref. D'Ambrosio and Portolés (1997) for a thorough discussion) $a_V \simeq -0.72$ and $b \simeq 0.8 - 0.9$, in good agreement with phenomenology.

$\underline{K_L \to \mu^+\mu^-.}$ To fully exploit the potential of $K_L \to \mu^+\mu^-$ in probing short–distance dynamics it is necessary to have a reliable control on its long–distance amplitude. However the dispersive contribution generated by the two–photon intermediate state cannot be calculated in a model independent way and it is subject to various uncertainties. The branching ratio can be generally decomposed as $B(K_L \to \mu^+\mu^-) = |\Re e\mathcal{A}|^2 + |\Im m\mathcal{A}|^2$, and the dispersive contribution can be rewritten as $\Re e\mathcal{A} = \Re e\mathcal{A}_{long} + \Re e\mathcal{A}_{short}$. Within the Standard Model $\Re e\mathcal{A}_{short}$ has been evaluated at NLO by Buchalla and Buras (1994) and is proportional to $(1.3-\rho)$, where ρ is the usual CKM parameter in the Wolfenstein convention. The measurement of $B(K_L \to \mu^+\mu^-)$ by Heinson et al. (1995) is almost saturated by the absorptive amplitude leaving very little room for the dispersive contribution : $|\Re e\mathcal{A}_{exp}|^2 = (-1.0 \pm 3.7) \times 10^{-10}$ or $|\Re e\mathcal{A}_{exp}|^2 < 5.6 \times 10^{-10}$ at 90% C.L. Thus an upper bound on $|\Re e\mathcal{A}_{long}|$ can be used to set a lower bound on ρ.

To estimate $\Re e\mathcal{A}_{long}$, D'Ambrosio et al. (1997) proposed a low energy parameterization of the $K_L \to \gamma^*\gamma^*$ form factor that include the poles of the lowest vector meson resonances with arbitrary residues

$$f(q_1^2, q_2^2) = 1 + \alpha\left(F(q_1^2) + F(q_2^2)\right) + \beta F(q_1^2)F(q_2^2)\,;\, F(q^2) = q^2/(q^2 - m_V^2) \quad (2)$$

α and β, expected to be $\mathcal{O}(1)$ by naive dimensional chiral power counting, are in principle directly accessible by experiment in $K_L \to \gamma\ell^+\ell^-$ and $K_L \to e^+e^-\mu^+\mu^-$. Up to now there is no experimental information on β, whereas $\alpha = -1.63 \pm 0.22$. Note that an accurate theoretical determination of β requires the knowledge of the strong Pseudoscalar-Vector-Vector vertex.

The form factor defined in Eq. (2) goes as $1 + 2\alpha + \beta$ for $q_i^2 \gg m_V^2$ and, as long as $1 + 2\alpha + \beta \neq 0$, an ultraviolet cutoff is needed to regularize the contribution to $\Re e\mathcal{A}_{long}$. However, for $q_i^2 \gg m_V^2$ one can use perturbative QCD to estimate the $K_L \to \gamma^*\gamma^*$ vertex. The explicit QCD calculation show a mild behavior of the form factor at large q^2, consistent with result of the

FMV model. Using α_{exp} and the QCD constraint D'Ambrosio et al. (1997) estimate

$$\rho > -0.42 \quad (90\% C.L.)\ . \tag{3}$$

This bound could be very much improved if α, β and possibly higher order contributions to the $K_L \to \gamma^* \gamma^*$ form factor were measured more accurately.

$\underline{K_L \to \pi^0 e^+ e^-.}$ This process is being searched for as a signal of direct $\Delta S = 1$ CP violation. We analyze the three components of the decay: 1) direct CP violation through one-photon exchange, 2) CP violation through the mass matrix and 3) CP-conserving (two-photon) contributions. The primary weak Hamiltonian responsible for the transition has the form

$$\mathcal{H}_W^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \left[C_{7V}(\mu) Q_{7V} + C_{7A} Q_{7A} \right], \tag{4}$$

where $Q_{7V} = (\overline{s}d)_{V-A}(\overline{e}e)_V$ and $Q_{7A} = (\overline{s}d)_{V-A}(\overline{e}e)_A$. The decay rate is

$$B(K_L \to \pi^0 e^+ e^-)_{\rm dir} = 4.16 (\Im m \lambda_t)^2 (y_{7A}^2 + y_{7V}^2), \tag{5}$$

$\Im m \lambda_t = \Im m V_{td} V_{ts}^* = |V_{ub}||V_{cb}| \sin\delta = A^2 \lambda^5 \eta$, and $V_{td} = |V_{ub}| \sin\delta$. A, λ, η refer to the Wolfenstein parametrization of the CKM matrix. Using the results of Buras et al. (1994) for y_{7A} and y_{7V}, one gets the branching ratio

$$B(K_L \to \pi^0 e^+ e^-)_{\rm dir} = 2.32 \times 10^{-12}. \tag{6}$$

For the second component, we review the formalism for analyzing mass matrix CP violation, first set forth by Ecker et al. (1987b) and Ecker et al. (1988). This amounts to a prediction for the decay rate for $K_S \to \pi^0 e^+ e^-$, since the mass matrix effect is defined by

$$A(K_L \to \pi^0 e^+ e^-)|_{\rm MM} \equiv \epsilon A(K_S \to \pi^0 e^+ e^-). \tag{7}$$

The uncertainties in this method appear to be so large that they will obscure the direct CP violation unless it is possible to measure the $K_S \to \pi^0 e^+ e^-$ rate, which could become possible at DAΦNE. In particular, one cannot extract all the parameters needed from the available experimental data. Using additional assumptions instead, one gets for the mass-matrix contribution

$$B(K_L \to \pi^0 e^+ e^-)_{\rm MM} = 0.37 \times 10^{-12}. \tag{8}$$

The CP-conserving amplitude remains also somewhat uncertain, but present indications are such that there may be a sizable CP-violating asymmetry in the e^+, e^- energies from the interference of CP-conserving and CP-violating amplitudes. This may potentially be useful in determining whether direct CP violation is present. This component proceeds through a CP-conserving two-photon intermediate state. If we ignore the electron mass, the form of the amplitude will be

$$A(K_L \to \pi^0 e^+ e^-)_{\rm CPC} = G_8 \alpha^2 K p_K \cdot (k - k')(p_K + p_\pi)^\mu \overline{u} \gamma_\mu v, \tag{9}$$

where K is given by (Donoghue and Gabbiani (1995))

