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The $\Delta I = 1/2$ rule and other matrix elements[†‡]

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Abstract

Recent work by J.Prades and myself on $K \to \pi\pi$ is described. The method we use to consistently connect long and short distances is described and numerical results for the $\Delta I = 1/2$ rule and on B_6, the parameter relevant for the strong part of ϵ'/ϵ, are given in the chiral limit.

1. Introduction

The qualitative feature that $\Gamma(K^0 \to \pi^0\pi^0) \gg \Gamma(K^+ \to \pi^+\pi^0)$ is one of the oldest problems in kaon physics, the $\Delta I = 1/2$ rule. The isospin-2 final state amplitude A_2 is much smaller than the isospin-0 amplitude A_0, experimentally $|A_0/A_2| = 22.1$, while simple W-exchange naively predicts a ratio of $\sqrt{2}$. The work presented here has been published in [1] and presented in [2]. A review of Kaon physics is in [4] and in the talks presented at Kaon99[3].

The underlying standard model process is the exchange of a W-boson but the large difference in the Kaon and W-mass enhances normally suppressed contributions by large factors $\ln(m_W^2/m_K^2) \approx 10$. At the same time, at low energies the strong interaction coupling α_S becomes very large which requires us to use non-perturbative methods at those scales.

The resummation of large logarithms at short-distance can be done using renormalization group methods. At a high scale the exchange of W-bosons is replaced by a sum over local operators. For weak decays these start at dimension 6. The scale can then be lowered using the renormalization group. The short-distance running is now known to two-loops [5, 6] (NLO) which sums the $(\alpha_S \ln(m_W/\mu))^n$ and $\alpha_S\,(\alpha_S \ln(m_W/\mu))^n$ terms. A review of this can be found in the lectures by A. Buras [7].

The major remaining problem is to calculate the matrix elements of the local operators at some low scale. I will address some progress on this issue in this talk. The main method was originally proposed in Ref. [8] arguing that $1/N_c$ counting could be used to systematically calculate the matrix elements. Various improvements have since been introduced. The correct momentum routing was introduced in [9]. The use of the extended Nambu-Jona-Lasinio model as an improved low energy model was introduced for weak matrix elements in [10] and a short discussion of its major advantages and disadvantages can be found in [11]. The results obtained were encouraging but a major problem remained. At NLO order the short-distance running becomes dependent on the precise definition of the local operators. This dependence should also be reflected in the calculations of the matrix elements as well as a correct identification of the scale of the renormalization group in the matrix element calculation. The more precise interpretation of the scheme of [8] introduced in [10] was shown there at one-loop to satisfy the latter criterion. I present in the next section how this method also satisfies the latter at NLO and how it solves the first problem as well. We call this method the X-boson method. The third section describes the numerical results we obtained in [1] for the $\Delta I = 1/2$ rule in the chiral limit. The results obtained there are also reported here in the more standard B_6, defined here with respect to our X-boson scheme.

Other recent work on matrix elements is the work of [12] and [13] using the $1/N_c$ method as well. A more model dependent approach is [14].

2. The X-boson method

The basic idea is that we know how to hadronize currents or at least that this is a tractable problem. So we replace the effect of the local operators of $H_W(\mu) = \sum_i C_i(\mu) Q_i(\mu)$ at a scale μ by the exchange of a series of colourless X-bosons at a

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low scale μ. The scale μ should be such that the $1/N_c$ suppressed contributions have no longer large logarithmic corrections. Let me illustrate the procedure in the case of only one operator and neglecting penguin contributions. In the more general case all coefficients become matrices.

$$C_1(\mu)(\bar{s}_L\gamma_\mu d_L)(\bar{u}_L\gamma^\mu u_L) \Longleftrightarrow X_\mu\left[g_1(\bar{s}_L\gamma^\mu d_L)+g_2(\bar{u}_L\gamma^\mu u_L)\right]. \quad (1)$$

Summation over colour indices inside brackets is understood. We now determine g_1, g_2 as a function of C_1. This is done by equalizing matrix elements of C_1Q_1 with the equivalent ones of X-boson exchange. The matrix elements are at the scale μ chosen such that perturbative QCD methods can still be used and thus we can use external states of quarks and gluons. To lowest order this is simple.

