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Low Energy Constants from $K_{\ell 4}$ Form-Factors

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We have calculated the form-factors F and G in $K \to \pi\pi\ell\nu$ decays ($K_{\ell 4}$) to two-loop order in Chiral Perturbation Theory (ChPT). Combining this together with earlier two-loop calculations an updated set of values for the L_i^r, the ChPT constants at $\mathcal{O}(p^4)$, is obtained. We discuss the uncertainties in the determination and the changes compared to previous estimates.

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1. The theoretical study of $K \to \pi\pi l\nu$ decays, $K_{\ell 4}$ decays, provides an interesting possibility to test our understanding of the long-distance dynamics of the strong sector in the Standard Model. Perturbative QCD can not be applied at energies lower than the spontaneous symmetry scale, $\Lambda_\chi \sim m_\rho$. One has to resort to an effective approach to the full theory, Chiral Perturbation Theory (ChPT) [1], to obtain a reliable model independent description of physical processes. The main purpose in the study of $K_{\ell 4}$ decays is twofold: *i*) This decay is one of the cleanest sources of $\pi\pi$ pairs at low-energies and thus provides us with the possibility to check $\pi\pi$-scattering near threshold. *ii*) The form-factors themselves are also directly of interest. They provide a direct test of our understanding of the three-flavour sector and are in addition one of the major inputs to determine the needed constants to predict other quantities. The early history can be found in the review [2]. $K_{\ell 4}$ have been treated in the context of ChPT at one-loop [3] and in a dispersively improved one-loop representation [4]. These studies delivered the standard values of the low-energy constants L_1^r, L_2^r and L_3^r.

On the other hand, $\pi\pi$-scattering has been studied at two-loop order in two-flavour ChPT, direct [5] and via dispersive methods [6]. The main remaining uncertainty on the prediction of $\pi\pi$-scattering is the value of the low-energy constants. In [5] the standard values from [4] were used. A Roy equation analysis using high energy $\pi\pi$-data, gave significantly different values for these constants [7]. There are several possible sources for this discrepancy: the Omnès representation of [4] is not sufficient for $K_{\ell 4}$; the presence of large systematic effects in the $\pi\pi$-scattering data base and Roy analysis; or large corrections to the one-loop relation between two- and three-flavour ChPT constants. Therefore, and since new experiments are underway, a full two-loop calculation of $K_{\ell 4}$ is necessary. A first step was the estimate of the double logarithm corrections of $\mathcal{O}(p^6)$ to $K_{\ell 3}$ and $K_{\ell 4}$ [8]. It was shown there they could be large. In addition pions in the $I = 0$, S-wave have strong final state interactions. We describe here first results of the full $K_{\ell 4}$ $\mathcal{O}(p^6)$ calculation.

2. $K_{\ell 4}$ decays are described by the matrix element

$$M^{ij} = \frac{G_F}{\sqrt{2}} V_{us}^* \, \bar{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_\ell) \times \langle \pi^i(p_i)\pi^j(p_j)|V^\mu - A^\mu|K(k)\rangle \,. \quad (1)$$

Lorentz invariance allows to parametrize the hadronic part by four form-factors

$$\begin{aligned} \langle \pi^i\pi^j|A^\mu|K\rangle &= -\frac{i}{m_K}\left[P^\mu F^{ij} + Q^\mu G^{ij} + L^\mu R^{ij}\right] , \\ \langle \pi^i\pi^j|V^\mu|K\rangle &= -\frac{1}{m_K^3}\epsilon^{\mu\nu\alpha\beta} L_\nu P_\alpha Q_\beta H^{ij} \,, \end{aligned} \quad (2)$$

with $P = p_i + p_j, Q = p_i - p_j$ and $L = p_\ell + p_\nu$. p_i is the momentum of π^i. All four form-factors are dimensionless functions of three variables, $s_\pi = P\cdot P, u_\pi = (k-p_i)^2$ and $t_\pi = (k-p_j)^2$. They can be decomposed, into a part symmetric or antisymmetric under $t_\pi \leftrightarrow u_\pi$ interchange, corresponding respectively to the isospin 0 or 1 part for F, R and isospin 1 or 0 for G, H. These processes are dominated by F and G. H has a fairly small influence and a one-loop calculation, as performed in [9], is sufficient to reach $\mathcal{O}(p^6)$ since it comes through the odd-intrinsic-parity sector. R only contributes proportional to m_ℓ^2 and can thus be neglected in the decays with an electron.