$$K = \frac{B(x)}{16\pi^2 m_K^2}\left[\frac{2}{3}\ln\left(\frac{m_\rho^2}{-s}\right) - \frac{1}{4}\ln\left(\frac{-s}{m_e^2}\right) + \frac{7}{18}\right], \tag{10}$$

and B is the only form factor relevant in the limit of vanishing m_e. Using the estimate of B by Cohen et al. (1993) one gets

$$B(K_L \to \pi^0 e^+ e^-)_{\rm CPC} = 4.89 \times 10^{-12}. \tag{11}$$

A slightly smaller result is obtained in (D'Ambrosio and Portolés (1997)) due to the inclusion of the corrections suggested by Kambor and Holstein (1994):

$$0.1 < B(K_L \to \pi^0 e^+ e^-)_{\rm CPC} \times 10^{-12} < 3.6. \tag{12}$$

$\underline{K_S \to \pi^0 e^+ e^- \text{ and } K^\pm \to \pi^\pm l^+ l^-}$. Contrary to $K_L \to \pi^0 e^+ e^-$, these processes are dominated by long–distance effects. The amplitudes can be calculated using the parametrization of Ecker et al. (1987b), Ecker et al. (1988):

$$\begin{aligned} A(K_S \to \pi^0 e^+ e^-) &= -\frac{G_8\alpha}{4\pi} d_S (p_K + p_\pi)^\mu \overline{u}\gamma_\mu v \qquad (13)\\ A(K^+ \to \pi^+ l^+ l^-) &= -\frac{G_8\alpha}{4\pi} d_+ \left[(p_K + p_\pi)^\mu - \frac{(m_K^2 - m_\pi^2)}{q^2} q^\mu\right] \overline{u}\gamma_\mu v \end{aligned}$$

with

$$\begin{aligned} d_S &\equiv \Re e\, w_S + 2\phi_K(q^2), \qquad d_+ \equiv w_+ + \phi_K(q^2) + \phi_\pi(q^2),\\ w_+ &= \frac{64\pi^2}{3}\left(N_{41}^r - N_{15}^r + 3L_9^r\right) - \frac{1}{3}\ln\frac{m_K^2}{\mu^2};\; w_S = w_+ + 32\pi^2\left(N_{15}^r - 2L_9^r\right),\\ \phi_i(q^2) &= \frac{m_i^2}{q^2}\int_0^1 dx \left[1 - \frac{q^2}{m_i^2}x(1-x)\right]\ln\left[1 - \frac{q^2}{m_i^2}x(1-x)\right], \end{aligned} \tag{14}$$

where the N_i^r's are the renormalized couplings of the $\mathcal{O}(p^4)$ $\Delta S = 1$ weak Lagrangian in the basis of Ecker et al. (1993). w_+ is extracted from rate and/or the spectrum of $K^\pm \to \pi^\pm l^+ l^-$, however recently Adler et al. (1997b) seems to confirm a value of w_+ in $K^+ \to \pi^+\mu^+\mu^-$ 2σ's away from the value extracted from $K^+ \to \pi^+ e^+ e^-$.

$\underline{K_L \to \pi^0\gamma e^+ e^-}$ This decay, which occurs at a *higher* rate than the nonradiative process $K_L \to \pi^0 e^+ e^-$, can be a background to CP violation studies using the reaction in the previous section. It is of interest in its own right in the context of CHPT, through its relation to the decay $K_L \to \pi^0\gamma\gamma$. Using the framework of the calculation performed by Cohen et al. (1993) for $K_L \to \pi^0\gamma\gamma$, one can provide a straightforward $\mathcal{O}(p^6)$ calculation. This is the generalization to $k_1^2 \neq 0$ of the original chiral calculation of Cappiello et al. (1993), Cohen et al. (1993). Here k_1 is the momentum of the off-shell photon. The branching ratio obtained by Donoghue and Gabbiani (1997) is:

$$\mathrm{B}(K_L \to \pi^0 \gamma e^+ e^-) = 2.3 \times 10^{-8}. \tag{15}$$

The behavior of the $K_L \to \pi^0 \gamma e^+ e^-$ amplitude mirrors closely that of the process $K_L \to \pi^0 \gamma\gamma$. This reaction should be reasonably amenable to experimental investigation in the future. It is 3–4 orders of magnitude larger than the reaction $K_L \to \pi^0 e^+ e^-$, which is one of the targets of experimental kaon decay programs, due to the connections of the latter reaction to CP studies. The regions of the distributions where the experiment misses the photon of the radiative process can potentially be confused with $K_L \to \pi^0 e^+ e^-$ if the resolution is not sufficiently precise. In addition, since the π^0 is detected through its decay to two photons, there is potential confusion related to misidentifying photons. The study of the reaction $K_L \to \pi^0 \gamma e^+ e^-$ will be a valuable preliminary to the ultimate CP tests.

3.3 π

For $\pi^0 \to \gamma\gamma$ we discussed the influence of the anomaly, CHPT calculations for this process and the ones with one or two off-shell photons, Dalitz or double Dalitz decays exist up to order p^6 (Anomaly (1985)). The corrections are mainly the change of F_0, the chiral limit decay constant, to F_π and a small contribution to the slope. The main contributions are those coming from the p^6 Lagrangian. VMD, the chiral quark model and the Nambu–Jona-Lasinio model (Bijnens and Prades (1994)) have all been used to estimate these constants. The slope agrees reasonably well in these three estimates. A remeasurement of this slope would be useful. The present measurements rely on an extrapolation from high $Q^2 > 1\ GeV^2$ to small values. The main uncertainties on the theoretical prediction are now the value of F_{π^0} where we have to remove isospin breaking and electromagnetic corrections from the measured F_{π^+} and the quark mass corrections to the decay rate. The latter are large in the chiral quark model but unlike most other cases the ENJL model predicts in this case a vastly smaller correction of about $+0.7\%$ (Bijnens and Prades (1994)). Notice that both of these corrections tend to increase the decay width.

The other decay we discussed was the recent two-loop calculation of $\pi \to e\nu\gamma$ (Bijnens and Talavera (1997)). The results of this work were presented in (Bijnens (1997)).

4 Production II

4.1 $a_\mu = \frac{g_\mu - 2}{2}$ and $\alpha(M_Z^2)$

The high precision muon $g-2$ measurements are excellent candidates for probing the electro-weak sector of the Standard Model (SM) and new physics scenarios beyond the Standard Model. The present experimental value is Bailey (1977):

$$a_\mu = (g_\mu - 2)/2 = 11\,659\,230(85) \times 10^{-10} \ (\mu^\pm \text{ average}). \tag{16}$$

The muon $g-2$ experiment E821 at BNL (Lee Roberts (1992)) plans to reach $\pm 40 \times 10^{-11}$. This is sufficient to observe the electro-weak contributions to a_μ if the precision of the hadronic corrections is improved. The SM prediction of a_μ can be written as $a_\mu = a_\mu^{QED} + a_\mu^{hadronic} + a_\mu^{e.w.}$. The dominant pure QED contribution to a_μ is known up to $O(\alpha/\pi)^5$ (Kinoshita (1996)). SM electro-weak contributions are known up to two loops (see Czarnecki and refs. therein). Numerical values are in Table 2.

