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Some thoughts on how to match Leading Log Parton Showers with NLO Matrix Elements[1]

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Abstract

We propose a scheme that could offer a convenient Monte Carlo sampling of next-to-leading-order matrix elements and, at the same time, allow the interfacing of such parton configurations with a parton-shower approach for the estimation of higher-order effects. No actual implementation exists so far, so this note should only be viewed as the outline of a possible road for the future, submitted for discussion.

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1 Introduction

One of the main themes of particle physics is the strive for an increased accuracy in the description of physical processes. This is required to test the current standard model in detail, and often also to control background to searches for new physics.

A main road to improvements is the perturbative higher-order calculations of physical processes. For most processes, currently this means next-to-leading order (NLO), i.e. one order higher than the Born-level of the process, either by the emission of one more parton or by the inclusion of one-loop corrections to the Born graph. In principle, the perturbative expansion is a well-defined and successful technique but, for QCD processes, the large α_s value makes the perturbative series only slowly convergent. This problem is especially severe in the collinear region, where the emission rate is increasing as one approaches the non-perturbative régime. Finite total cross sections are obtained only by a cancellation between large positive real and large negative virtual contributions. Therefore higher-order matrix elements (ME's) are not of much use to describe the substructure of jets, apart from very crude features.

Parton showers (PS's) have complementary strengths. By a resummation of the large logarithmic terms, e.g. into Sudakov form factors, it is possible to obtain a reasonable description also in regions of large α_s values. Formally, for most generators, this resummation is only certified to leading logarithmic (LL) accuracy, but in reality many of the expected next-to-leading log improvements are already included, such as exact energy-momentum conservation, angular ordering, and optimal scale choice for α_s. Furthermore, the cross section for any n-parton configuration is always positive definite. Finally, it is possible to terminate the parton showers at some process-independent lower cut-off Q_0 and attach a — thereby also process-independent — non-perturbative hadronization model for physics below that scale.

The main PS weakness, on the other hand, is the crude treatment of wide-angle parton emission, where many Feynman diagrams may contribute with comparable strengths, and the final rate therefore may depend on detailed interference effects not present in the PS language. For some simple processes it has been possible to improve the showers by explicit NLO ME information in this region, thereby obtaining an improved description of the process [1]. In general, however, this approach does not appear tractable, and one would like to find other methods to combine the advantages of the ME answer at large parton separations with the PS one at small separation (and with hadronization models for scales below that).

In this note we will present some thoughts on a more general — although maybe less beautiful — ME/PS matching strategy that could be used for a larger set of NLO processes. To be specific, we will consider the example where the leading-order process is of the $2 \to 2$ type, i.e. producing two high-$p_\perp$ jets, as observed in HERA photoproduction events. The NLO corrections then contain both $2 \to 3$ processes and virtual corrections to the $2 \to 2$ ones.

2 The NLO parton configurations

The first step is to use the ME's to set up the starting configuration, either 2- or 3-"jet" events, where "jet" is a misnomer denoting that any nearby partons have been clustered. The two event classes are distinguished by some jet resolution criteria, i.e. by requiring

that none of the partons in a $2 \to 3$ configuration are found in a soft or collinear region. This can be defined, e.g., in terms of minimal $p_\perp$ and $R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$, or minimal invariant masses between partons.

While it is straightforward to 'cut out' the appropriate phase space regions from the 3-"jet" final state, to remain with a finite and positive differential cross section everywhere else, the consequences for the 2-"jet" configurations are more complicated. Here one will now receive contributions from
(1) the leading-order $2 \to 2$ ME's,
(2) the virtual terms, including (counter)terms coming from the scale-dependent parton distributions, also $2 \to 2$, and
(3) those $2 \to 3$ parton configurations that are rejected as such by the criteria above, and thus should be reclassified as 2-"jet" events.

Divergences from soft and collinear emissions should cancel between the two latter event classes. In the extreme divergent region, the three-body phase space naturally reduces to the two-body one, and so the singularities can be cancelled analytically. The finite pieces are often integrated numerically as three separate contributions, that are only combined in the end to give the correct cross section. If this strategy is applied in an event generator, it becomes necessary to work with events with negative weight, from the virtual corrections term. This is possible, but known to be a very unstable procedure in practice. For instance, it requires the hadronization model to be continuous in the limit that two partons are brought closer and eventually merged to one. This is true for the string model [2], although some fine print sets a practical limit, but many other hadronization models are flawed in this respect from the onset.