In the remainder we concentrate on the $ij = +-$ channel. The others can be derived from it using isospin relations. The older available experiments are compatible with the $K^+ \to \pi^+\pi^- e^+\nu_e$ experiment [10]. The most recent other measurement is $K_L \to \pi^\pm\pi^0 e^\mp\nu$ [11].

The experiment [10] relied on a partial wave analysis of the form-factors. The analysis only kept s and p waves, the effect of d-waves as well as of s_ℓ was within the measurement errors and neglected in the further analysis. They used the parametrization

$$F = f_s e^{i\delta_s} + f_p e^{i\delta_p} + \text{D-wave} + \dots\,,$$
$$G = g e^{i\delta_p} + \text{D-wave} + \dots\,, \qquad (3)$$

where δ_i are the i-wave strong two-pion final state phase shifts, and f_s, f_p and g are defined to be real functions with argument s_π. They then divided the data in 5 energy bins, observed an s_π dependence in f_s and one compatible with it for g. The final fit was performed using

$$f_s(q^2) = f_s(0)\left(1+\lambda_f q^2\right)\,, \quad g(q^2) = g(0)\left(1+\lambda_g q^2\right)\,,$$
$$q^2 = (s_\pi - 4m_\pi^2)/(4m_\pi^2)\,. \qquad (4)$$

with $\lambda_f = \lambda_g$. Furthermore, the f_p value was compatible with zero. The results obtained, with $\sin\theta_C = 0.22$, are

$$f_s(0) = 5.59 \pm 0.14\,, \quad g(0) = 4.77 \pm 0.27\,,$$
$$\lambda_f = 0.08 \pm 0.02\,. \qquad (5)$$

Using isospin the result of [11] is $g(0) = 5.50 \pm 0.50$ which using PDG procedures [12] leads to a combined

$$g(0) = 4.93 \pm 0.31\,. \qquad (6)$$

Note that [11] neglected f_p which can account for up to 5% of the value of $g(0)$.

3. As mentioned above, earlier calculations gave an indication of potentially sizeable corrections at $\mathcal{O}(p^6)$. In order to obtain the full correction we have evaluated both form factors, F and G, at next-to-next-to-leading (NNLO) order in ChPT following a diagrammatic approach. One faces the evaluation of a large number of diagrams but only one new topology, shown in Fig. 1, appears w.r.t. the vector-vector and axial-vector–axial-vector two point-functions [13]. It involves a new set of integrals, the vertex-integrals, that can be obtained in terms of a two-parameter integral representation [14].

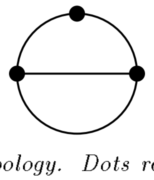

FIG. 1. *Vertex topology. Dots refer to strong vertices or current insertions.*

The full evaluation leads to rather long expressions. More than half of the complexity stems from the topology of Fig. 1 due to the large number of possible mass and momentum combinations. We have performed several checks on our full expressions: *i)* As a basic test of the algebraic programs we recover the one-loop expressions [3,4]. *ii)* All non-local divergences cancel when adding the full set of diagrams together with wave function renormalization. *iii)* The polynomial divergences also cancel against the counter-terms determined in general for the even-intrinsic parity $\mathcal{O}(p^6)$ sector [15]. *iv)* The double logarithms terms are in agreement with those of [8]. *v)* Some diagrams involving three-point one-loop integrals can also be obtained via the renormalization of the one-loop graphs, and hence the three-point integrals should cancel in the final result. *vi)* The sunset-type integrals encountered in [14] are in agreement with those already calculated in [13] using different methods. *vii)* The two-particle discontinuities of the vertex-type integrals have been checked using the Cutkosky rules below the three-particle thresholds. *viii)* Our results satisfy the isospin relation $M^{+-} = M^{-0}/\sqrt{2} + M^{00}$. In view of these checks, we trust our calculations of the matrix-elements.

To obtain the final answer, we shift the bare quantities –masses and decay constants– to the renormalized physical ones and add the part coming from the $\mathcal{O}(p^6)$ Lagrangian as determined in [16].