The three classes of dominant hadronic contributions are: (I) hadronic vacuum polarization (HVP) at order $(\alpha/\pi)^2$, (II) higher order corrections to HVP ($(\alpha/\pi)^3$) and (III) light-by-light (LL) ($(\alpha/\pi)^3$). (II) has been numerically computed in Kinoshita (1985) with the value $-90(5) \times 10^{-11}$, while a recent analytical estimate (Krause) gives $-101(6) \times 10^{-11}$. We focus here on some recent developments in the determination of (I) and open questions in (III), further details on the latter are given by Bijnens (1997). The most precise determination of the HVP contribution to a_μ (see de Rafael, Pallante for an alternative theoretical estimate) and $\alpha^{-1}(M_Z^2)$ is extracted from $R(s) = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ through the following dispersion relations ($\Delta\alpha^{(5)}_{had}$ is for five light quarks (u,d,s,c,b)):

$$\left[a_\mu^{HVP}; \Delta\alpha^{(5)}_{had}(M_Z^2)\right] = \left[\left(\frac{\alpha m_\mu}{3\pi}\right)^2; -\frac{\alpha M_Z^2}{3\pi}\right] \int_{4m_\pi^2}^{\infty} ds\, \frac{R(s)\left[\hat{K}(s); 1\right]}{[s^2; s(s - M_Z^2 - i\epsilon)]}\,. \tag{17}$$

$\hat{K}(s)$ smoothly increases from 0.63 to 1 with s. a_μ^{HVP} is dominated by the low energy $2\pi, \rho$ region, while $\Delta\alpha^{(5)}_{had}$ is dominated mainly by $\sqrt{s} = [2, 40]$ GeV. Alemany (1997) use recent ALEPH data on hadronic τ decays (Aleph (1997)) (2π and 4π channels) and the $R(s)$ data to improve the determination of a_μ^{HVP} and $\Delta\alpha^{(5)}_{had}$. Hadronic τ decay data in the $\pi^0\pi^-$ channel agree well with the corresponding $e^+e^- \to (\pi^+\pi^-)^{I=1}$ data (and are more accurate) and with CHPT predictions near threshold (Aleph (1997), Alemany (1997)).

The largest uncertainties for a_μ^{HVP}(Alemany (1997)) still come from the ρ-meson region followed by the $\sqrt{s} = [2.125, 40]$ GeV region. The third comes from the 4π and the $KK\pi\pi$ channels, the former mainly because of experimental discrepancies in $e^+e^- \to \pi^+\pi^-2\pi^0$. The inclusion of τ data highly improves the precision in a_μ^{HVP}, which is dominated by the 2π channel. The value of (Alemany (1997)) is compatible with most previous estimates. We therefore quote their value for a_μ^{HVP} in Table 2. The inclusion of τ data does not improve the determination of $\Delta\alpha^{(5)}_{had}$, since this is dominated by the $2 - 40$ GeV region. The τ data slightly increase previous values from e^+e^- data only, leading to $\alpha^{-1}(M_Z^2) = 128.878(85)$(Alemany (1997)).

The main uncertainties in the LL contribution (Bijnens et al. (1995), Hayakawa (1995)) come from the lack of knowledge of the off-shell behaviour of the vertices $P\gamma^*\gamma^*$ and $P'P'\gamma^*\gamma^*$, where $P = \pi^0, \eta, \eta'$ and $P' = \pi^\pm, K^\pm$.

Both are responsible for about half the error of Bijnens et al. (1995). The present discrepancy between the estimate in (Bijnens et al. (1995)) $-92(32)\times 10^{-11}$ and the one in (Hayakawa (1995)) $-79.2(15.4)\times 10^{-11}$ (see Bijnens (1997) for a discussion) comes mainly from the model dependence of the $P'P'\gamma^*\gamma^*$ vertex in the meson-loop diagram. Recently, CLEO published data for the meson-photon transition form factors with one photon highly off-shell ($\gamma^*(Q^2)\gamma \to P$) and for space-like photons with Q^2 from 1.5 to 9 (π^0), 20 (η) and 30 (η') GeV^2 (CLEO). Their data support a pole dominance picture and an asymptotic behaviour A/Q^2 suggested by theory (Lepage (1979)) but not with the suggested coefficient A. In addition, the form factor with two off-shell photons still needs to be studied theoretically and experimentally.

Adding together the SM contributions, we obtain a_μ as shown in table 2:

Table 2. Updated SM prediction of a_μ. Errors added in quadrature

Type	$a_\mu^{SM}(\times 10^{11})$	Reference
QED	116 584 705.7(1.9)	Kinoshita (1996)
Weak	151(4)	Czarnecki
Hadronic-HVP	7011(94)	Alemany (1997)
Hadronic-h.o.	-90(5)	Kinoshita (1985)
Hadronic-LL	-92(32)	Bijnens et al. (1995)
Total	116 591 686(100)	

Muon $g-2$ experiments also allow to explore new physics scenarios beyond the Standard Model and are able to put constraints on new physics scales which are competitive/complementary with limits from present accelerators. From the prediction of a_μ^{SM} of table 2 and the experimental value (16) the allowed window for new physics is $-11.0\times 10^{-9} < \delta a_\mu < 23.3\times 10^{-9}$. Experimental and theoretical errors have been added in quadrature. This already imposes stringent contraints on different SUSY scenarios (Moroi). Assuming the window for new physics from $BNL-E821$ to be $|\delta a_\mu| < 40\times 10^{-11}$, we list in table 3 the accessible limits on new physics. Muon $g-2$ experiments are highly preferrable to accelerator measurements for establishing $W^\pm$ compositeness and a value for $g_W - 2$, they are competitive for SUSY, various Higgs scenarios and μ compositeness, but not competitive for exploring extra W, Z bosons scenarios.

4.2 VV and AA chiral sum rules to p^6

In this section we discuss a new calculation of the isospin and hypercharge axialvector current propagators ($\Delta^{\mu\nu}_{\mathrm{A}33}(q^2)$ and $\Delta^{\mu\nu}_{\mathrm{A}88}(q^2)$) to two loops in

Table 3. Limits on new physics scales with a window for of $|\delta a_\mu| \leq 40 \times 10^{-11}$. The typical contribution to δa_μ is also shown for each scenario. SUSY treated in the large $\tan\beta$ limit.

