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Models of Low Energy Effective Theory applied to Kaon Non-leptonic Decays and Other Matrix Elements[1,2]

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Abstract. In this talk I describe work on computing non-leptonic matrix elements consistently with both long and short distance contributions included. On the simpler example of the π^+-π^0 mass difference I explain in detail the matching procedure and the difference between various low-energy models. I then explain the new difficulties in non-leptonic Kaon decays and how the matching here can in principle be done in the same way when scheme dependences are correctly accounted for. In the end I summarize the results J. Prades and I obtain for the $\Delta I = 1/2$ rule and B_6.

INTRODUCTION

The problem of describing non-leptonic decays is a very old one and is still not fully solved today. In this talk I will describe the large N_c method first suggested in a series of papers by Bardeen, Buras and Gérard [1] and later extended by several other authors. I will illustrate most of the problems and solutions on the example of the charged and neutral pion mass difference and afterwards show how this method can be extended systematically to the case of non-leptonic weak decays as well. The main results described there are those of [2] and also descibed in [3].

The subject of this meeting was hadronic physics, so why are we interested in these extra quantities. They provide a very strong test of our understanding of the strong interaction at all length scales. Our present knowledge of the strong interaction can be summarized as:

Short Distance: This is the perturbative QCD domain and here QCD has had many successes, we count this region as understood.

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(Very) Long Distance: This is the Chiral Perturbation Theory (CHPT) regime [4]. Many successes again and basically understood.

Intermediate Distance: This is the domain of models supplemented with various arguments, sum rules, lattice QCD results, etc. and is the most difficult.

In the type of observables covered in this talk all three regimes are important. We consider processes with incoming and outgoing hadrons but with an internally exchanged photon or weak boson. The difficulty now resides in the fact that even if the external hadrons have all low momenta we need to integrate over all momenta of the internal γ or W^+. This means that all regimes come into play and that they need to be connected properly to each other. The last is known as matching.

The main part is in Sect. I where I show how we can explain the mass difference, $m^2_{\pi^+} - m^2_{\pi^0}$ using this class of methods. Here we can also see how the model approach and the correct answer agree. Sect. II then covers the extra problems involved in non-leptonic weak decays and how the X-boson method of [2] can be used to solve those. Finally I present numerical results for this case and conclusions.

I A SIMPLE EXAMPLE: THE π^+-π^0 MASS DIFFERENCE.

This non-leptonic matrix element has several features that make it simpler.

1. We can neglect m_u and m_d to a rather good approximation. This then allows current algebra to relate the electromagnetic mass difference to a vacuum to vacuum matrix element only [5]. This can then be related to the measured hadronic cross-sections in electron-positron annihilation so in this case we know the correct answer.

2. There are no large masses involved so there are no large logarithms that need resummation.

3. The photon itself provides for an easy identification of correct scales.

Basically the procedure is now to evaluate

$$m^2_{em} = -\langle M|e^2 \int \frac{d^4q}{(2\pi)^4} \frac{J_\mu(q)J_\nu(-q)}{q^2} \left(g_{\mu\nu} - \xi \frac{q_\mu q_\nu}{q^2}\right) |M\rangle \,. \tag{1}$$

where M stands for the meson under consideration and J_μ for the electromagnetic current. ξ is a gauge parameter. The procedure is now as follows:
1:We rotate the integral over photon momenta in Eq. (1) to Euclidean space. This has two advantages, in Euclidean space thresholds and poles are smoothed out making treatment of these easier and Euclidean space momenta have all components small if q^2_E is small. The latter allows for a simpler identification of long and short-distance than in Minkowski space.
2: The final step is now to set

$$\int d^4q_E = \int_0^\infty q_E^3 dq_E \int d\Omega = \underbrace{\int_0^\mu q_E^3 dq_E \int d\Omega}_{\text{long-distance}} + \underbrace{\int_\mu^\infty q_E^3 dq_E \int d\Omega}_{\text{short-distance}} \tag{2}$$

and perform both integrals separately. Notice that the scale μ is just a splitting scale in the integral and is not directly related to any subtraction scale in the calculation itself. Therefore, if both the long-distance (from 0 to μ) and the short-distance are calculated with high enough precision the final result should be independent of the value of μ. We check this by varying μ in all our calculations, i.e. we check the matching.