4. One of the main problems is how to deal with the coefficients of the $\mathcal{O}(p^6)$ Lagrangian. Here we follow the philosophy of most of the other $\mathcal{O}(p^6)$ calculations and estimate them using resonance exchange. This worked well at $\mathcal{O}(p^4)$ [17]. In the second paper of [18] this was found to agree reasonably well at $\mathcal{O}(p^6)$ for the form-factors considered there. We use the notation as defined in [13] but include also terms which did not contribute there. Specifically we use for the vector nonet matrix V_μ

$$\mathcal{L}_V = -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}m_V^2\langle V_\mu V^\mu\rangle - \frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle$$
$$- \frac{ig_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + f_\chi\langle V_\mu[u^\mu,\chi_-]\rangle$$
$$+ i\alpha_V\langle V_\mu[u_\nu, f_-^{\mu\nu}]\rangle \qquad (7)$$

and for the axial-vector nonet A_μ

$$\mathcal{L}_A = -\frac{1}{4}\langle A_{\mu\nu}A^{\mu\nu}\rangle + \frac{1}{2}m_A^2\langle A_\mu A^\mu\rangle - \frac{f_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle$$
$$+ \gamma_A^{(1)}\langle A_\mu u_\nu u^\mu u^\nu\rangle + \gamma_A^{(2)}\langle A_\mu\{u^\mu, u_\nu u^\nu\}\rangle\,. \qquad (8)$$

We have chosen a representation where the vectors and the axial-vectors only start contributing to the mesonic Lagrangian at $\mathcal{O}(p^6)$. For the scalar nonet we take

$$\mathcal{L}_S = \frac{1}{2}\langle\nabla_\mu S\nabla^\mu S\rangle - \frac{1}{2}m_S^2\langle S^2\rangle + c_d\langle S u^\mu u_\mu\rangle + c_m\langle S\chi_+\rangle\,.$$

We fixed the parameters as much as possible from experiment or the comparison with $\mathcal{O}(p^4)$ [5,17]. For the remainder we use the values predicted by the Nambu-Jona-Lasinio model [19]. The specific inputs used are $m_V = 0.77$ GeV, m_A =1.23 GeV, m_S =0.98 GeV, $f_V = 0.20$, $f_\chi = -0.025$, $g_V = 0.09$, $\alpha_V = -0.014$, $f_A = 0.1$, $\gamma_A^{(1)} = 0.006$, $\gamma_A^{(2)} = -0.01$, $c_m = 42$ MeV and $c_d = 32$

MeV. These are used in all the fits of Table I, except fit 6 where we kept only f_V, f_χ and g_V as nonzero couplings.

5. ChPT in the meson sector has as parameters at $\mathcal{O}(p^2)$ F_0, B_0 and the quark masses $\hat{m} = m_u = m_d$ and m_s. The latter only appear multiplied with B_0 for a total of 3 parameters. At $\mathcal{O}(p^4)$, we have $L_1^r, \ldots, L_{10}^r$ [1] and at $\mathcal{O}(p^6)$ there are 90 additional parameters [16]. In the two-flavour sector the equivalent of L_9^r and L_{10}^r has been determined to NNLO [18]. L_{10}^r has no influence on the other quantities considered here and the influence of L_9^r is small. We will therefore use the standard values of [1,4]. For the $\mathcal{O}(p^6)$ we use the resonance saturation approximation described above.

As input we use $F_\pi = 92.4$ MeV, $F_K/F_\pi = 1.22 \pm 0.01$ and the physical masses of pion, kaon and eta [12] in addition to $f_s(0)$, $g(0)$ and λ_f. The final input is the ratio $m_s/\hat{m}$. We further assume that the parameters L_4^r and L_6^r are small because they vanish in the large N_c-limit. When calculating the form-factors for $K^+ \to \pi^+\pi^-e^+\nu$, F_π and F_K we use the physical charged masses $m_{\pi^+} = 139.56995$ MeV; $m_{K^+} = 493.677$ MeV and $m_\eta = 547.30$ MeV. When comparing with the quark-mass ratios we use the π^0-mass, $m_{\pi^0} = 134.9764$ MeV, the physical η-mass and $m_K^2 = 1/2(m_{K^+}^2 + m_{K^0}^2 - 1.8(m_{\pi^+}^2 - m_{\pi^0}^2)) = 494.53$ MeV. This removes the electromagnetic mass corrections including estimated corrections to Dashen's theorem [20].