Type	δa_μ	Limit	Quality
SUSY	$\tan\beta\,\alpha m_\mu^2/(\sin^2\theta_W m_{susy}^2)$	>120-130 GeV	competitive
W_R	$m_\mu^2/m_{W_R}^2$	> 250 GeV	>300-450 from $p\bar{p}$
Z'	$m_\mu^2/m_{Z'}^2$	> O(100) GeV	>120-130 pres. lim.
Light-Higgs	-$O(10^{-3}g)$	>O(300) GeV	competitive
Heavy Higgs	-$O(g)$	> O(500) GeV	competitive
μ compositness	m_μ^2/Λ^2	>4-5 TeV	competitive
Excited μ	m_μ/m_μ^*	> 400 GeV	competitive
$(g_W-2)/2$		≤ 0.02	dominant (LEP II≤ 0.2)
$W^\pm$ compositness	m_W/Λ	> 2 TeV	dominant
	m_W^2/Λ^2	>400 GeV	dominant

$SU(3) \times SU(3)$ CHPT. (Golowich and Kambor (1997a), Golowich and Kambor (1997b)) This completes work done for the corresponding quantities involving vector currents, (Golowich and Kambor (1995), Golowich and Kambor (1996)) which was reviewed in the plenary talk by Bijnens. (Bijnens (1997))

The motivation to consider these quantities is twofold. First, it becomes possible to formulate new chiral sum rules, valid to second order in quark mass. Second, these sum rules allow one to fix certain coupling constants of the order p^6 chiral lagrangian (LEC). They are given as integrals over moments of the spectral functions of vector and axialvector currents. Since both sides of these sum rules are physical observables, the determination of the coupling constants does not depend on the renormalization scale. This kind of uncertainty, inherent in the often used method of resonance saturation, is therefore avoided. Moreover, it is possible to check the validity of the principle of resonance saturation at order p^6 (Golowich and Kambor (1996)) To our knowledge, this is the first time this goal has been achieved in the nonanomalous sector of ChPT. Below, we shall give an example where an order p^6 LEC is seen to be *not* saturated by the *lowest* lying resonance, but rather gets substantial contributions from the region above this resonance.

Besides the application to chiral sum rules discussed here, this work yields a set of additional results, among which are: i) a large number of constraints on the set of beta functions of $\mathcal{O}(p^6)$ counterterms, ii) predictions for threshold behaviour of the 3π, $\bar{K}K\pi$, $\bar{K}K\pi$, $\eta\pi\pi$, *etc* axialvector spectral functions, iii) an extensive analysis of the so-called 'sunset' diagrams; since we work in chiral SU(3), the case of unequal masses has to be considered, iv) mass corrections to the Das-Mathur-Okubo sum rule (Das, Mathur and Okubo (1967)), and v) a complete two-loop renormalization of the masses and decay constants of the pion and eta mesons. This final item places the axialvector problem at the heart of two-loop studies in $SU(3)\times SU(3)$ ChPT.

The $SU(3)$ axialvector current propagators are defined as

$$\Delta^{\mu\nu}_{\mathrm{A}ab}(q^2) \equiv i\int d^4x\; e^{iq\cdot x}\; \langle 0|T\left(A^\mu_a(x)A^\nu_b(0)\right)|0\rangle \qquad (a,b=1,\dots,8) \quad (18)$$

and have spectral content

$$\frac{1}{\pi}\mathcal{I}m\;\Delta^{\mu\nu}_{\mathrm{A}ab}(q^2) = (q^\mu q^\nu - q^2 g^{\mu\nu})\rho^{(1)}_{\mathrm{A}ab}(q^2) + q^\mu q^\nu \rho^{(0)}_{\mathrm{A}ab}(q^2)\;, \qquad (19)$$

where $\rho^{(1)}_{\mathrm{A}ab}$ and $\rho^{(0)}_{\mathrm{A}ab}$ are the spin-one and spin-zero spectral functions. The tensor structure of Eq. (19) motivates the usual decomposition,

$$\Delta^{\mu\nu}_{\mathrm{A}ab}(q^2) = (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi^{(1)}_{\mathrm{A}ab}(q^2) + q^\mu q^\nu \Pi^{(0)}_{\mathrm{A}ab}(q^2)\;\;. \qquad (20)$$

$\Pi^{(1)}_{\mathrm{A}ab}$ and $\Pi^{(0)}_{\mathrm{A}ab}$ are the spin-one and spin-zero axialvector polarization functions. We consider $a=b=3$ (isospin) and $a=b=8$ (hypercharge).

We have determined the propagator through two-loop order

$$\begin{aligned}\Delta^{\mu\nu}_{\mathrm{A}aa}(q^2) = {} & (F_a^2 + \hat{\Pi}^{(0)}_{\mathrm{A}aa}(q^2))g^{\mu\nu} - \frac{F_a^2}{q^2-M_a^2}q^\mu q^\nu \\ & + (2L^{\mathrm{r}}_{10} - 4H^{\mathrm{r}}_1 + \hat{\Pi}^{(1)}_{\mathrm{A}aa}(q^2))(q^\mu q^\nu - q^2 g^{\mu\nu})\;, \end{aligned} \qquad (21)$$

where F_a^2, M_a^2 are now renormalized at two-loop level, L^{r}_{10}, H^{r}_1 are $\mathcal{O}(p^4)$ counterterms which appear in the one-loop analysis and $\hat{\Pi}^{(0,1)}_{\mathrm{A}}(q^2)$ are finite two-loop functions. These latter contain contributions from the order p^6 counterterm lagrangian in terms of renormalized LEC's. Note that there are kinematic poles at $q^2=0$ in both polarization functions defined in Eq. (20), although the sum $\Pi^{(1)}_{\mathrm{A}} + \Pi^{(0)}_{\mathrm{A}}$ is free of such poles.

