LU TP 97-36
November 1997

What happens at the end of the QCD cascades?

Bo Andersson[1]

Department of Theoretical Physics, Lund University,
Sölvegatan 14A, S-223 62 Lund, Sweden

Abstract

This is a progress report on recent work done in Lund to describe the final steps in the QCD coherent bremsstrahlung cascades, with particular emphasis on the occurrence of a size parameter $c = 11/6$ in phase space. Two gluons, being interacting vector particles, will due to helicity conservation effectively stay apart a distance c, counted in generalised rapidity (a notion which I will define in the text but essentially corresponds to rapidity measured along the color flux lines of the field between the gluons). The same size c also occurs in the β-function as a measure of the region, inside which a virtual gluon splitting via reabsorbtion in the next step of perturbation theory will correspond to a "loss-term" in the Callan-Symanzik equations. Our conclusion is that at the end of the cascades when the dipole masses (M_d) are such that $\log(M_d^2/\Lambda^2) \simeq nc$ with n a few units, then an ordered field of a helix character will emerge, i.e. a set of final "screwy gluons" are emitted in an ordered way in rapidity and azimuthal angle around the dipole axes so that the color lines are turning around.

[1] bo@thep.lu.se

What happens at the end of the QCD cascades?

Bo Andersson[a†]

[a]Department of Theoretical Physics,
University of Lund,
Sölvegatan 14A, 22362 Lund, Sweden

This is a progress report on recent work done in Lund to describe the final steps in the QCD coherent bremsstrahlung cascades, with particular emphasis on the occurrence of a size parameter $c = 11/6$ in phase space. Two gluons, being interacting vector particles, will due to helicity conservation effectively stay apart a distance c, counted in generalised rapidity (a notion which I will define in the text but essentially corresponds to rapidity measured along the color flux lines of the field between the gluons). The same size c also occurs in the β-function as a measure of the region, inside which a virtual gluon splitting via reabsorbtion in the next step of perturbation theory will correspond to a "loss-term" in the Callan-Symanzik equations. Our conclusion is that at the end of the cascades when the dipole masses (M_d) are such that $\log(M_d^2/\Lambda^2) \simeq nc$ with n a few units, then an ordered field of a helix character will emerge, i.e. a set of final "screwy gluons" are emitted in an ordered way in rapidity and azimuthal angle around the dipole axes so that the color lines are turning around.

1. Introduction

Let me first of all confess that some parts of this talk contain statements that have not been proved as of now. My only excuse is that we have a set of results which all of them indicate that the basic ideas are sound. At the moment we (which means me, Gösta Gustafson and our postdoc Peter Sutton) are looking for experimental observables which can settle if the predicted effects actually occur. As this is a complicated thing it will take some time before we will come back to try to prove "everything".

I will use the timelike cascades in (perturbative) QCD as a theoretical laboratory to explore the ideas. Thus we will consider the result of a (large) energy excitation in the vacuum, resulting in the production of color-anticolor together with the ensuing bremsstrahlung radiation. This will via the ordinary Lund way of thinking lead to the production of a set of color dipoles which are moving apart in a fan-like way. In this way we have emphasized *the field links*, i.e. the dipoles between the bremsstrahlung quantas. If one instead considers the production of the quantas themselves, they will correspond to *the excitations in between the field links* and there is obviously a duality between the two ways of describing the process. Further if we instead would have considered the probing of the hadronic wave function towards larger and larger virtualities (i.e. the "space-like cascades") it is shown in the Linked Dipole Model, [1], that a very similar picture will emerge and essentially everything stated in this talk can be taken over.

I will during the talk compare the behaviour of the QCD fields to the corresponding behaviour in QED in order to emphasize the differences between a non-abelian and an abelian gauge theory. Actually, almost all our findings are independent of the particular gauge group.

The basic results stem from the fact that there is a *field self-interaction in the nonabelian theories* , i.e there is a coupling between three massless vector fields (there is also to preserve the symmetry a four-gluon coupling but it does not play any role for the main physics). This does not occur in QED, where the *photons are chargeless.* This feature of the nonabelian theories will, due to the *helicity conservation of the coupling*, result in a diminished phase space region for gluon emission with a typical conar size corresponding

†Work done in Lund together with G. Gustafson and P.Sutton

to $\delta y \equiv c = 11/6$ around each gluon (and c is a number independent of the gauge group).