In order to compare with expression (4) we have to pull out some of the kinematic behaviour encoded in the matrix elements. We set

$$\begin{aligned} f_s(0) &= F(s_\pi, s_\ell, \cos\theta_\pi = 0)\,, \\ g(0) &= G(s_\pi, s_\ell, \cos\theta_\pi = 0)\,, \\ \lambda_f &= \left(\frac{|F(s'_\pi, s_\ell, \cos\theta_\pi = 0)|}{|F(s_\pi, s_\ell, \cos\theta_\pi = 0)|} - 1\right)\frac{4m_\pi^2}{s'_\pi - s_\pi}\,. \end{aligned} \tag{9}$$

We use $s_\pi = (2m_\pi + 1\ MeV)^2$ to avoid numerical integration problems very near threshold, $s'_\pi = (336\ MeV)^2$ and $s_\ell = 0$. $f_s(0)$, $g(0)$ and λ_f are fitted with the experimental errors quoted in (5).

F_K/F_π is calculated with the formulas of [13] and required to be 1.22 ± 0.01. The lowest order meson masses, m_{0i}^2, are calculated from the physical ones using the formulas of [13] and we require $m_s/\hat{m} = (2m_{0K}^2 - m_{0\pi}^2)/m_{0\pi}^2 = (3m_{0\eta}^2 - m_{0\pi}^2)/(2m_{0\pi}^2) = 24$ with an error of 10%.

The errors quoted above are the source of the errors quoted in our main fit in Table I. They do not include an estimate of the theoretical uncertainties due to higher orders. The error procedure is the same as in [4], since we use the same data the errors have thus not changed.

We first display our main fit and the canonical values. The $\mathcal{O}(p^4)$ fit illustrates the change due to the $\mathcal{O}(p^6)$ corrections. It is similar to the one-loop fits performed in [4], the difference due mainly to using $L_4^r = 0$ and $L_6^r = 0$. The remainder of the table shows the variation with the choices of input. Fit 2 is with a different choice of input quark-mass ratio, $m_s/\hat{m} = 26$, fit 3 with a different choice of $s_\ell = 0.1$ GeV, fit 4 of s'_π, fit 5 with $L_4^r = -0.3\ 10^{-3}$ and $L_6^r = -0.2\ 10^{-3}$. Setting $L_9^r = 0$ does not change results within the accuracy displayed. Fit 6 is with only the vector meson part in the resonance estimate. As can be seen the main contribution comes from the vector-exchange with the experimentally determined quantities. The rest is from the scalar sector. The axial-vectors contribute only for $s_\ell \neq 0$. Fit 7 and 8 are with two different values of the renormalization scale, $\mu = 0.5$ GeV and $\mu = 1.0$ GeV. Our full result is μ-independent as it should be, the difference is due to the fact that we estimated the value of the $\mathcal{O}(p^6)$ coefficients at the scale μ. In fit 9 we present the result with the $g(0)$ value of [11] included.

6. In conclusion, we have performed a NNLO calculation of the F and G form-factors in $K_{\ell 4}$ decays. We then used this calculation together with earlier calculations of masses and decay constants to update the ChPT parameters at $\mathcal{O}(p^4)$ with a certain range of choices for the fitting of the available $K_{\ell 4}$ data and the estimates of the $\mathcal{O}(p^6)$ parameters. None of the variations of input change the parameters outside the experimentally determined error except the rather extreme case of $\mu = 0.5$ GeV.

When looking at the $K_{\ell 4}$ calculation it can be seen that for the form-factor F the estimate of higher orders of [4] was in the right direction. The value of L_2^r has changed even more than in the unitarization estimate. The other parameters stayed essentially within the estimated errors but the decrease in L_5^r is somewhat larger than naively expected.

The errors should be taken with caution, L_7^r and L_8^r are strongly anti-correlated and L_3^r is strongly anti-correlated with L_1^r and L_2^r. If we choose as fit variables $X_1 = L_2^r - 2L_1^r - L_3^r$, $X_2 = L_2^r$ and $X_3 = (L_2^r - 2L_1^r)/L_3^r$ [4], X_3 is little correlated with the others. We obtain

$$X_3 = 0.09^{+0.07}_{-0.10}\,. \tag{10}$$

The large N_c-prediction $|X_3| \ll 1$ is obviously well satisfied.