The derivation of chiral sum rules proceeds by first obtaining dispersion theoretic expressions for the various polarization functions. The numbers of necessary subtractions is obtained from the asymptotic behavior ($s\to\infty$) of the spectral and polarization functions, which follows from perturbative QCD, (Floratos, Narison and Pich (1979), Narison and de Rafael (1980)). Entire sequences of chiral sum rules are obtained by evaluating arbitrary derivatives of such dispersion relations at $q^2=0$. Examples are (a=3,8)

$$\frac{1}{n!}\left[\frac{d}{dq^2}\right]^n \left(\Pi^{(1)}_{\mathrm{V}aa} - \Pi^{(1+0)}_{\mathrm{A}aa}\right)(0) = \int_0^\infty ds\frac{(\rho^{(1)}_{\mathrm{V}aa} - \rho^{(1+0)}_{\mathrm{A}aa})(s)}{s^{n+1}},\; n\ge 0 \quad (22)$$

$$\frac{1}{n!}\left[\frac{d}{dq^2}\right]^n \hat{\Pi}^{(0)}_{\mathrm{A}aa}(0) = \int_0^\infty ds\frac{\bar{\rho}^{(0)}_{\mathrm{A}aa}(s)}{s^n}, \quad n\ge 1 \qquad (23)$$

$$\frac{1}{(n-1)!}\left[\frac{d}{dq^2}\right]^{n-1} \hat{\Pi}^{(1)}_{\mathrm{A}aa}(0) - \frac{1}{n!}\left[\frac{d}{dq^2}\right]^n \hat{\Pi}^{(0)}_{\mathrm{A}aa}(0) = \int_0^\infty ds\frac{\rho^{(1)}_{\mathrm{A}aa}(s)}{s^n}. \quad (24)$$

Eq. (24) is valid for $n \geq 2$ and $\bar{\rho}^{(0)}_{\mathrm{A}aa}(s) \equiv \rho^{(0)}_{\mathrm{A}aa}(s) - F_a^2\delta(s - M_a^2)$.

As an application, consider the sum rule of Eq. (24) with $n = 2$ and isospin flavour. Explicitly (Golowich and Kambor (1997a))

$$4\left(2B_{32}^{(0)} - B_{33}^{(0)}\right)(\mu) - \frac{0.204 + \log\frac{M_\pi^2}{\mu^2} + \frac{5}{4}\log\frac{M_K^2}{\mu^2}}{3072\pi^4 F_\pi^2} = \int_0^\infty ds\ \frac{\rho^{(1)}_{\mathrm{A}33}(s)}{s^2}\ . \quad (25)$$

The first term inside the parentheses on the LHS arises from a scale-independent two-loop contribution. $2B_{32}^{(0)} - B_{33}^{(0)}$ is a renormalized coupling constant of the $O(p^6)$ counterterm lagrangian. To estimate it, we must evaluate the right-hand-side of Eq. (25). The dominant contribution to $\rho_{\mathrm{A}33}(s)$ arises from the $\mathrm{J}^{\mathrm{PC}} = 1^{++}$ a_1 resonance. But there is also some structure at higher energy due mainly to $n_\pi \geq 5$ multiparticle states, and fairly rapidly thereafter asymptotic behaviour sets in. Employing the fits of Ref. (Donoghue and Golowich (1994)) we obtain $(2B_{32}^{(0)} - B_{33}^{(0)})(M_{a_1}) \simeq 0.0044\ \mathrm{GeV}^{-2}$ with an error bar of about 15 %. The $\mathcal{O}(p^6)$ counterterm dominates the other terms on the LHS of Eq. (25), showing the importance of a full ChPT calculation as compared to a chiral-log treatment. However, the a_1 resonance contributes only $\simeq 0.0030\ \mathrm{GeV}^{-2}$, thus the principle of lowest lying resonance saturation is violated at the level of 30 % for this combination of $O(p^6)$ LEC's.

A phenomenological analysis of further chiral sum rules, including mass corrections to the Das-Mathur-Okubo sum rule (Das, Mathur and Okubo (1967)), is under investigation.

4.3 Meson-meson scattering in a nonperturbative method

In $O(p^4)$ χPT the amplitude *matrix* is obtained as $T \simeq T_2 + T_4 + ...$

We are proposing a method where (Oller et al. (1997))

$$T \simeq T_2 \cdot [T_2 - \mathrm{Re}T_4 - T_2 \cdot \mathrm{Im}G \cdot T_2]^{-1} \cdot T_2 \quad (26)$$

with G satisfying $T_2 \cdot \mathrm{Im}G \cdot T_2 = \mathrm{Im}T_4$.

If the complete $O(p^4)$ calculations are available, it reduces to:

$$T \simeq T_2 \cdot [T_2 - T_4]^{-1} \cdot T_2 \quad (27)$$

which is nothing but the generalization of the Inverse Amplitude Method (Truong (1988)) to coupled channels. However, at present, not all the $O(p^4)$ calculations for the $\pi\pi, \pi K, K\bar{K}, \pi\eta$ and $K\eta$ channels have been performed.

Nevertheless, we can use a similar approach to the Lippmann-Schwinger equations (Kaiser et al. (1995)), using

$$G = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{[q^2 - m_{i1}^2 + i\epsilon][(P-q)^2 - m_{i2}^2 + i\epsilon]} \quad (28)$$

to approximate:

$$\mathrm{Re}T_4 \simeq T_4^{tree} + T_2 \cdot \mathrm{Re}G \cdot T_2 \quad (29)$$

and thus we get

$$T \simeq T_2 \cdot [T_2 - T_4^{tree} - T_2 \cdot G \cdot T_2]^{-1} \cdot T_2 \tag{30}$$

In Fig. 2 we present the results of applying this method to meson-meson interactions below 1.2 GeV. The fit has 7 free parameters (the chiral parameters in the χPT $O(p^4)$ Lagrangian, but in a cut-off regularization scheme). It reproduces the data (see references in (Truong (1988)) and (Kaiser et al. (1995))) with a remarkable success up to 1.2 GeV, including six resonances: the $\sigma, \rho, K^*, f_0, a_0$ and ϕ. The reasons why such an extremely simple method works so well are still under study, together with possible applications to other processes (Oller et al. (1997)).

5 Other: Divergences at order p^6

As discussed at length in this working group, there are now several calculations available in the literature at order p^6 in CHPT. Unlike the calculations at order p^4, these have not been checked against the complete calculation of the divergences at the level of the generating functional, since until now this was not known at this order. In this talk I have described the results of this rather lengthy calculation that we (G. Colangelo, J. Bijnens and G. Ecker[1]) have just completed.