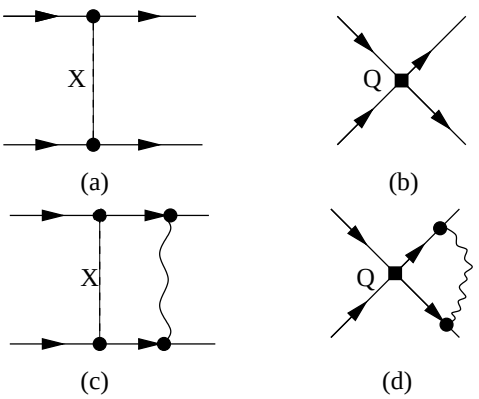

Figure 1. The diagrams needed for the identification of the local operator Q with X-boson exchange in the case of only one operator and no Penguin diagrams. The wiggly line denotes gluons, the square the operator Q and the dashed line the X-exchange. The external lines are quarks.

The tree level diagram from Fig. 1(a) is set equal to that of Fig. 1(b) leading to

$$C_1 = g_1 g_2/M_X^2. \quad (2)$$

At NLO diagrams like those of Fig. 1(c) and 1(d) contribute as well leading to

$$C_1\left(1+\alpha_S(\mu)r_1\right) = \frac{g_1g_2}{M_X^2}\left(1+\alpha_S(\mu)a_1+\alpha_S(\mu)b_1\log\frac{M_X^2}{\mu^2}\right). \quad (3)$$

At this level the scheme-dependence disappears. The left-hand-side (lhs) is scheme-independent. The right-hand-side can be calculated in a very different renormalization scheme from the lhs. The infrared dependence of r_1 is present in precisely the same way in a_1 such that g_1 and g_2 are scheme-independent and independent of the precise infrared definition of the external state in Fig. 1.

One step remains, we now have to calculate the matrix element of X-boson exchange between meson external states. The integral over X-boson momenta we split in two

$$\int_0^\infty dp_X \frac{1}{p_X^2-M_X^2} \Longrightarrow \int_0^{\mu_1} dp_X \frac{1}{p_X^2-M_X^2}+\int_{\mu_1}^\infty dp_X\frac{1}{p_X^2-M_X^2} \quad (4)$$

The second term involves a high momentum that needs to flow back through quarks or gluons and leads through diagrams like the one of Fig. 1(c) to a four quark-operator with a coefficient

$$\frac{g_1g_2}{M_X^2}\left(\alpha_S(\mu_1)a_2+\alpha_S(\mu_1)b_1\log\frac{M_X^2}{\mu^2}\right). \quad (5)$$

The four-quark operator thus needs to be evaluated only in leading order in $1/N_c$. The first term we have to evaluate in a low-energy model with as much QCD input as possible. The μ_1 dependence cancels between the two terms in (4) if the low-energy model is good enough and all dependence on M_X^2 cancels out to the order required as well. Calculating the coefficients r_1, a_1 and a_2 gives the required correction to the naive factorization method as used in previous $1/N_c$ calculations.

It should be stressed that in the end all dependence on M_X cancels out. The X-boson is a purely technical device to correctly identify the four-quark operators in terms of well-defined products of nonlocal currents.