A Short-Distance

The short-distance contribution was first calculated in [6] using the sum rule by Das et al. [5]. It was later rederived using the Operator Product expansion in [7]. The diagrams in Fig. 1 depict the main contributions. Performing the photon integral leads to a set of four-quark operators that can be evaluated in leading $1/N_c$ since we can then apply factorization. The result is [6,7]

$$\left(m_{\pi^+}^2 - m_{\pi^0}^2\right)_{SD} = \frac{3\alpha_S\alpha}{\mu^2} F^2 B_0^2 = \frac{3\alpha_S\alpha}{\mu^2} \frac{\langle\bar{q}q\rangle^2}{F^2}\,, \tag{3}$$

with F the pion decay constant in the chiral limit and B_0 the parameter in lowest order CHPT describing the quark condensate.

B Long-Distance

In the previous subsection we could use perturbative QCD but that is not possible in the long distance domain. So here we have to put in the things we know and try various models.

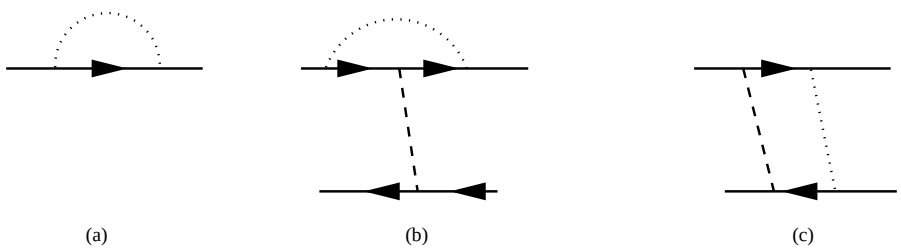

FIGURE 1. The three short-distance contributions, (a) electromagnetic quark-mass corrections. (b) Penguin-like diagrams (c) Box Diagrams. The dashed line is a gluon, the dotted line a photon and the full lines are quarks.

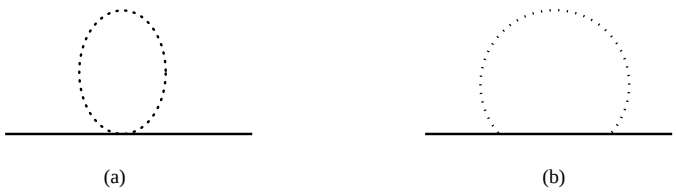

FIGURE 2. The long-distance contributions to $\left(m^2_{\pi^+} - m^2_{\pi^0}\right)$. The dotted line is a photon and the full lines are pions.

CHPT: This can be done for μ rather small and leads to

$$\left(m^2_{\pi^+} - m^2_{\pi^0}\right)_{LD} = \frac{3\alpha}{4\pi}\left(\mu^2 + \frac{2L_{10}}{F^2}\mu^4\right) . \tag{4}$$

The first term was first done in [6] and the chiral correction in [8,9]. The two contributing diagrams are depicted in Fig. 2. Something that is important is that the gauge dependence only cancels between the two diagrams in Fig. 2.

Chiral Quark Model: This was done in [10] and gives only a marginal improvement. Note that we cannot use the usual dimensional regularization here but must use the cut-off in the photon propagator. There is the additional problem that at first sight only the equivalent of the diagram of Fig 2(a) appears, which is a two-loop diagram, and the result is not gauge invariant. Only after the equivalent of (b) is added, which is a three-loop diagram, does the gauge dependence cancel as required [10].

With Vector-axial-vector Mesons: We have to include here Weinberg's constraint on the couplings to obtain a unique result otherwise the result will be very dependent on the specific model used. E.g. a hidden gauge model with only vector mesons is still quadratic in μ^2 but with a negative coefficient. Using Weinberg's constraints leads to

$$\left(m^2_{\pi^+} - m^2_{\pi^0}\right)_{LD} = \frac{3\alpha}{2\pi}M_V^2 \log\left[\frac{M_V^2 + \mu^2}{M_A^2 + \mu^2}\frac{M_A^2}{M_V^2}\right] . \tag{5}$$

But beware of partial results. Using a linear vector representation only even gave a quartic dependence on μ [8]. The result in (5) for $\mu \to \infty$ is basically the result of [5] and was also obtained in [11]. It has also several nice features. Expanding in μ for small μ reproduces the CHPT result with the meson dominated value for L_{10}. For large μ it goes as $1/\mu^2$ so it can match on very well to the earlier short-distance result.

ENJL This basically coincides with the previous result and was first obtained in [12].