The generating functional Z is defined as:

$$e^{\frac{i}{\hbar}Z[f]} = \mathcal{N} \int \mathcal{D}\phi e^{\frac{i}{\hbar}S[\phi,f]} \quad , \tag{31}$$

where $\mathcal{N}$ by definition ensures $Z[0]=0$, and $S[\phi,f]$ contains a series of local actions which are the coefficients of an expansion in powers of $\hbar$:

$$S[\phi,f] = S_0 + \hbar S_1 + \hbar S_2 + O(\hbar^3) \quad , \quad S_i = \int dx \mathcal{L}_i \quad , \tag{32}$$

where f stands for a set of external fields. Accordingly, the generating functional $Z[f]$ can be calculated as an expansion in $\hbar$:

$$Z[f] = Z_0 + \hbar Z_1 + \hbar Z_2 + O(\hbar^3) \quad , \tag{33}$$

where $Z_0 = S_0$. While for Z_1 we know an explicit, closed expression (in terms of the logarithm of the determinant of a differential operator), that also makes the calculation of the divergent part trivial, the calculation of Z_2 is much more cumbersome. It requires the calculation of the diagrams shown in Fig. 3, whose exact meaning we cannot define here for lack of space. We refer the interested reader to (Jack and Osborn (1982)). A complete description of this calculation and of various subtleties connected to the renormalization at

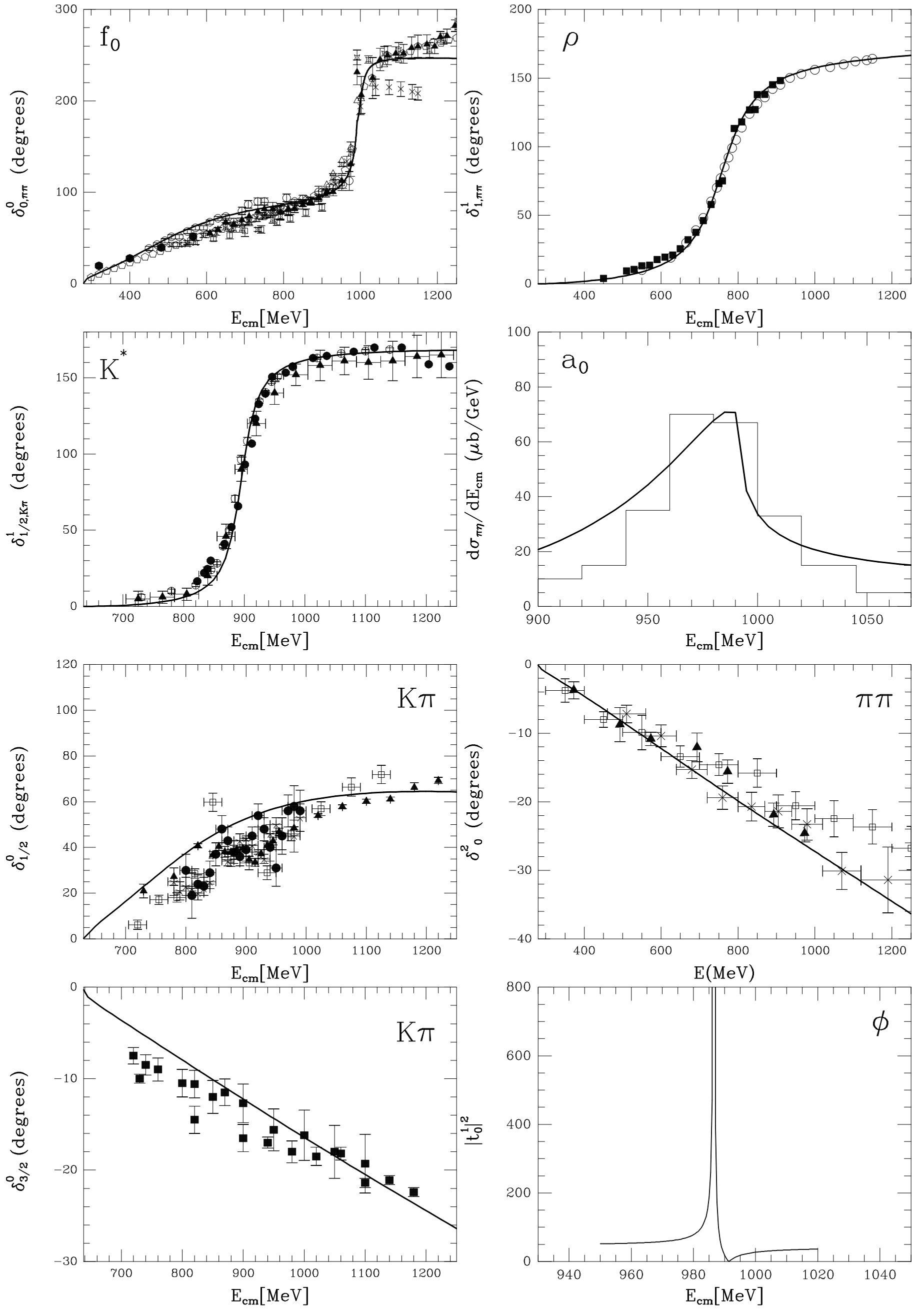

Fig. 2. The fit to the various meson scattering described in the text

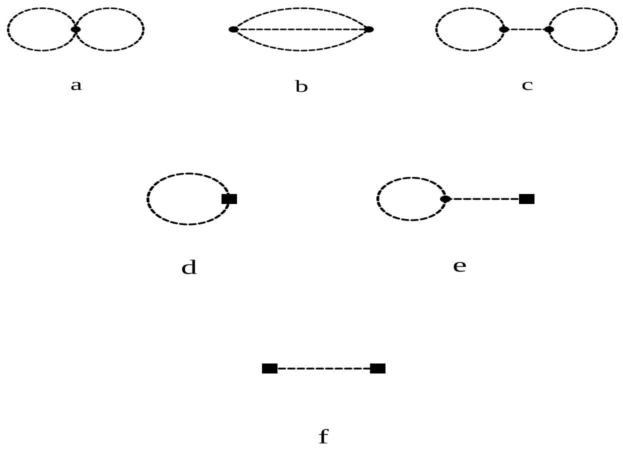

Fig. 3. Diagrams that contribute to Z_2. Vertices with dots stand for vertices coming from S_0 (*i.e.* vertices of order p^2 in CHPT), whereas vertices with a box stand for vertices coming from S_1 (*i.e.* vertices of order p^4 in CHPT).

order p^6 in CHPT (already discussed in part also in Bijnens et al. (1997a)) will be given in a forthcoming article (Bijnens et al. (1997b)).

In such lengthy calculations, any kind of check is most welcome. In this case the main one is: all divergences that are not polynomials in the external fields must cancel in the end. These kind of divergences are however present in single diagrams, and therefore the fact that they must cancel when all diagrams are summed up represents a very thorough check on the calculation.