On the other hand, the same mechanism also implies that the radiation of a new gluon with (generalised) transverse momentum $k_\perp$ from a dipole with mass M_d will result in two new dipoles, that move apart in the cms of the original one. While the rapidity region available for gluon emission in the original dipole cms is $(\Delta y)_1 = \log(M_d^2) - c$, the (generalised) rapidity region available for gluon emission from the two new dipoles will be $(\Delta y)_2 = (\Delta y)_1 + \log(k_\perp^2) - c$. Evidently there is an increase in the phase space for gluon emission as long as $\log(k_\perp^2) > c$. This means that hard gluon emission is favorable for further emissions (although suppressed by the generally small size of the running coupling). But the main thing is that *there is a competition between an increase and a decrease in the available phase space.*

At the end of the cascades we feel that there is an effective cutoff (we have actually already exploited these features in connection with the Discrete QCD Model, [2]) when the dipole masses are so small that there is no further real gluon emissions possible due to helicity conservation. At this level, however, there is another possibility available and that is to produce an ordered field exploiting both the rapidity and the azimuthal angle degrees of freedom. This configuration, which was named the "screwy gluon" option by Bj, corresponds to a helix shaped color field configuration turning around the final dipole axes. It will be presented and the possibility for its emergence will be discussed. We are at the moment in Lund investigating whether it has some experimental consequences in terms of observables.

In Section 2 I will present the necessary material to understand the Dipole Cascade and the emergence of the number c and in Section 3 the reasons why we think that the Screwy Gluon option is very probable. I have (at the time of this talk) no predictions for experimental observables but I hope to be able to tell you more about it next time.

2. The Dipole Cascades; Increase and Decrease of Phase Space

Bremsstrahlung is an almost classical effect and it is a necessary feature of all gauge field theories. In all of them a charged particle is surrounded by an extended Coulomb field. If there is a sudden change in the state of the charge (a "local interaction") then it takes some time for the fields to readjust and there will be variations resulting in outward moving radiation, i.e. what we call bremsstrahlung. The same happens if we make a sudden large energy insertion in the vacuum, thereby producing a charge-anticharge pair (I will call them e^-e^+ in QED and either $q\bar{q}$ or $3\bar{3}$ color charges in QCD). Then it is necessary to "build up the Coulomb fields" around the outward moving quanta and there will be radiation of a similar kind.

For a sufficiently local interaction the bremsstrahlung spectrum will be of a dipole character, i.e. there is a direction (the dipole axes) but no extension (no formfactor suppression from an extended charge distribution in space-time). In this case the inclusive distribution of the number of gauge particles sent out in phase space is

$$dn = \bar{\alpha}\frac{dk_\perp^2}{k_\perp^2}dy(\frac{d\phi}{2\pi}) \times Polsum \tag{1}$$

where $\bar{\alpha}$ is the effective coupling, $k_\perp$, y and ϕ the transverse momentum, rapidity and azimuthal angle (the azimuthal angle dependence is usually neglected) and the final factor ($Polsum$), corresponding to the spin couplings, will be further discussed below. We will briefly consider the precise definitions before we consider the implications. The effective coupling is for QED and for QCD given by

$$\bar{\alpha}_{QED} = \frac{\alpha}{\pi} \simeq \frac{1}{137\pi}$$
$$\bar{\alpha}_{QCD} = \frac{N_c\alpha_s}{2\pi} \simeq \frac{6}{11\log(k_\perp^2/\Lambda^2)} \tag{2}$$

Actually at LEP1 the QED coupling has increased to $1/128\pi$ (which is according to the rules for the running of that quantity). The occurrence of the number of colors and the factor 1/2 in the QCD coupling is due to early conventions but the result that *the running is governed by* $1/c = 6/11$ *is a basic gauge group independent result* (I have neglected the $-2n_f/3$ which should accompany 11 in the denominator because it is a small effect related to the possibility of gluon splitting $g \to q\bar{q}$) and we will come to it later).