Instead we would propose a rearranged procedure, inspired by the so-called subtraction method [3]. (Which does not exclude the use of the competing phase-space slicing method to handle the collinear regions.) In analogy with this method, the third term is subdivided in two:
(3a) a strongly simplified matrix element, that reproduces the correct behaviour in the singular regions, but away from this can be chosen in a convenient way that allows simple integration over the extra 3-body phase space variables not present in the 2-body phase space (for fixed incoming partons), and
(3b) the difference between the full and the approximate expressions, that is everywhere finite, but messy to integrate analytically.

In a Monte Carlo context, this would work as follows:

- Pick a desired $2 \to 2$ parton configuration, at random (but biased to the regions of large cross sections, of course).
- Evaluate the differential cross section contributions from the parts (1) and (2+3a) for this configuration. By the notation (2+3a) we imply that the singular contributions now explicitly cancel between (2) and (3a). We expect (1)+(2)+(3) to be clearly positive, in the sense that, were (2)+(3) anywhere to become negative of the same order as (1) is positive, one would be entering a collinear/soft region of large higher-order corrections, better described by showers. Therefore the parameters of the clustering algorithm must be chosen so as to avoid this. Provided that the approximate form in (3a) is not very badly chosen, also (1)+(2+3a) should always be positive, although this is not strictly required.
- Pick a 3-body phase-space point in the soft/collinear regions that, by the jet resolution criteria, should be classified as the 2-"jet" configuration picked above.
- Evaluate the difference (3b) in this point, and multiply by the integral of the extra

3-body phase space variables.

- Add (1)+(2+3a)+(3b) together to obtain the cross section for the two-"jet" configuration. With a reasonable separation into (3a) and (3b), the grand sum should always be a non-negative number, that can then be used to accept/reject events in order to obtain a final event sample with unit weight.

The key feature here is that, by the Monte Carlo nature of it, one and the same two-"jet" configuration will be assigned different (non-negative) weights each time the cross section is evaluated, since the associated three-parton configuration will differ, but it is arranged so that the average converges to the right cross section.

3 The PS interface

A parton shower is organized in terms of some evolution variable, such that emissions are ordered to give e.g. a decreasing angle, transverse momentum or mass [3]. Often the upper limit of evolution is set to cover the full phase space of emissions, but some other maximum can also be indicated. For instance, if the ME regularization has been defined in terms of some minimum invariant mass (or angular) scale between resolved partons, a natural complement is a shower evolution in mass (or angle) from this scale downwards. However, such a match would never be perfect, so one would always need the capability to reject unwanted branchings in a shower. Fortunately, an acceptance/rejection step is part of shower algorithms anyway, so one only needs to add further rejection criteria matching the NLO ME cuts.

The method would therefore be as follows:

- In two-"jet" events, the evolution is started from some conveniently large scale, chosen so that no allowed phase-space regions are excluded. When a potential new emission has been selected by the shower algorithm, the resulting new parton configuration is tested by the "jet" clustering algorithm. Any emission that gives a three-"jet" classification is rejected and the evolution is continued downwards. This scheme should be applied both for the initial- and final-state showers. Ambiguities could arise, e.g. if one emission from the initial and one from the final state happen to overlap, so that they together define a third "jet", although individually they do not. Occurrences of this kind are formally of higher order, so one is free to pick any sensible strategy. One extreme would be only to apply cuts to each shower separately, another to insert a final clustering test that makes use of all partons to accept/reject the full shower treatment.
- In three-"jet" events, in principle all further emission is allowed, also such that leads to four or more "jets". However, if one were to allow emissions at scales harder than the ones in the basic $2 \to 3$ graph itself, there is a manifest risk of doublecounting in the jet cross section. Again, this could be avoided by applying a veto to emissions. In [4] an explicit algorithm is presented, that sets up a final-state shower from a given parton configuration, in a sensible way that avoids (or at least minimizes) doublecounting. A similar approach should be possible for the initial-state showers.

4 Outlook

If we want to improve the precision of NLO QCD tests in the future, new strategies need to be developed. This note is one such proposal. The main point is an alternative

phase-space sampling/integration strategy for NLO ME's. It would lead to events with positive definite weights, that therefore could be better interfaced to parton showers and hadronization models, and thus more realistically compared with data.

Clearly many details need to be settled, to make this a working proposition. While it should not be necessary to recalculate any of the NLO corrections to a process, the code for the evaluation of cross sections need to be restructured compared with current practice. Especially, the phase-space generation machinery must be rewritten significantly. By comparison, the modifications required in existing parton-shower algorithms appear more straightforward.

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