Extensions of the above exists for nonzero-quark masses [7,13] and references therein and also with more large N_c arguments to underpin the matching [9].

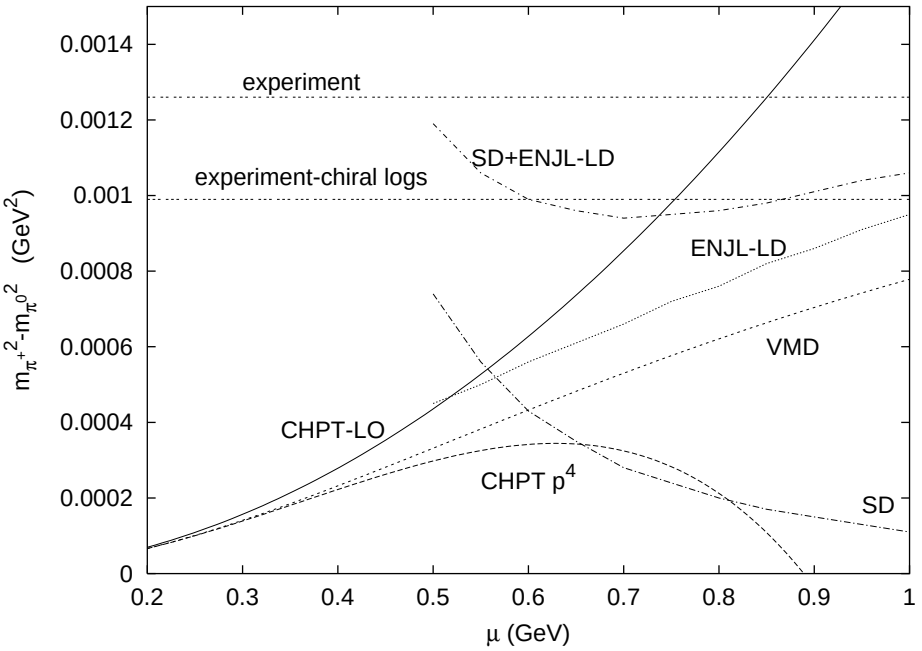

FIGURE 3. The long-distance contributions and short-distance contribution compared with the experimental values.

C Discussion

Numerical results are shown in Fig 3 for all cases. The experimental value and the one with the sub-leading in $1/N_c$ subtracted are shown as the horizontal lines. The subtracted part is the chiral logarithm contribution as estimated in [13]. Notice that CHPT starts deviating quickly from the VMD and ENJL results. The CHPT result is only reliable up to about 500 MeV. The VMD result and the ENJL result basically coincide here, the difference is due to the precise input values. Both these curves also follow essentially the correct answer as obtained from electron-positron annihilation and the sum-rule of [5].

Notice that the ENJL model has the correct matching on to the low μ CHPT result and is a considerable improvement over it at higher μ. Notice also the almost perfect agreement with the estimated part leading N_c part of the mass-difference.

From this section we can conclude:

1. Different Low energy models give quite different results and we have to use short-distance constraints and phenomenological inputs to improve the long-distance contribution to above the regime where CHPT is applicable.

2. CHPT alone for the long-distance regime is as a first guestimate acceptable but start differing from the correct answer at a scale of about 500 MeV.

3. Even for this low-momentum dominated observable the short-distance contributions are sizable at scales around 800 MeV.

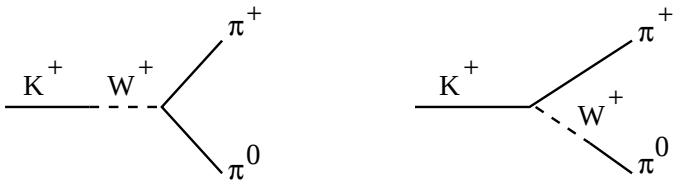

FIGURE 4. The naive W^+ exchange contribution to $K^+ \to \pi^+\pi^0$.