The results of this calculation are best expressed in terms of a basis at order p^6. Such a basis has been constructed by Fearing and Scherer (1996) for the $SU(3) \otimes SU(3)$ case. Since we wanted to express our results for a generic number of light flavours N and also for $N = 3$ and 2, we have worked out our own basis (in fact two different versions of it). Our final list contains 115 independent terms in the general N case. We are implementing all possible trace relations for the $N = 3$ and $N = 2$ case and will then be able to compare to the basis chosen by Fearing and Scherer (1996).

6 Conclusions and Acknowledgments

The discussions in this working group show that there is progress and good future prospects both in theory and experiment in this area. We thank the organizers for a well run and efficient meeting. F.G. and G.I. thank the organizers for providing financial support for the attendance of this workshop.

[1] We also enjoyed the collaboration of J. Gasser in early stages of this project. We gratefully acknowledge his very important contribution.

References

Adler, R. et al., CPLEAR Collaboration (1997): Phys. Lett. B407 (1997) 193.
Adler, S. et al., E787 Collaboration (1997b): Preprint BNL-64627, hep-ex/9708012.
Adler, S. et al., E787 Collab. (1997c), Phys. Rev. Lett. 79 (1997) 2204
Alemany, R. M. Davier and A. Höcker, LAL 97-02, hep-ph/9703220.
ALEPH Coll., CERN-PPE/97-013, January 28, 1997.
Alkofer, R. and C. Roberts, Phys. Lett. B369(1996) 101
Amsler, C. hep-ex/9708025
Anomaly (1985): Donoghue, J., Holstein, B, Lin, Y: Phys. Rev. Lett. 55 (1985) 2766
Bijnens, J., Bramon, A, Cornet, F.: Phys. Rev. Lett. 61 (1988)
Bijnens, J., Bramon, A, Cornet, F.: Z. Phys. C46 (1990) 599
Donoghue, J., Wyler, D.: Nucl. Phys. B316 (1989) 289
Ametller, Ll., Bijnens, J., Bramon, A., Cornet, F: Phys. Rev. D45 (1992) 986
Bijnens, J.: Int. J. Mod. Phys. A8 (1993) 3045
Antipov, Y.N. et al., Phys. Rev. D36 (1987) 21
Antonelli, V. et al.: Nucl. Phys. B469 (1996) 181.
Bailey, J. et al., Phys. Lett. B 68 (1977) 191; F.J.M. Farley, E. Picasso, in "Quantum Electrodynamics", ed. T. Kinoshita, World Scient. 1990.
Barnett, R. et al. (1996): *Review of Particle Properties*, Phys. Rev. D54 (1996) 1.
Bergström, L., Massó, E., Singer, P. (1983-1990): Phys. Lett. B131 (1983) 229; Phys. Lett. B249 (1990) 141.
Bernstein, A., Kaiser, N. (1997): Working group on Electromagnetic Production of Goldstone Bosons, these proceedings
Bijnens, J. A. Bramon and F. Cornet, Phys. Lett. B237 (1990) 488
Bijnens, J. (1997): these proceedings
Bijnens, J., Prades, J.: Z. Phys. C64 (1994) 475
Bijnens, J. E. Pallante and J. Prades (1995), Phys. Rev. Lett. 75 (1995) 1447; 75 (1995) 3781 (E); Nucl. Phys. B474 (1996) 379.
Bijnens, J., Talavera, P.: Nucl. Phys. B489 (1997) 387
Bijnens, J. G. Colangelo, G. Ecker, J. Gasser and M. Sainio (1997a), hep-ph/9707291, Nucl. Phys. B, in press.
J. Bijnens, G. Colangelo and G. Ecker (1997b), work in progress.
Bondar, B. et al., Phys. Lett. B356 (1995) 8
Browder, T. et al, CLEO collab., CLNS-97-1484, June 1997, hep-ex/9706005
Buchalla, G., Buras, A.J. (1994): Nucl. Phys. B412 (1994) 106.
Buchalla, G., Buras, A.J. (1996): Phys. Rev. D54 (1996) 6782.
Buras, A.J. et al. (1994): Nucl. Phys., B423 (1994) 349.
Calén, H. et al., Nucl. Instr. Meth. A379 (1996) 57.
Calén, H. et al., Phys.Rev.Lett. 79 (1997)2642.
Calén, H. et al., preprint TSL/ISV-97-0181.
Cappiello, L., D'Ambrosio, G. (1988): Nuovo Cimento 99A (1988) 155.
Cappiello, L., D'Ambrosio, G., Miragliuolo, M. (1993): Phys. Lett. B298 (1993) 423.
CLEO Collaboration, CLNS 97/1477, hep-ex/9707031
Czarnecki, A. B. Krause and W. Marciano, Phys. Rev. D52 (1995) R2619; Phys. Rev. Lett. 76 (1996) 3267;
S. Peris, M. Perrottet and E. de Rafael, Phys. Lett. B355 (1995) 523;
A. Czarnecki, B. Krause, hep-ph/9606393.
T.V. Kukhto et al., Nucl. Phys. B371 (1992) 567.