With these values for the low-energy constants we can see how the various quantities behave. The numbers correspond to $\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ contributions.

$$\begin{aligned} m_{\pi^0}^2/m_{\pi^0}^2|_{\exp} &= 0.767 + 0.003 + 0.230\,, \\ m_K^2/m_K^2|_{\exp} &= 0.766 - 0.015 + 0.249\,, \\ m_\eta^2/m_\eta^2|_{\exp} &= 0.817 - 0.098 + 0.281\,, \\ F_K/F_\pi &= 1.000 + 0.160 + 0.060\,, \\ F_\pi/F_0 &= 1.000 + 0.138 - 0.110\,, \\ f_s(0) &= 3.78 + 1.21 + 0.60\,, \end{aligned}$$

$$g(0) = 3.78 + 0.98 + 0.01\,,$$
$$\lambda_f = 0.000 + 0.002 + 0.078\,. \qquad (11)$$

These numbers were calculated before rounding the L_i^r to the number of significant digits given in Table I. As was already observed in [4,8,21] the slope is almost entirely from $\mathcal{O}(p^6)$-effects.

Making use of the central values of the main fit in Table I together with the $\mathcal{O}(p^4)$ relations between the two- and three-flavour low-energy constants we estimate $\bar{l}_1$ and $\bar{l}_2$

$$\bar{l}_1 = -0.3(-0.9)\,, \quad \bar{l}_2 = 5.3(5.5)\,. \qquad (12)$$

The values in brackets are those using the values of fit 9. We remind the reader that the relation used has possibly large $\mathcal{O}(p^6)$ corrections. These should be compared respectively with -1.7 and 6.1 using the L_i^r from [4] and with $\bar{l}_2 = 4.2$ [7].

In conclusion, we have performed a full $\mathcal{O}(p^6)$ calculation of $K_{\ell 4}$ decays. We then performed a full refit of the $\mathcal{O}(p^4)$ parameters of ChPT and discussed the changes and the validity of a large-N_c prediction.

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TABLE I. Results for $L_i^r(\mu)$ for the various fits described in the main text. Notice that L_4^r, L_6^r and L_9^r are input, $L_4^r = 0, L_6^r = 0$ and $L_9^r = 6.90\ 10^{-3}$. Errors are fitting errors described in the text. All $L_i^r(\mu)$ values quoted have been brought to the scale $\mu = 0.77$ GeV. The standard values are $m_s/\hat{m} = 24$, $s'_\pi = 0.336$ GeV and $s_\ell = 0$.

	Main Fit	[1,4]	$\mathcal{O}(p^4)$	fit 2	fit 3	fit 4	fit 5	fit 6	fit 7	fit 8	fit 9
$10^3\ L_1^r$	0.61±0.23	0.37±0.23	0.46	0.61	0.59	0.57	0.54	0.53	0.52	0.74	0.74
$10^3\ L_2^r$	0.92±0.24	1.35±0.23	1.49	0.92	0.88	0.94	0.85	0.70	1.35	0.87	1.05
$10^3\ L_3^r$	−3.21±1.01	−3.5 ±0.85	−3.18	−3.21	−3.11	−3.25	−2.88	−2.66	−3.44	−3.12	−3.80
$10^3\ L_5^r$	0.91±0.15	1.4 ±0.5	1.46	0.89	0.91	0.90	0.95	0.94	1.62	0.69	0.84
$10^3\ L_7^r$	−0.36±0.16	−0.4 ±0.2	−0.49	−0.29	−0.36	−0.37	−0.37	−0.36	−0.36	−0.34	−0.37
$10^3\ L_8^r$	0.64±0.19	0.9 ±0.3	1.08	0.51	0.64	0.64	0.67	0.64	0.78	0.58	0.64
changed			$\mathcal{O}(p^4)$	$m_s/\hat{m}$	$\sqrt{s_\ell}$	$\sqrt{s'_\pi}$	$L_4^r; L_6^r$	V_μ	μ	μ	$g(0)$
quantity				26	0.1	0.293	$-0.3; -0.2$	only	0.5	1.0	4.93
Unit					GeV	GeV	10^{-3}		GeV	GeV	