3. Numerical results and conclusions

We now use the X-boson method with r_1 as given in [5] and $a_1 = a_2 = 0$, the calculation of the latter are in progress, and $\mu = \mu_1$. For B_K we can extrapolate to the pole for the real case ($\hat{B}_K$) and in the chiral limit ($\hat{B}_K^\chi$) and for $K\to\pi\pi$ we can get at the values of the octet (G_8), weak mass term (G_8') and 27-plet (G_{27}) coupling. We obtain

$$\hat{B}_K = 0.69\pm0.10\,;\ \hat{B}_K^\chi = 0.25\text{–}0.40\,;\ G_8 = 4.3\text{–}7.5\,;\ G_{27} = 0.25\text{–}0.40 \text{ and } G_8' = 0.8\text{–}1.1\,, \quad (6)$$

to be compared with the experimental values $G_8 \approx 6.2$ and $G_{27}\approx 0.48$ [1, 16].

In Fig. 2 the μ dependence of G_8 is shown and in Fig. 3 the contribution from the various different operators.

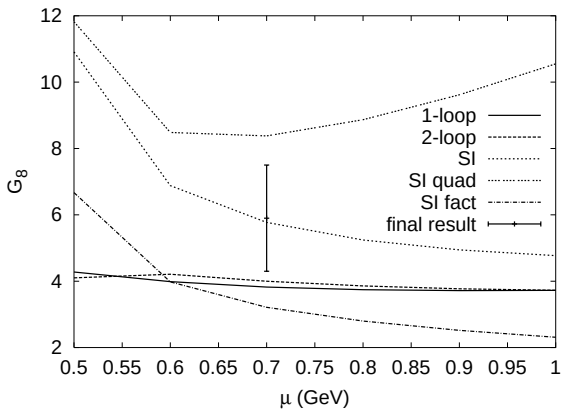

Figure 2. The octet coefficient G_8 as a function of μ using the ENJL model and the one-loop Wilson coefficients, the 2-loop ones and those including the r_1 (SI). In the latter case also the factorization (SI fact) and the approach of [12] (SI quad) are shown.

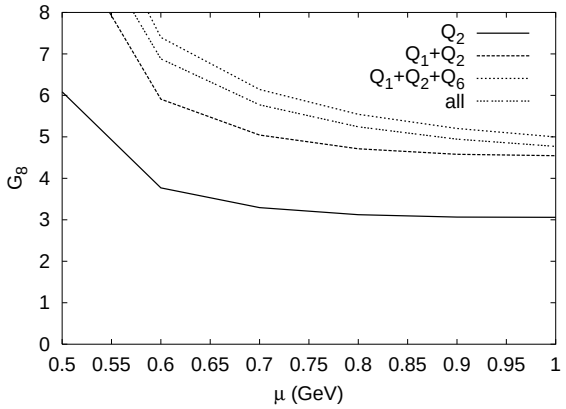

Figure 3. The composition of G_8 as a function of μ. Shown are Q_2, Q_1+Q_2, $Q_1+Q_2+Q_6$ and all 6 Q_i. The coefficients r_1 are included in the Wilson coefficients.

Fig. 3 shows that the contribution from Q_6 to the $\Delta I = 1/2$ rule is rather small. It is the penguin like contributions from Q_2 that are the major contributions. From our numerics we can also extract the value from B_6 that follows from our calculation. In Table 1 we give the value as a function of the matching scale μ for the calulation in CHPT and the one using the ENJL model. We have normalized here to the large N_c value since the pure factorizable value of B_6 is ill-defined in the chiral limit[1]. The enhancement away from lower values was also seen in the most recent paper of [12].

I showed how the X-boson method allows to correctly treat NLO scheme dependence and that using that method and the ENJL model at low energies reproduces the $\Delta I = 1/2$ rule *quantitatively* without any free parameters. The results for B_6 are encouraging with respect to the experimental value of ϵ'/ϵ

Table 1. B_6 as a function of μ using CHPT and the ENJL model. Numbers are calculated using the results of [1].

μ (GeV)	0.6	0.7	0.8	0.9	1.0
CHPT	1.19	0.93	0.70	0.50	0.36
ENJL	2.27	2.16	2.11	2.11	2.14

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