Cohen, A.G., Ecker, G., Pich, A. (1993): Phys. Lett. B304 (1993) 347.
D'Ambrosio, G. et al. (1994): Phys. Rev. D50 (1994) 5767; (E) D51 (1994) 3975.
D'Ambrosio, G. et al. (1995): in *The Second DAΦNE Physics Handbook*, Eds. Maiani, L. Pancheri, G., Paver, N. (Frascati, 1995), p. 265.
D'Ambrosio, G. et al. (1996): Phys. Lett. B380 (1996) 165; Z. Phys. C76 (1997) 301.
D'Ambrosio, G., Isidori, G. (1996): Preprint INFNNA-IV-96/29, hep-ph/9611284.
D'Ambrosio, G., Portolés, J. (1996): Phys. Lett. B389 (1996) 770; (E) B395 (1997) 389.
D'Ambrosio, G., Portolés, J. (1997): Nucl. Phys. B492 (1997) 417.
D'Ambrosio, G., Isidori, G., Portolés, J. (1997): preprint INFNNA-IV-97/40, hep-ph/9708326.
Das, T., V.S. Mathur and S. Okubo, Phys. Rev. Lett. 19 (1967) 859
Donoghue, J.F., Gabbiani, F. (1995): Phys. Rev. D54 (1995) 2187.
Donoghue, J.F., Gabbiani, F. (1997): Phys. Rev. D56 (1997) 1605.
Donoghue, J.F. and E. Golowich, Phys. Rev. D49 (1994) 1513
Drechsel, D.(1997): Working Group on Hadronic Form Factors, these proceedings
Ecker, G., Pich, A., de Rafael, E. (1987a): Phys. Lett. B189 (1987) 363.
Ecker, G., Pich, A., de Rafael, E. (1987b): Nucl. Phys. B291 (1987) 692.
Ecker, G., Pich, A., de Rafael, E. (1988): Nucl. Phys. B303 (1988) 665.
Ecker, G., Pich, A., de Rafael, E. (1990): Phys. Lett. B237 (1990) 481.
Ecker, G., Kambor, J., Wyler, D. (1993): Nucl. Phys. B394 (1993) 101.
Ecker, G. (1995): Prog. Part. Nucl. Phys. 35 (1995) 1.
Ecker, G. (1997): these proceedings.
Fearing, H. and S. Scherer, Phys. Rev. D53 (1996) 315.
Floratos, E.G, S. Narison and E. de Rafael, Nucl. Phys. B155 (1979) 115
Gasser, J. (1997): these proceedings
Gasser, J. and H. Leutwyler, Nucl. Phys. B250 (1985) 465
Georgi, H. Phys.Rev. D49 (1994) 1666
Golowich, E. and J. Kambor, Nucl. Phys. B447 (1995) 373
Golowich, E. and J. Kambor, Phys. Rev. D53 (1996) 2651
Golowich, E. and J. Kambor, 'Chiral Sum Rules to Second Order in Quark Mass', Phys. Rev. Lett. (to be published); hep-ph/9797341
Golowich, E. and J. Kambor, 'Two-loop Analysis of Axialvector Current Propagators in Chiral Perturbation Theory', hep-ph/9710214
Hayakawa, M. T. Kinoshita and A. Sanda (1995), Phys. Rev. Lett. 75 (1995) 790; Phys. Rev. D54 (1996) 3137;
M. Hayakawa and T. Kinoshita, KEK-TH530, hep-ph/9708227
Heinson, A.P. et al., E791 Collaboration (1995): Phys. Rev. D51 (1995) 985.
Herrera-Siklody, P. et al.: Nucl.Phys.B497 (1997) 345 and hep-ph/9710268
Holstein, B Phys. Rev. D53 (1996) 4099
Jack, I. and H. Osborn, Nucl. Phys. B207 (1982) 474.
Jegerlehner, F. hep-ph/9606484, and refs. therein;
S. Eidelman, F. Jegerlehner, Z. Phys. C67 (1995) 585;
T. Kinoshita, B, Nizić, Y. Okamoto, Phys. Rev. D 31 (1985) 2108;
W.A. Worstell, D.H. Brown, Phys. Rev. D 54 (1996) 3237.
Kaiser, N. P. B. Siegel and W. Weise, *Nucl. Phys.* **A594** 325 (1995). J. A. Oller and E. Oset, *Nucl. Phys.* **A620** 438(1997).

Kambor, J., Missimer, J., Wyler, D. (1989): Nucl. Phys. B346 (1990) 17.
Kambor, J., Missimer, J., Wyler, D. (1991): Phys. Lett. B261 (1991) 496.
Kambor, J., Donoghue, J.F., Holstein, B.R., Missimer, J., Wyler, D. (1992): Phys. Rev. Lett. 68 (1992) 1818.
Kambor, J., Holstein, B.R. (1994): Phys. Rev. D49 (1994) 2346.
Kinoshita, T. B. Nizić, Y. Okamoto, Phys. Rev. D 31 (1985) 2108.
Kinoshita, T. Rep. Prog. Phys. 59 (1996) 1459.
Kitching, P. et al., E787 Collaboration (1997): BNL-64628, hep-ex/9708011.
Krause, B. hep-ph/9607259.
Lee Roberts, B. Z. Phys. C 56 (Proc. Suppl.) (1992) 101;
see also `http://www.phy.bnl.gov/g2muon/home.html`
Lepage, G. S.J. Brodsky, Phys. Lett. B 87 (1979) 359;
Manohar, A. Phys. Lett. B 244 (1990) 101;
J.M. Gérard, T. Lahna, Phys. Lett. B 356 (1995) 381.
Leutwyler, H. Phys. Lett. B374 (1996) 181 and hep-ph/9709408
Littenberg, L. (1989): Phys. Rev. D39 (1989) 3322.
Lowe, J. for the BNL E865 experiment, 1997.
Meißner, U.-G., Sevior, M. (1997): Working Group on $\pi\pi$ and πN interactions , these proceedings
Miskimen, R. K. Wang and A. Yegneswaran (Spokespersons), *Study of the Axial Anomaly using the* $\gamma\pi^+ \to \pi^+\pi^0$ *Reaction near Threshold, TJNAF-E94015*
Moinester, M (1997): these proceedings
Moroi, T. Phys. Rev. D 53 (1996) 6565;
M. Carena, G.F. Giudice, C.E.M. Wagner, hep-ph/9610233.
Narison, S. and E. de Rafael, Nucl. Phys. B169 (1980) 253
Novikov, V.A. et al. (1977): Phys. Rev. D16 (1977) 223.
O'Dell, V., KTeV collab., *Status and New Results of the KTeV Rare Kaon Program*, MIST workshop, May 1997, Fermilab.
Oller, J.A., E. Oset and J. R. Peláez, in preparation, 1997.
Pallante, E. Phys. Lett. B341 (1994) 221
Peris, S. Phys. Lett. B324 (1994) 442
Peris, S. and E. de Rafael, Phys. Lett. B348 (1995) 539
Pich, A., de Rafael, E. (1991): Nucl. Phys. B358 (1991) 311.
de Rafael, E. (1994): *Chiral Lagrangians and kaon CP violation*, in *CP Violation and the limits of the Standard Model* TASI94 Proceedings (Boulder, 1994).
de Rafael, E. Phys. Lett. B322 (1994) 239
Rosselet, L. et al., Phys. Rev. D15 (1977) 574
Stern, J. (1997): these proceedings
Truong, T.N. *Phys. Rev. Lett.* **61** (1988) 2526, ibid **67**, 2260 (1991); A. Dobado, M. J. Herrero and T. N. Truong, *Phys. Lett.* **B235** 129 (1990); A. Dobado and J. R. Peláez, *Phys. Rev.***D47**, 4883 (1992), ibid *Phys. Rev.***D56** (1997) 3057; T. Hannah, *Phys. Rev.***D52** 4971(1995), ibid **D54** 4648 (1996), ibid**D55** 5613 (1997) and this proceedings in Drechsel (1997).
Ueda, T. Phys. Rev. Lett. 66 (1991) 297 and Ueda, T. Phys. Lett. B292 (1992) 228
van Kolck U. (1997): these proceedings
VEPP 2M, new unpublished data, Novosibirsk
Zou, Y. et al. (1996): Phys. Lett. B369 (1996) 362.