II KAON NON-LEPTONIC DECAYS

One of the difficult unresolved problems is to understand the origin of the $\Delta I = 1/2$ rule. The underlying process is W^+-exchange leading to an operator of the quark-structure $(\bar{s}u)(\bar{u}d)$ which has both isospin 1/2 and isospin 3/2 pieces. If we assume the W^+ couples directly to hadrons the process $K^+ \to \pi^+\pi^0$ goes simply via the diagrams in Fig. 4, but there are no such diagrams for $K^0 \to \pi^0\pi^0$ because of charge conservation. So we would expect that $\Gamma(K^+ \to \pi^+\pi^0) \gg \Gamma(K^0 \to \pi^0\pi^0)$. The experimental numbers are $\Gamma(K^+ \to \pi^+\pi^0) = 1.1\ 10^{-14}$ MeV and $\Gamma(K^0 \to \pi^0\pi^0) = \frac{1}{2}\Gamma(K_S \to \pi^0\pi^0) = 2.3\ 10^{-12}$ MeV, precisely the opposite. Translated into isospin amplitudes for the decays, see e.g. [14] for the precise definitions, we obtain $|A_0/A_2|_{\rm exp} = 22.1$ .The problem is not due to chiral corrections since using the estimate of [14,15] we can extract them and get

$$|A_0/A_2|_{\rm chiral} = 16.4 \underbrace{= \sqrt{2}}_{\rm naive} . \tag{6}$$

where the last number is the one using naive W^+-exchange as depicted in Fig. 4. In the notation used in [2,14] we have

$$A_0 = C(9G_8 + G_{27})\sqrt{6}/9\ F_0(m_K^2 - m_\pi^2) \quad A_2 = C10G_{27}\sqrt{6}/9\ F_0(m_K^2 - m_\pi^2) \tag{7}$$

which after subtracting the estimated chiral corrections from experiment yields

$$G_8 = 6.2 \pm 0.7 \quad G_{27} = 0.48 \pm 0.06 . \tag{8}$$

Both G_8 and G_{27} are equal to one in the W^+-exchange limit, the constant C was chosen to have this. We thus have to explain the large deviation from 1 using the corrections suppressed by $1/N_c$.

This is not a hopeless task as the sub-leading corrections coming from the diagrams in Fig. 1 with the photon replaced by the gluon are of order

$$\frac{\alpha_S}{N_c} \log \frac{M_W^2}{\mu^2} \tag{9}$$

compared to the leading contribution and this is in fact larger than one.

Luckily we know how to resum this type of logarithms [16]. At a high scale we can replace the effect of W^+-exchange by a sum of local operators by virtue of

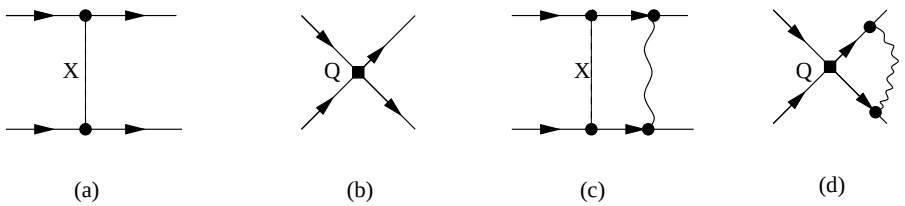

FIGURE 5. The diagrams needed for the identification of the local operator Q with X-boson exchange in the case of only one operator and no Penguin diagrams. The wiggly line denotes gluons, the square the operator Q and the dashed line the X-exchange. External lines are quarks.

the operator product expansion. We can then use the whole renormalization group machinery to run this sum over local four-quark operators down to a low scale μ_R. This is explained in great detail in [16]. The end result is

$$H_W = \sum_{i=1,10} C_i(\mu_R) Q_i \tag{10}$$

with a series of known coefficients $C_i(\mu_R)$, the Wilson coefficients. The final answer is then the matrix element of this sum over four-quark operators, $\langle \pi\pi | H_W | K \rangle$.

But here we have two problems:

1. In the previous section the short and long-distance contributions were separated via the photon momentum. Here we have to link this somehow to the scale μ_R appearing in the weak Hamiltonian H_W.

2. To next-to-leading order in the renormalization group the coefficients $C_i(\mu_R)$ also depend rather strongly on the precise definition of the local four quark operators Q_i in QCD perturbation theory.

In [18] we showed how the method of [1] supplemented with the correct momentum routing [6,17] solved problem 1. In [2] and in the various already published talks [3] we showed how a careful identification across the long-short-distance boundary is also possible in this case. The basic idea is to go at the scale μ_R back from the local four-quark operators to the exchange of a series of X-bosons. These X-bosons can then be treated in exactly the same way as we did the photon in the previous section, thus allowing a correct calculation at all length scales.