The transverse momentum and the (dipole cms) rapidity are defined in a Lorentz invariant way in terms of the squared masses of the final state partons (the emitters are conventionally indexed 1 and 3 and the emitted gluon 2):

$$\begin{aligned} s_{ij} &= (k_i + k_j)^2 = 2k_ik_j \\ &= 2k_{ti}k_{tj}(cosh(\Delta y)_{ij} - cos(\Delta\phi)_{ij}) \\ s_{12} + s_{23} + s_{13} &= s \\ s_{12} = s(1-x_3), s_{23} &= s(1-x_1) \\ k_\perp^2 &= \frac{s_{12}s_{23}}{s} \\ y &= \frac{1}{2}\log\left(\frac{s_{12}}{s_{23}}\right) \end{aligned} \tag{3}$$

The requirement of energy momentum conservation determines the phase space

$$k_\perp \cosh(y) \leq \frac{\sqrt{s}}{2} \tag{4}$$

and it can be conveniently approximated as $|y| < (L-\kappa)/2$ with the variables $\mathrm{L} \equiv \log(s/\Lambda^2)$ and $\kappa \equiv \log(k_\perp^2/\Lambda^2)$.

This means that the phase space is the interior of a triangular region (the Lund Triangular Phase Space, which I have described and depicted in earlier talks) in the (y,κ)-plane with the height (in κ for $y=0$) and the width (in y for $\kappa = 0$) equal to L. The inclusive density inside the triangle is in this Leading-Log Approximation given by the effective coupling $\bar{\alpha}$ according to Eq (1). The rapidity range, $L-\kappa$, is of course the length of a hyperbola spanned between the emitters in a space-time (or energy-momentum space) picture (at the scale $k_\perp^2$).

For an exclusive statement like e.g. the probability dP to emit the first gluon with a certain (κ_1, y_1) (first meaning no gluon with larger transverse momentum) it is necessary to multiply the inclusive formula in Eq (1) with a Sudakov formfactor containing the probability not to emit above κ_1:

$$\begin{aligned} Sud(L,\kappa_1) &= \exp(-\int_{\kappa_1}^{L} dn) \\ dP(q,g_1,\bar{q}) &= dn(\kappa_1,y_1)Sud(L,\kappa_1) \end{aligned} \tag{5}$$

The probability to emit two gluons is a complicated expression (covering one and a half pages in Nucl PhysB) but it can to about a percentage error all over phase space be approximated by

$$\begin{aligned} dP(q,g_1g_2,\bar{q}) &= dP(q,g_1,\bar{q}) \times \\ &\times[dP(qg_2g_1) + dP(g_1g_2\bar{q})] \end{aligned} \tag{6}$$

in easily understood notations. Thus contrary to QED where the chargeless photons still leave the e^-e^+ current as the single emitter, the 8-charge gluon (g_1) in QCD changes the original $q\bar{q}$ dipole into two dipole emitters, one between the qg_1 and one between the $g_1\bar{q}$, and *each can independently emit the second gluon (g_2).* The requirement for the validity of the approximation in Eq (6) is that $k_{\perp 1} \geq k_{\perp 2}$ or else the indices are exchanged.

The two independent dipoles are moving apart (with the g_1 "corner" connecting them) and this means that they have together a larger rapidity range for the emission of g_2, i.e. the original hyperbola length $L = \log(s)$ is exchanged for two hyperbolas with combined length $\log(s_{qg_1}) + \log(s_{g_1\bar{q}}) = L + \log(k_{\perp 1}^2)$.

From anyone of the two new dipoles we may then emit the second gluon, thereby producing three independent dipole emitters and the process can be continued towards more dipoles (with smaller masses). The corresponding generalised rapidity range counted along the colorlines from the jth to the $j+1$st gluon in color order will be $\Delta y = \sum_j \log(s_{jj+1}) = L + \sum_j \log(k_{\perp j}^2)$ (we have defined this so-called λ-measure in an infrared stable way in[3]). The main point is that all the gluonic excitations can be described each with a well-defined λ-value for its position along the color flow.