So we replace, using a single operator as an example

$$C_1 Q_1 = (\bar{s}_L \gamma_\mu d_L)(\bar{u}_L \gamma_\mu u_L) \Longleftrightarrow X_\mu \left[g_1 (\bar{s}_L \gamma^\mu d_L) + g_2 (\bar{u}_L \gamma^\mu u_L) \right] . \tag{11}$$

Using the tree level diagrams of Fig. 5 this gives $C_1 = g_1 g_2 / M_X^2$. If we now include the one loop diagrams we obtain instead:

$$C_1 \left(1 + \alpha_S(\mu_R) r_1\right) = \frac{g_1 g_2}{M_X^2} \left(1 + \alpha_S(\mu) a_1 + \alpha_S(\mu_R) b_1 \log \frac{M_X^2}{\mu_R^2} \right) . \tag{12}$$

On the l.h.s. the scheme dependence disappears but there is a dependence in r_1 on the choice of external states. The exact same dependence in a_1 cancels this.

We now split the integral over the X-boson momentum as in the previous section

$$\int_0^\infty dp_X \Longrightarrow \int_0^\mu dp_X + \int_\mu^\infty dp_X \tag{13}$$

In the final answer all M_X dependence drops out, the logarithm proportional to b_1 shows up in precisely the same way in the evaluation of the short distance part of (13) which is proportional to $g_1 g_2/M_X^2 \{\alpha_S(\mu) a_2 + \alpha_S(\mu) b_1 \log(M_X^2/\mu^2)\}$ The coefficients r_1, a_1 and a_2 give the corrections to the naive $1/N_c$-method.

We now use the X-boson method described above and put $\mu = \mu_R$. The low energy part can be calculated using CHPT, this is the approach used originally by [1] and presently pursued by [19] without including the corrections due to the change in scheme when going to the long-distance part. Their results coincide with ourss when we restrict our results to their approximations. We obtain [2]

$$\begin{aligned} B_K &= 0.6\text{—}0.8 \quad B_K^\chi = 0.25\text{—}0.40 \quad G_8 = 4.3\text{—}0.75 \\ G_{27} &= 0.25\text{—}0.40 \quad G_8' = 0.8\text{—}1.1 \quad B_6(\mu = 0.8 \text{ GeV}) \approx 2.2 \end{aligned} \tag{14}$$

B_K is the bag-parameter relevant for $K^0 - \overline{K^0}$ mixing at the physical quark masses and B_K^χ the same in the chiral limit. The quark mass corrections are quite sizable. The results for G_8 and G_{27} are obtained without any free input and agree within the uncertainties of the method with the experimental values. We conclude that we basically now have a first principle understanding of the $\Delta I = 1/2$ rule. We discuss the various contributions below. G_8' is the coefficient of the weak mass term, it contributes at leading order to processes like $K_L \to \gamma\gamma$ [14] and is often forgotten in those analyses. Finally B_6 is much larger than used in all the analysises of the recent experimental results for ϵ'/ϵ [20] which is very encouraging.

The final result for G_8 is depicted in Fig. 6. We have shown the one-loop result (1-loop), the two-loop result with NDR Wilson coefficients (2-loop) and the two-loop results with correction for the long-distance scheme added (SI) using our results for the long distance part. We also showed what naive factorization with the SI Wilson coefficients would give and what the method of [19] would give in the chiral limit with the same Wilson coefficients (SI quad).

If we look at the various contributions to G_8 we see in Fig. 7 that the contribution of Q_1 and Q_2 are both large and fairly constant while Q_6 contributes 20% or less. If we look inside the calculation we see that the difference with the G_{27} evolution is mainly given by the long-distance Penguin-like contributions to Q_2. The behaviour of B_6 is more difficult, it is ill-defined in the chiral limit in the factorizable approximation [2] and we can thus only define it with respect to the full large N_c limit. Calculating it in CHPT only then gives fairly low values as is visible in the second line of Table 1. Adding higher order corrections we immediately obtain a strongly enhanced value as is obvious from the second line in Table 1.

TABLE 1. B_6 as a function of μ using the results of [2].