Before we continue we return to Eq (1) and in particular consider the polarisation sum contribution (which has up to now been neglected and put equal to unity). Its precise properties depends upon if we are dealing with a $q\bar{q}$, qg ($g\bar{q}$) or gg dipole but it stems from the spin couplings between the emitter(s) (it is essentially only sensitive to the closest emitter) and the new gluon. *These couplings have the property in all gauge theories that they are helicity conserving.*

This means that if a spin-1/2 parton emits a spin-1 parton, keeping its helicity, then the spin-1 "child" must go apart from the "mother"-particle and the same goes even more for a spin-1 emitting a spin-1. To estimate the effect we consider (for fixed $k_\perp$ or κ) the available rapidity range:

$$\int_{y_{min}}^{y_{max}} dy(Polsum) = L - \kappa - c + O(k_\perp^2/s) \qquad (7)$$

where $c = 11/12 + 11/12$, $3/4 + 11/12$ or $3/4 + 3/4$ if the emitters are gg, qg or $q\bar{q}$. The quantities c are written as sums to show that a spin-1 (g) emitter and a spin-1/2 (q or $\bar{q}$) emitter has an empty conar region of size 11/6 and 3/2, respectively, surrounding it. (In order to understand the result we note that in terms of the x-variables introduced in Eq (3) the $(Polsum)$ factor is $(x_1^{n_1} + x_3^{n_3})/2$ with $x_{1,3} = 1 - k_\perp \exp(\pm y)/\sqrt{s}$ and $n_{1,3}$ equal to 2 or 3 for $q(\bar{q})$ and g, respectively. $y_{max,min}$ are determined from the energy momentum requirement in Eq (4).

It is worth while noticing that the gluon splitting process $g \to q\bar{q}$, where a spin-1 emits two spin-1/2 partons, does not behave as the earlier mentioned processes. The process $g \to gg$ behaves as dy and therefore corresponds to a wide spread in rapidity $\simeq L - \kappa$. There is no z or $(1-z)$ pole in the splitting function for the $g \to q\bar{q}$ case so it behaves instead as $dy_r \exp(-|y_r|)$ and the effective distance in relative rapidity y_r is 2/3 between the q and $\bar{q}$ in order to conserve "helicity". The gluon splitting process is only of the order of a few percent compared to the gluon emission process and can therefore in general be neglected.

Thus the Dipole Cascade Model will produce a fan-like set of dipoles at each level, which in the LLA increases the phase space (the generalised rapidity along the color lines) for further emissions. But if the influence from the $(Polsum)$ term is included (Modified LLA) there is at each emission also an empty conar region around a really emitted parton.The first part dominates at large energies and large transverse momenta but the second takes over at the end of the cascades. The fact that the MLLA correction is also sufficient to take into account (most) of the recoil corrections in the emissions is very gratifying.

3. The Reabsorption and the Screwy Gluon Field

The Callan-Symanzik equations are designed to describe the change in a production matrix element $\mathcal{M}$ if one changes the observation scale, i.e. if we consider the change in going from κ to $\kappa - d\kappa$ in the Dipole Cascade. There are then a set of dynamical possibilities for the matrix element to change, i.e. each dipole rapidity range will increase $L(M_d^2, k_\perp^2) \equiv \log(M_d^2/\Lambda^2) - \kappa \to L(M_d^2, k_\perp^2) + d\kappa$, some dipole(s) may decay by "real" gluon emission etc. But the Callan-Symanzik equation also contains the next-order, in the coupling, possibility that an already emitted quantum is reabsorbed by the inverse process, i.e. if we consider gluon emission $g \to gg$ as the real process there is evidently the inverse process $gg \to g$ (possible if the two gluons are going sufficiently close in phase space).

Vacuum quantum fluctuations are virtual and short-lived and do not necessarily conserve the quantum numbers. In particular the helicity quantum number, that we have discussed above, need not conserved. Actually when we change $\kappa \to \kappa - d\kappa$ there is in the Callan-Symanzik equation the contributions (there are more contributions depending upon the variables we are investigating but we do not need them for this discussion)

$$\frac{\partial \mathcal{M}}{\partial \kappa} - \beta(\alpha)\frac{\partial \mathcal{M}}{\partial \alpha} + \cdots = 0 \qquad (8)$$