μ (GeV)	0.6	0.7	0.8	0.9	1.0
CHPT	1.19	0.93	0.70	0.50	0.36
ENJL	2.27	2.16	2.11	2.11	2.14

III CONCLUSIONS

a The X-boson method in combination with large N_c arguments allows to correctly identify quantities across theory boundaries assuming we can identify currents across the boundary.

b The mass difference $m^2_{pi^+} - m^2_{\pi^0}$ is well described by these methods with a surprisingly large short-distance contribution.

c The $\Delta I = 1/2$ rule is now quantitatively understood to about 30% with NO free input. This calculation passes *all* requirements usually asked of in this context but there are many technical subtleties.

d $B_6 \approx 2.2$ is good news for those trying to explain the observed values of ϵ'/ϵ within the standard model.

e This program has been quite successful but we need new ideas to calculate more complex processes.

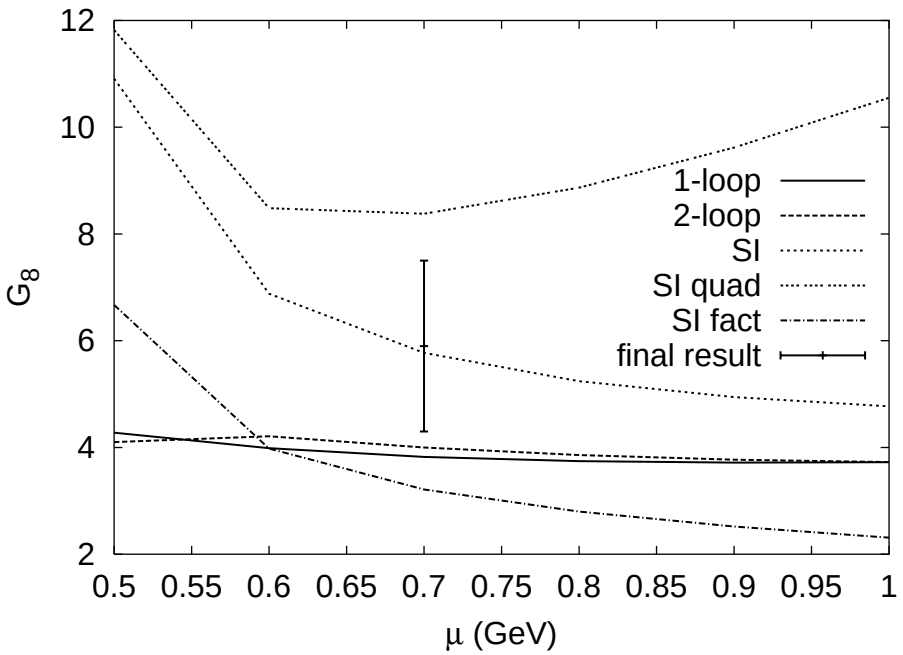

FIGURE 6. The octet coefficient G_8 as a function of μ using the ENJL model and the one-loop Wilson coefficients, the 2-loop ones and those including the r_1 (SI). In the latter case also the factorization (SI fact) and the approach of [19] (SI quad) are shown.

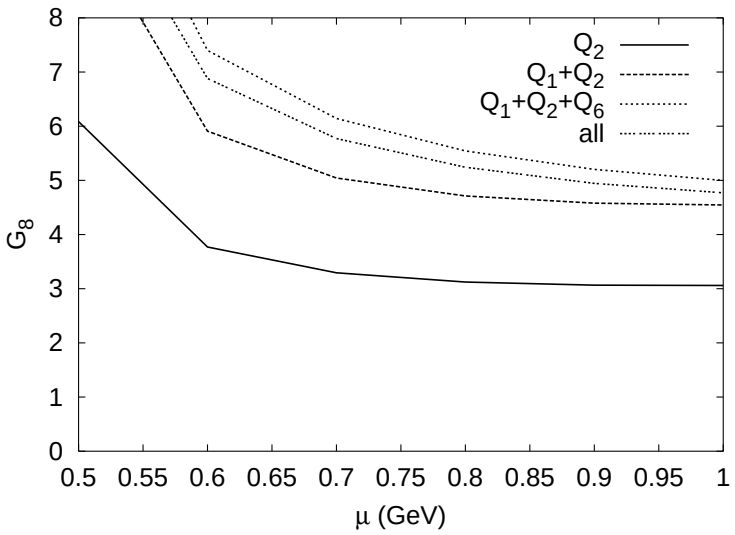

FIGURE 7. The composition of G_8 as a function of μ. Shown are Q_2, Q_1+Q_2, $Q_1+Q_2+Q_6$ and all 6 Q_i. The coefficients r_1 are included in the Wilson coefficients.

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