(as κ decreases the sign in front of the β function is different from the conventional one). A useful way to write the contribution from the β term in QCD is

$$-\beta(\alpha_s)\frac{\partial \mathcal{M}}{\partial \alpha_s} = (\frac{11}{6}\frac{3\alpha_s}{2\pi} - \frac{2}{3}\frac{n_f\alpha_s}{4\pi})\alpha_s\frac{\partial \mathcal{M}}{\partial \alpha_s} \qquad (9)$$

In this way we have clarified *the gain-loss character of the β term.* The operator $\alpha_s\partial\mathcal{M}/\partial\alpha_s$ is a number operator, i.e. for any function $\mathcal{M} = \sum \alpha_s^n m_n$ it provides the number n of possible insertions, the quantities $3\alpha_s/2\pi$ and and $n_f\alpha_s/4\pi$

are the couplings for $gg \to g$ and $q\bar{q} \to g$ (and the inverse processes) while 11/6 and 2/3 corresponds to the effective (generalised) rapidity ranges available for these reabsorption processes for a given κ.

The first part of Eq (9), i.e. the inverse of the gluon emission term, is a loss term because if we have emitted two gluons at κ and "counted them", we find at $\kappa - d\kappa$ only one gluon when they are reabsorbed again. The second part (which occurs also in QED and always is proportional to the number of fermions) is a gain term because if we have split a gluon at κ and reabsorbed the $q\bar{q}$ at $\kappa - d\kappa$ we have gained one gluon compared to the situation at κ.

In other words, the loss-region we introduced before around each emitted gluon, 11/6, is accounted for in the sense that gluons emitted inside this region are effectively reabsorbed at the next order in perturbation theory. (Evidently there is not a completely fixed number for this rapidity range, there are exponential tails allowing emission inside and reabsorbtion outside the cones but the number(s) c are useful estimates of the effects and the remaining terms $O(k_\perp^2/s)$ in Eq (7) are generally small).

The QCD perturbative cascades are slow processes and the average behaviour can be well described in terms of a hydro-dynamical flow together with gaussian fluctuations around the mean. But when one comes down to the region where the dipole masses fulfil $\log(M_d^2/\Lambda^2) \simeq nc$ with n a few units, then the recoil contributions are generally very noticable (actually so large that the coherence criteria become fuzzy, i.e. the recoils are generally of the same order or larger than the emissions so that κ-ordering becomes impossible).

At this level it is evident that many different diagrammatic contributions start to become of the same order and in the final result one must take into account the interference effects. I will now present a possible scenarium which may take over at this level, i.e. a semi-classical gluon field in which the field excitations occur in an ordered way.

If we go back to (the first line in) Eq (3) we note that when the "true" rapidity differences Δy are of order unity, then the "distance", d between two gluons in phase space is also influenced by the azimuthal angular difference $\Delta\phi$:

$$d^2 = 2(cosh(\Delta y) - \cos(\Delta\phi)) \simeq$$
$$((\Delta y)^2 + (\Delta\phi)^2)(1 + \frac{(\Delta y)^2 - (\Delta\phi)^2}{12} + \cdots) \quad (10)$$

As the emission probability for n color-ordered gluons is governed by

$$\prod_{j=1}^{n-1} \frac{\bar{\alpha}}{s_{jj+1}} \propto \prod_{j=1}^{n-1} \frac{\bar{\alpha}}{d_{jj+1}^2} \equiv P_{\mathcal{D}} \quad (11)$$

it is accordingly for sufficiently large $\bar{\alpha}$ favorable to emit many gluons but also to emit them so that the distances d between them become small.

In the celebrated "lego-plots" in rapidity y and azimuthal angle ϕ, which Bj prefers in his "plumbing scenario" compared to the Triangular Phase Space, usually used in the Lund description of the bremsstrahlung, we can investigate the result of such a toy model. The rules are 1) Require that the minimum distance d_m between two gluons is $d_m = 11/6$ (helicity conservation and no reabsorption on the next level) 2)Find the maximal contribution $P_{\mathcal{D}}$ from the possible distributions of gluons $\mathcal{D}$.

The range in ϕ is fixed, equal to 2π, (note that ϕ is a periodical variable such that a point with $\phi = 0$ is equivalent to $\phi = 2\pi$ for fixed y) but the result depends upon (besides the minimum distance $d_m = c$) the size of the effective coupling $\bar{\alpha}$ and the range in rapidity ℓ. When the coupling is large there is a tendency to emit many gluons but the condition 1) above means that the number is finite. Actually when the rapidity size ℓ is sufficiently small (we identify this with the dipole sizes $\log(M_d^2/\Lambda^2)$ at the end of the cascade, i.e. in general a few units of c) then there is a major contribution from a particular close-packing distribution $\mathcal{D}_s$ (as of now we have no precise value for the largest ℓ, which is allowed for a given value of $\bar{\alpha}$).

This distribution $\mathcal{D}_s$ can be described in the following way. There are lines in the (ϕ, y) lego-plot with a constant $dy/d\phi = c/2\pi \equiv f$ and all the gluons are emitted along them at a distance c in color-order (actually it is a single line, which if it starts at $y = 0, \phi = 0$ will at $y = c$ come back at $\phi = 0$ etc.). Evidently this corresponds to a color field which is turning around the axis and there are $2\pi/c \simeq 3.5$ gluons per turn and as each such takes up c units in rapidity there will be $2\pi/c^2 \simeq 1.9$ gluons per rapidity unit.

Actually, there is a continuous field correspondence to the situation described above. Consider an energy-momentum space vector current, described by the space-like four-vector $k(t)$. It is defined along curves $\mathcal{C}(t)$ in the $(y\phi)$ legoplot plane (t parametrises the points along the curve(s) $\mathcal{C} \equiv [y(t), \Phi(t)]$ and Φ is a multiple of the azimuthal angle ϕ). We further assume that the current is emitting a continuous stream of gluons dk/dt along the curve. The gluons are lightlike, i.e. $(dk/dt)^2 = 0$ while the current in the general case has a varying virtuality $k^2 = -m^2(t)$. A general formula for $k(t)$ is (in lightcone components and transverse components):

$$k = m[cos(\theta)(\exp(y), -\exp(-y), 0, 0) + \sin(\theta)(0, 0, \cos(\Phi), \sin(\Phi))] \qquad (12)$$

where the sine and cosine of the angle $\theta(t)$ corresponds to the relative weight of the transverse and longitudinal (space) vector components of k. The condition of an everywhere lightlike derivative then leads directly to a differential equation:

$$\frac{1}{2}\left[\frac{d(2\theta)}{dt}\right]^2 + (1 - cos(2\theta) = (1 - cos(2F)) - 2\left[\frac{d\log(m)}{dt}\right]^2 \qquad (13)$$

Here we have introduced the angle F as the direction of the unit tangent vector to the curve $\mathcal{C}$ in the legoplot plane, i.e. $(dy/dt, d\Phi/dt) = (sin(F), cos(F)$ (which means that the parameter t is identified with the length element along the curve). The Eqs (12) and (13) are very general (and we recognise the pendulum equation for the angle 2θ). But if we specialise to the case considered above where the gluons are emitted along straight lines with the small angle f in the legoplot we may go to the small-angle limit of Eq (13) to obtain a harmonic oscillator eqation with a simple cosine solution. In this way we end up with the result that $t = \phi$ and $y = f\phi$ (with proper boundary conditions and

$$\frac{dk}{d\phi} = m(\exp(y), \exp(-y), f\cos(\phi), f\sin(\phi)) \qquad (14)$$

These kind of fields are similar to those studied in the 80's (during the time when "dimensional reduction" was popular) and are almost identical to those described by N.K Nielsen as "gauge fields" on a string sheet (a two-dimensional surface embedded in four-dimensional space-time).

I will end at this point with the remark that our present phenomenological findings are that $m \simeq \Lambda\exp(-c/2)$ (i.e. with the ordinarily used Λ in the perturbative cascades the "final" dipole masses are just where we expected them to be) and that the only other number in the present considerations $c = 11/6$ is fixed by the considerations presented above.

REFERENCES

1. B. A., G. Gustafson and J. Samuelsson, Nucl. Phys. B467 (1996) 443.
2. B. A., G. Gustafson and J. Samuelsson, Nucl. Phys. B463 (1996) 217.
3. B. A., G. Gustafson and B. Söderberg, Nucl. Phys. B264 (1986) 29